M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Renewable Bio-economic and Impulsive Models on Biological Systems

Paper – DSE - 07 (B25)

Time : 2 hours Full Marks : 40

Part – I (24 marks)

The figures in the margin indicate full marks.

Symbols / Notations have their usual meaning.

Answer ***any three*** from the following four questions.

1. Consider the linear **T**-periodic system with usual standard assumptions with fixed moments of impulsive effect

$$\frac{dx}{dt} = P(t)x, \quad t \neq t_k$$
$$\Delta x = B_k x, \quad t = t_k$$

Suppose $\lim_{t\to\infty} t_k = \infty$, and let, $X(t)$ be the fundamental matrix of solutions of the system in $\mathbb{R}_+$. Then prove that

a) For any constant matrix $M \in C^{nxn}$, $X(t)\bar{M}$ is also a solution of the system.

b) If $Y : \mathbb{R} \to C^{nxn}$ is a solution of the system, there exists a unique matrix $\bar{M}$ such that $Y(t) = X(t)\bar{M}$.

c) If $Y(t)$ is a fundamental solution matrix, then $\det \bar{M} \neq 0$.

d) If all the assumptions mentioned above hold true then prove that each fundamental matrix of the system can be represented in the form

$$X(t) = \varnothing(t)e^{\Lambda t},\ t \in \mathbb{R}.$$ 2+1+1+4

2. Find the disease-free equilibrium and basic reproduction number R_0 of a Leprosy disease mathematical model. Also, discuss thoroughly the sensitivity analysis of the parameters used in your formulated model and hence, evaluate the most sensitive parameters that can be used as control parameters. 1+3+4

3. a) Define feedback control. Formulate two different mathematical models expressing the definition of positive and negative feedback control for gaining insight of the biological interpretation with suitable assumptions.

   b) Describe the mathematical model formulation for pest management in Jatropha Curcas with integrated pesticides and find the equilibrium points of this system. 1+4+3

4. Formulate the mathematical model by considering the effect of awareness programs in controlling the disease HIV.

Also, describe the optimal control induced mathematical model and find the value of $u_1^*(t)$, $u_2^*(t)$ as optimal solution pairs. 2+6

Part – II (16 marks)

Answer ***any two*** questions.

1. a) Discuss the Gordon's principle results based on parabolic yield-effort curve for the logistic model of renewable resource harvesting.

   b) What do you mean by economic overfishing? 5+3

2. Construct a suitable mathematical model for combined harvesting of two ecologically independent fish populations and explain the harvesting policy with respect to the model. 8

3. Discuss the Schaefer model for exploitation of a fish population and its optimal harvest policy for different values of the discount rate. 8