Ref. No.: Ex/PG/SC/CORE/PHY/TH/107/2022

M. Sc. Physics Examination 2022

First Year, Second Semester

SOLID STATE PHYSICS

(Subject code: PG/SC/CORE/PHY/TH/107)

Time: Two hours Full marks: 40

Answer any FOUR questions.

1. (a) Consider a system of fixed volume in thermodynamic equilibrium under the magnetic field H at finite temperature T. Show that in equilibrium, the entropy, S and the magnetic induction, B of this system can be obtained by using the expressions,

$$S = -\left(\frac{\partial G}{\partial T}\right)_{\mathrm{H}}, \quad \mathrm{B} = -\left(\frac{\partial G}{\partial \mathrm{H}}\right)_{T},$$

respectively, where G is the Gibbs potential.

(b) Show that the Gibbs potential for the superconducting state, G_s is related to that of the normal state, G_n by the equation,

$$G_s(T,\mathrm{H}) = G_n(T,\mathrm{H}) + \frac{1}{2}\mu_0\left(H^2 - H_c^2(T)\right).$$

Draw the variations of G_s and G_n with the magnetic field when $T < T_c$.

(c) Explain that the superconducting state to normal metallic state transition is second order at the points $T = 0$ and $T = T_c$, and otherwise first order, since the critical field of transition is

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right].$$

(d) Find the expression for specific heat difference between normal and superconducting states and obtain the Rutgers formula. Draw the variation of specific heat with temperature. [2+(2+1)+2+(2+1)=10]

2. (a) Show that in thermodynamic equilibrium, magnetic susceptibility, χ, can be defined as

$$\chi = -\frac{\mu_0}{\mathcal{V}}\frac{\partial^2 \mathcal{F}}{\partial \mathrm{B}^2}.$$

Symbols have their usual meaning.

(b) State the Bohr-van Leeuwen theorem.

(c) Using the classical Hamiltonian of a charged particle,

$$H_c(\mathrm{r},\mathrm{p}) = \frac{1}{2m}\left[\mathrm{p} - q\mathrm{A}(\mathrm{r})\right]^2 + q\phi(\mathrm{r}),$$

prove the Bohr-van Leeuwen theorem. Symbols have their usual meaning.

(d) Explain the outcome of this theorem from physical point of view in terms of circular and skipping orbits. [2+1+5+2=10]

[Turn over

3. (a) Prove the Sommerfeld expansion,

$$\int_0^{\infty} f(E)\,\frac{\partial F}{\partial E}\,dE = F(E_F) + \frac{\pi^2}{6}\,(k_B T)^2 \left(\frac{\partial^2 F}{\partial E^2}\right)_{E_F},$$

where $f(E)$ is the Fermi-Dirac distribution function and $F(E)$ is a regular function and such that $F(0) = 0$.

(b) By using the density of states, $g(E) = \frac{3}{2} n (E_F(0))^{-\frac{3}{2}} \sqrt{E}$, and Fermi level

$$E_F(T) \approx E_F(0) \left[1 - \frac{\pi^2}{12} \left(\frac{T}{T_F}\right)^2\right],$$

for the three-dimensional free electron systems, derive the expression of Pauli paramagnetic susceptibility at finite temperature, $\chi_P(T)$. Symbols have their usual meanings.

[4+6=10]

4. (a) State and prove the Bloch's theorem..

(b) Show that the Schrödinger equation of a single electron, $H(\mathrm{r})\,\Psi(\mathrm{r}) = E\,\Psi(\mathrm{r})$, where the Hamiltonian, $H(\mathrm{r}) = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathrm{r})$ is invariant under the lattice translation of any Bravais vector R, *i. e.*, $H(\mathrm{r} + \mathrm{R}) = H(\mathrm{r})$, can be expressed in reciprocal space as

$$\left(\frac{\hbar^2}{2m}(\mathrm{k} - \mathrm{G})^2 - E\right) c_{\mathrm{k-G}} + \sum_{\mathrm{G'}} V_{\mathrm{G'-G}}\; c_{\mathrm{k-G'}} = 0.$$

Symbols have their usual meaning. [(2+3)+5=10]

5. (a) Define and derive structure factor from X-ray scattering of unit cell. Show that structure factor is independent of the shape and size of the unit cell.

(b) Determine the structure factor of β-ZnS and indicate the cases where intensity can be observed. The atomic positions of Zn and S in the unit cell of β-ZnS are given in the following table.

Zn	$(0\,0\,0)$	$(0\,\frac{1}{2}\,\frac{1}{2})$	$(\frac{1}{2}\,0\,\frac{1}{2})$	$(\frac{1}{2}\,\frac{1}{2}\,0)$
S	$(\frac{1}{4}\,\frac{1}{4}\,\frac{1}{4})$	$(\frac{1}{4}\,\frac{3}{4}\,\frac{3}{4})$	$(\frac{3}{4}\,\frac{1}{4}\,\frac{3}{4})$	$(\frac{3}{4}\,\frac{3}{4}\,\frac{1}{4})$

[5+5=10]

6. Obtain the dispersion relation for a linear monatomic lattice and describe its vibrational spectrum. [10]

7. (a) Explain why crystals cannot posses a long range five-fold symmetry.

(b) Why only fourteen Bravais lattice exist? Which of the seven crystal systems has the maximum number of Bravais lattice? Prove using a diagram the equivalence of a base centered tetragonal with primitive tetragonal.

(c) Assuming hard incompressible spheres, draw the projection of (111) plane of an fcc unit cell, and hence calculate the packing fraction of that plane.

[3+(1+1+2)+3=10]