2. If K^a is a killing vector of a manifold then derive the Killing equation: $K_{a;b} + K_{b;a} = 0$

   Determine the condition for the Killing vector K^a to be orthogonal to a family of space-like hyper surfaces. 3+7

3. Give an explicit derivation of the Newtonian limit to the Einstein field equations. Hence find the value of the coupling constant. 8+2

4. Define clearly the manifolds over which the Lagrangian and Hamiltonian dynamics are defined for a physical system. What do you mean by Hamiltonian vector field in phase space? Hence define the Poisson bracket between two functions in phase space. Using simpletic structure in phase space deduce the condition for Canonical transformation. 2+2+2+2+2

5. Write down the differential equation for the geodesic in a manifold with arbitrary parameter. Derive the first integral of the geodesic and give geometrical interpretation to it. Show that geodesic is a straight line both for E^2 and E^3. 2+3+1+2+2

6. Derive the FLRW line element starting from cosmological principle. Write down the Einstein field equations for this line element and also the conservation of the energy-Momentum tensor for perfect fluid. Derive the first law of thermo-dynamics from this conservation equation. 6+2+1+1

Ex/SC/MATH/PG/DSE/TH/06/B10/2022

M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Application of Differential Geometry to Relativity and Mechanics

Paper – DSE - 06 (B10)

Time : Two hours Full Marks : 40

Answer ***any Four*** questions.

All questions carry equal makrs.

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

1. a) Using general Lorentz transformation (relative velocity v is along any arbitrary direction), find the expression for velocity $\vec{u}'$ of a particle as observed in S' frame if $\vec{u}$ denotes the velocity of that particle in S frame. Hence show that if $|\vec{u}|$, $|\vec{v}|$ be both less than c then $|\vec{u}'| < c$. 2+4

   b) Show that the set F of all linear transformation $L = \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix}$ from R^2 to itself and characterized by the fact $l_{11} > 0$, $\det L > 0$ and satisfying relation $L^{-1}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} L = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ forms a group under composition of maps. Using this result prove that the 2D Lorentz transformation form a group. 3+1