4. Why is Rosenbrock method called the method of rotating coordinates?

Minimize $(x_1-2)^4+(x_1-2x_2)^2$ by Rosenbrock method by taking initial point as (0, 3·0).

5. Using Steepest Descent method, minimize $f=x_1^2+x_2^2+2gx_1+2hx_2+c$ starting from the point $\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$.

6. Write down the properties of penalty function.

Minimize $f(x_1,x_2)=\frac{1}{3}(x_1+1)^3+x_2$, subject to $x_1\geq 1$, $x_2\geq 0$, using exterior penalty function method with the calculus method of unconstrained minimization.

Ex/SC/MATH/PG/CORE/TH/12/2022

M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Optimization and Control Theory

Paper – Core-12

Time : Two hours Full Marks : 40

Use a separate answer script for each Part.

Part – I (24 Marks)

Attempt ***any three*** questions out of the following five:

3×8=24

1. a) Define the rocket railroad car problem of control.

   b) Define Pontryagin Maximum principle.

   c) Introducing and using the maximum principle, find out the optimal control & production of consumption of a basic model. 2+2+4=8

2. a) Define the controllability matrix $G=G(M,N)$.

   b) Prove that $rank(G)=n$ if and only if $0\in C^0$ Where rank of G is defined as the number of linearly independent rows/column of G. 2+6=8

3. a) Obtain the state transition matrix of the following discrete-time system :

$$x(K+1)=G\,x(K)+H\,u(K)$$
$$y(K)=C\,x(K)$$

Where, $G = \begin{pmatrix} 0 & 1 \\ -0{\cdot}16 & -1 \end{pmatrix}$, $H = (1 \quad 1)^T$, $C = (1 \quad 0)$.

Also, obtain the state $x(K)$ and the output $y(K)$ when the input $u(K) = 1$ for $K = 0, 1, 2$. Assume that the initial state is given by

$$x(0) = (x_1(0) \quad x_2(0))^T = (1 -1)^T.$$

b) Determine the inverse of the matrix $(z\,I - G)$ where

$$G = \begin{pmatrix} 0{\cdot}1 & 0{\cdot}1 & 0 \\ 0{\cdot}3 & -0{\cdot}1 & -0{\cdot}2 \\ 0 & 0 & -0{\cdot}3 \end{pmatrix}.$$

4+4=8

4. a) Consider the continuous-time system given as

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+a}.$$

Obtain the continuous-time state-space representation of the system. Also, find the discrete-time state-space representation and the pulse transfer function for the system.

b) Compute e^{At} for the maxtrix $A = \begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix}$ and also find the discretized version with delay in input for the system given as

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t)$$
$$y(t) = [1 \quad 0]\, x(t)$$

4+4=8

5. Consider the differential equation of the system $M\ddot{x} + F\dot{x} + Kx = 0$ with $x(0) = 1$ and $\dot{x}(0) = 1$. Find the condition for which the system will be asymtotically stable and hence, state the Lyapunov's first theorem from your analytical solution. [M, F and K as their usual notation] 8

Part – II (16 Marks)

Answer ***any four*** questions.

All questions carry equal marks.

1. Define a unimodal function. Maximize the following unimodal function

$$f(x) = \begin{cases} 2x/3, & x \le 3 \\ 5 - x, & x > 3 \end{cases}$$

in the interval [1, 4] by Fibonacci method using $n = 4$.

2. What are the roles of exploratory and pattern moves in the Hooke and Jeeves method?

Compute up to second iteration to minimize

$$f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

by Hooke and Jeeves pattern search method.

3. Use Nelder - Mead method to minimize $f(x, y) = x^2 - 4x + y^2 - y - xy$. Compute upto fourth iteration defining the initial simplex as $v_1 = (0,0)$, $v_2 = (1{\cdot}2, 0)$ and $v_3 = (0, 0{\cdot}8)$.