them for Rs. 3.6 each. He cannot return unsold newspapers. Daily demand has the following distribution:

No. of customers :	23	24	25	26	27	28	29	30	31	32
Probability :	0.01	0.03	0.06	0.10	0.20	0.25	0.15	0.10	0.05	0.05

If each day's demand is independent of the previous day's, how many papers should he order each day? 8

8. Consider a probabilistic order level system with uniform demand and no set up cost model with the following assumptions:

   i) t_p is the prescribed scheduling period.

   ii) Demand rate in a period t_p is constant.

   iii) z is the stock level to which the inventory is raised at the end of each period t_p.

   iv) X is the random demand in a period t_p and $f(x)$ is the probability density function of demand X.

   v) Lead time is zero.

   Show that the optimum order level z^* can be determined from the equation

$$\int_0^z f(x)\,dx + z\int_z^\infty \frac{f(x)}{x}\,dx = \frac{c_2}{c_1+c_2}$$

   where c_1 and c_2 denote respectively the holding and shortage costs per quantity per unit time. 8

M. Sc. Mathematics Examination, 2022

(2nd Year, 2nd Semester)

Production Planning and Inventory Control

Paper – DSE - 06 (B7)

Time : Two hours Full Marks : 40

The figures in the margin indicate full marks.

(Notations / Symbols have their usual meanings)

Answer ***any five***.

1. What is economic order quantity (EOQ)? Derive a simple EOQ formula and show that $\frac{K}{K^*} = \frac{1}{2}\left[\frac{Q^*}{Q} + \frac{Q}{Q^*}\right]$ where Q^* is the optimum value of Q, and K^* is the minimum cost under optimal procurement policy. 1+7

2. In a certain manufacturing situation, consider the followings:

   i) D is the uniform demand per year,

   ii) Production rate P is greater than the demand rate D,

   iii) Shortages are allowed and backlogged,

   iv) Lead time is zero.

   Show that the optimal production quantity of the manufacturing system is

$$Q^* = \sqrt{\frac{2c_3(c_1+c_2)}{c_1 c_2}}\sqrt{\frac{PD}{P-D}}$$

Find also the minimum average total cost of the system. 6+2

3. Using a (t, s_i) policy, examine an inventory system under the following assumptions:

   i) The system operates over a finite planning horizon H.

   ii) During the period H, there exists a total demand of D units.

   iii) The rate of demand R changes linearly with time t such that $R = at$, where $a(>0)$ is a constant.

   iv) c_1 and c_3 are respectively the unit carrying cost and ordering cost per order.

   Show that the optimum number of replenishments n^* satisfies the inequality

$$f\left(n^*-1\right) \leq \frac{c_1 DH}{c_3} \leq f\left(n^*\right)$$

   where $f(n) = \dfrac{n(n+1)}{(n+1)h(n) - nh(n+1)}$ and

   $h(n) = \dfrac{1}{2} + \dfrac{1}{6n}$. 8

4. Consider a shop which produces and stocks three items. The management desires never to have an investment in inventory of more than Rs. 15000. The items are produced in lots. The demand rate for each item is

constant and can be assumed to be deterministic. No back orders are to be allowed. The pertinent data for the items are given in the following table. The carrying charge on each item is 20% of average inventory valuation per annum. Determine the optimal lot size for each item.

Item	**1**	**2**	**3**
Demand rate (units/year)	1000	500	2000
Variable cost (Rs. per unit)	20	100	50
Set up cost per lot (Rs.)	50	75	100

8

5. Derive the expression for average total cost of the order level lot size system under appropriate assumptions. Hence find the optimal order quantity, order level and minimum average cost of the system. 5+3

6. Determine the optimal order quantity for a product for which the price breaks are as follows:

Quantity	**Unit cost (Rs.)**
$1 \leq q_1 < 500$	10
$500 \leq q_2 < 750$	9·25
$750 \leq q_3$	8·75

   The monthly demand for the product is 200 units. The cost of storage is 2% of the unit cost and the cost of ordering is Rs. 100. 8

7. A newspaper-boy buys papers for Rs. 2.6 each and sells