Draw the schematic diagram of the model and find the stability conditions of the endemic equilibrium of the above model. 8

4. The basic HIV model is governed by

$$\frac{dx}{dt} = S - \lambda xv - \mu x$$

$$\frac{dy}{dt} = \lambda xv - cy$$

$$\frac{dv}{dt} = Ncy - \gamma v$$

Find the underlying assumptions of this model. Draw the schematic diagram and also find the stability conditions of the endemic equilibrium of the above model. 8

M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Epidemiology and Eco-epidemiology

Paper – DSE (06 B-6)

Time : Two hours Full Marks : 40

The figures in the margin indicate full marks.

(Symbols and notations have their usual meanings)

Use separate answer script for each Part.

Part – I (Marks: 16)

Answer ***any one***.

1. Consider the following eco-epidemic model:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) - \alpha P(S + I),$$

$$\frac{dS}{dt} = \alpha\alpha_1 PS - dS - \lambda SI,$$

$$\frac{dI}{dt} = \alpha\alpha_1 PI - (d + v)I + \lambda SI.$$

a) Give a flow diagram of the model.

b) Find disease basic reproduction number around disease free planar equilibrium point of model. Prescribe the conditions for stability of disease free planar equilibrium. Give the biological interpretation of your results.

c) Study the global stability of the interior steady state.

2+(3+2+3)+6

2. a) Formulate a basic SIR model with frequency-dependent transmission, demography and vaccination with necessary assumptions.

   b) Determine different equilibrium points of your model.

   c) Find the disease eradication conditions of the model.

   d) Consider the following SIRV model:

$$\frac{dS}{dt} = -\lambda SI - \alpha,$$
$$\frac{dI}{dt} = \lambda SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I,$$
$$\frac{dV}{dt} = \alpha.$$

   Find the angle between the vaccinated and non-vaccinated trajectories at (S, I) and show that the angle has different signs on the left and right of the line $S = \frac{\gamma}{\lambda}$. 5+3+3+5

Part – II (Marks: 24)

Answer ***any three*** questions.

1. The basic SIR model with demography expressed as

$$\frac{dS}{dt} = \alpha N - \beta IS - \mu S$$
$$\frac{dI}{dt} = \beta IS - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

   Find the underlying assumptions of this model. Draw the schematic diagram and also find the stability conditions of the endemic equilibrium of the above model. 8

2. The dynamics is governed by

$$\frac{dS}{dt} = \mu N(1-P) - (\mu + \beta I)S$$
$$\frac{dE}{dt} = \beta IS - (\mu + \sigma)E$$
$$\frac{dI}{dt} = \sigma E - (\mu + \gamma)I.$$

   Find the underlying assumptions of this model and also find the stability conditions of the endemic equilibrium of the above model. 8

3. Write down a SIRS model with the following assumptions:

   i) The horizontal transmission follows the law of mass action where β is the rate of infection.

   ii) The infected population recovers at a rate γ.

   iii) The recovered population loses the immunity and again becomes at a rate δ.

   iv) No vertical transmission, no incubation periods are considered.

   v) No latent period.