Ex/PG/SC/CBS/PHY/TH/303/2022

M. Sc. Physics Examination, 2022

(2nd Year, 2nd Semester)

Quantum Field Theory

Paper – PG/SC/CBS/PHY/TH/303

Time : Two hours | Full Marks : 40

Use a separate answer script for each Group.

Group – A

Answer ***any one*** question.

1. a) Show that for a Schrodinger field, the field number operator and the field Hamiltonian operator commute with each other, irrespective of the particle statistics.

   b) A many-particle system is governed by a two-body interaction potential $U(x-x')$, and is given by the position-space Hamiltonian operator

$$\hat{H} = \int d^3x \hat{\Psi}^\dagger(x,t) H_0 \hat{\Psi}(x,t) + \frac{1}{2}\int d^3x \int d^3x' \hat{\Psi}^\dagger(x',t) U(x-x') \hat{\Psi}(x,t) \hat{\Psi}(x',t)$$

   [here H_0 is the usual single particle Schrodinger Hamiltonian]. Show that the operation of this second quantized Hamiltonian operator on a many-particle Fock state leads to a many-particle Schrodinger equation in the first quantized form. 10+10=20

[Turn over

2. a) Show how the Pauli exclusion principle follows from the anti-commutation rules obeyed by fermionic operators.

   b) Let $|vac\rangle$ represent the vacuum state in a many-body quantum system. Show that the state $\hat{\Psi}^{\dagger}(\vec{r}_1,t)\hat{\Psi}^{\dagger}(\vec{r}_2,t)|vac\rangle$ is an eigenstate of the field number operator with eigenvalue 2. Interpret your results for bosons and fermions separately.

   c) Show that the equations of motion obeyed by bosonic and fermionic operators for a quantized Schrodinger field resemble that obeyed by classical normal modes for a field of harmonic oscillators.

4+6+10=20

Group – B

Answer ***any four*** questions : 4×5=20

3. State Noether's theorem.

   Find the expression for Nother's charge.

   Give three examples of Noether's charge.

4. What do you mean by quantization fields?

   Describe necessary mathematical descriptions that describe this quantization.

   Using these quantization relations interpret the physical meaning of the expressions i) $\phi(\vec{x})|0>$, ii) $<0|\phi(\vec{x})|0>$.

5. Using the amplitude for a particle to propagate between two points of space-time and the principle of causality justify that every particle has corresponding antiparticle with the same mass and opposite quantum numbers.

6. Describe Feynmann description of propagator.

7. How the Hamiltonian is constructed in interacting field theories?

   Define S matrix.

   Then find the amplitude for transition from one state to another state at the late time $t \to \infty$.

8. Express mathematically the n point correlation function. Then using Wick's theorem express the above expression as the sum of the products of Feynmann propagators for $n = 4$.