M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Differential Geometry and Its Application II

Paper – 4.5 (B 2.13)

Time : Two hours Full Marks : 50

The figures in the margin indicate full marks.

(Notations and Symbols have their usual meanings)

Answer ***any five*** questions.

1. Define Killing vector field. When is a space said to be maximally symmetric? If K^a is a killing vector field of a manifold then derive the killing equation: $K_{a;b} + K_{b;a} = 0$

   State the condition for the killing vector K^a to be orthogonal to a family of space-like hypersurfaces. 2+2+4+2

2. Define Hamiltonian vector field. In the notion of differential geometry, define canonical transformation in phase space. Find the condition for it. Introduce the concept of generating function for canonical transformation and state its possible forms. 2+2+2+2+2

3. Define Einstein tensor. Prove that the divergence of the Einstein tensor vanishes. What is the physical interpretation of this relation? How many independent field equations are there in four dimensional space-time? What are the constrain equations? 1+3+2+2+2

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4. Using the differential geometric structure in phase space, introduce the notion of Poisson bracket between two functions in phase space. Show that

   i) $\vec{X}_H(K) = \{K, H\}$

   ii) $\left[\vec{X}_f, \vec{X}_g\right] = -\vec{X}_{\{f,g\}}$ 2+3+5

5. Derive the general Lorentz transformation. Deduce the transformation law for velocities of a particle in two inertial frame of references. What do you mean by proper time? 3+5+2

6. What is 'local inertial' co-ordinate system? If the metric coefficients are independent of any particular co-ordinate, then prove that, momentum along that direction is conserved. Show that acceleration vector is zero along the geodesic. 3+4+3

7. State the space-time metric for homogeneous and isotropic model of the universe. Describe the geometric structure of the space-time for three different choices of the curvature scalar. Write down Einstein equations and the conservation equation for this space-time model. 2+5+3