# M. Sc. Physics Examination, 2022

( 2nd Year, 2nd Semester )

## General Relativity and Cosmology (II)

## Paper – 404

Time : Two hours Full Marks : 40

Answer ***any four*** questions.

1. a) What is the cosmological principle? Show that the FLRW metric can be presented in the form

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

where $k = 0 \pm 1$ is the sign of spatial curvature. The symbols have their as usual meaning.

b) Calculate the numerical value of comoving ($r$), bolometric luminosity distance ($d_L$) and angular diameter distance ($d_A$) at redshift $z = 2$ for a flat Cosmological Constant dominated Universe with $h = 0{\cdot}7$. (1+5)+4

2. a) The binding energy of electron in the hydrogen atom equals to 13·6 eV. What is the temperature of Planck distribution, with this average photon energy?

b) Find the number densities of CMBR photons. Calculate the ratio of number density of baryons to number density of CMBR photons.

c) Find the ratio of thermal heat capacities of matter in the form of monotonic gas (hydrogen) and CMBR photons. What is the conclusion of this finding?

1+(3+2)+(3+1)

3. a) Find the temperature dependence for the Hubble parameter in the early flat radiation dominated Universe. What was the time dependence for temperature at the early stages of evolution of the Universe using Friedman equation? Determine the age of the Universe when its temperature was equal to 1 MeV. Calculate the time interval of the universe for cooling from the temperature $10^{12}$ K to $10^{11}$ K.

b) Calculate at which redshift corresponds the energy $k_B T = 0{\cdot}3$ eV of the photon thermal bath.

(2+4+1+2)+1

4. Obtain/Determine the Penrose diagram for the *de Sitter spacetime*, with the metric

$$ds^2 = -dt^2 + \frac{1}{H^2}\cosh^2(Ht)\left(d\chi^2 + \sin^2\chi d\Omega^2\right)$$

where $H > 0$ is a constant and $0 \le \chi \le \pi$. Here $(\chi, \theta, \phi)$ parametrizes the 3-sphere of radius $\chi$ and $\Omega$ denotes the solid angle. 10

5. In connection to Geodesic congruences define shear and expansion parameters. Calculate the effect of a shear



tensor on a circle of radius $r_0$. Derive Raychaudhuri's equation. 2+4+4

6. The fundamental equations of relativistic hydrodynamics are the Continuity equation, the Euler equation, the Energy equation, together with the equation of state (EOS). Taking the energy momentum tensor of a perfect fluid, derive the relativistic form of those fundamental equations. Further, find the expressions for the isotropic pressure and total energy density of the fluid. 6+4