problem by taking fundamental solution $w(x,\overline{x}) = -\frac{1}{2}|x-\overline{x}|$ with a parameter \overline{x} . [Hints. Convert the domain integrals into boundary integrals and obtain the boundary integral equation for a domain point, then assemble the matrix equation]. 10

3. Solve the integral equation

$$f(x) - \frac{1}{2} \int_0^1 (x+u) f(u) du = x$$

numerically by approximating the integral using Trapezoidal rule after dividing the interval [0, 1] into two equal parts. 10

Ex/SC/MATH/PG/4.1.1/2022

M. Sc. MATHEMATICS EXAMINATION, 2022

(2nd Year, 2nd Semester)

ADVANCED NUMERICAL ANALYSIS

PAPER – UNIT-4.1 (THEORY)

Time : One hour

Full Marks : 20

All questions carry equal marks.

Symbols and notations have their usual meanings.

Answer any two questions.

1. Consider a boundary value problem

 $\frac{d^2u}{dx^2} + u = x^2; \ 0 < x < 1$

subject to the boundary conditions u(0) = 0, u(1) = 0. Using piecewise linear elements with local shape functions $N_i^{[e]}(x) = \frac{x_{i+1} - x}{h_i}$, $N_{i+1}^{[e]}(x) = \frac{x - x_i}{h_i}$ for $x \in [x_i, x_{i+1}]$, taking two equal subintervals find the local stiffness matrix $K^{[e]}$ and the local load vector $F^{[e]}$ to construct a linear system KU = F, where $h_i = x_{i+1} - x_i$. 10

2. Consider a one-dimensional problem of Laplace's equation $u^n = 0$, $\forall x \in (0,1)$

with the boundary conditions u(0) = 100, u(1) = 0.

Use boundary element method to solve the above [Turn over