M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Fluid Mechanics II

Paper – DSE - 06 (B5)

Time : Two hours Full Marks : 40

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Answer question no. **6** and ***any Three*** from the rest.

1. State first law of thermodynamics. Show that the specific heat of a gas at constant volume is not equal to the specific heat of the gas at constant pressure. Hence derive the relation between specific heat at constant pressure and constant volume for perfect gas. Show that a quantity of heat added to an unit mass of gas is not a perfect differential. Derive the necessary and sufficient condition that the internal, enthalpy, free energy and Gibb's function for a gas are function of state. 1+3+2+3+4

2. Establish the relation $\frac{p_+}{p_-} = \frac{(\gamma+1)\rho_+ - (\gamma-1)\rho_-}{(\gamma+1)\rho_- - (\gamma-1)\rho_+}$.

   Here p_+, ρ_+ and p_-, ρ_- are the pressure and density of the gas after and before crossing the discontinuity surface and γ is an adiabatic constant.

   Show that the propagation speed of strong discontinuity surface always differ from local sound speed. Show that for steady motion the velocity of fluid normal to the discontinuity surface q_n at least on one side of the discontinuity surface exceeds the local sound velocity.

6+3+4

3. For steady two-dimensional gas motion in xy-plane derive the Bernoulli's equation in the form

$$\frac{1}{2}\vec{q}^2+\frac{\gamma}{\gamma-1}\Theta p^{\frac{\gamma-1}{\gamma}}=i_0(\psi)$$

where $\Theta=\frac{p}{\rho^\gamma}$.

What is critical sound speed? Show that the critical speed is always less than the sound speed in rest gas. Also show that $0\le\frac{q}{a_*}\le a_*\sqrt{\frac{\gamma+1}{\gamma-1}}$. 6+2+2+3

4. Establish the hodograph equations $\phi_\beta=\frac{\rho_0 q}{\rho}\psi_q$, $\phi_q=\frac{M^2-1}{\rho q}\psi_\beta$ for two dimensional irrotational motion of a compressible fluid. Hence derive Chaplygin's equation for the stream function for a steady two dimensional isentropic flow of a compressible fluid in the form

$$(\alpha_1-\tau)\psi_{\beta\beta}+4\alpha_1\tau^2(1-\tau)\psi_{\tau\tau}+4\alpha_1\tau\{1+(\alpha_2-1)\tau\}\psi_\tau=0.$$

7+6

5. For steady two-dimensional, irrotational barotropic motion of an ideal gas deduce that

$$\left(a^2-u^2\right)u_x-uv\left(u_y+v_x\right)+\left(a^2-v^2\right)v_y=0$$

Suppose a small perturbation is created in the fluid when a thin poorly bend body is placed in a homogeneous gas stream at a small angle of attack. Considering subsonic motion derive the following boundary value problem for the purturbed stream function

$\tilde{\Psi}_{xx}+\frac{1}{w^2}\tilde{\Psi}_{yy}=0$; $\tilde{\psi}=-U_\infty h_{1,2}(x)$ on $y=0\pm$, $a\le x\le b$, $\tilde{\psi}\to 0$ at infinity.

Hence derive the Prandtl Glauer rule connecting pressure coefficient for linearised subsonic flow of a gas and pressure coefficient for incompressible fluid flow.

2+6+5

6. Characterize subsonic flow and supersonic flow of a fluid in terms of Mach number. 1