3. a) Deduce the stress equation of motion of a continuum. Hence obtain the equilibrium equation of a continuum.

   b) The principal stresses at a point are $T_1=1$, $T_2=-1$ and $T_3=3$. If the stress at a point is given by

$$\left(T_{ij}\right)=\begin{pmatrix} T_{11} & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & T_{33} \end{pmatrix}.$$

   find the values of T_{11} and T_{33}. 5+5

4. Show that the maximum shearing stress is equal to half of the difference between the smallest and largest normal stresses and acts on a plane that bisects the angle between the directions of the largest and smallest principal stresses. 10

5. a) Derive the equation of continuity in Eulerian co-ordinate system. Give the physical interpretation of the equation of continuity.

   b) Derive the Euler's equation of motion for a perfect fluid. 5+5

6. Deduce the Navier-Stokes equation of motion for a viscous compressible fluid in the following form:

$$\frac{\partial\vec{q}}{\partial t}-\left[\vec{q}\times\text{curl}\vec{q}\right]=\vec{F}-\text{grad}Q+\frac{4}{3}\nu\text{ grad div}\vec{q}-\nu\text{ curl curl }\vec{q}$$

10

Ex/SC/MATH/PG/DSE/TH/01/B/2022

M. Sc. Mathematics Examination, 2022

(1st Year, 2nd Semester)

Mechanics of Continua

Paper – DSE-01B

Time : Two Hours Full Marks : 40

Symbols have their usual meanings.

Answer ***any four*** questions.

1. a) Show that a necessary and sufficient condition for the motion of a body is a rigid body motion is that $E_{ij}=0\ \left(i,j=x,y,z\right)$ throughout the body (E_{ij} are the components of the Lagrangian finite strain tensor).

   b) What do you mean by infinitesimal strain? Deduce the expression for the infinitesimal strain components $e_{ij}\left(i,j=x,y,z\right)$. 6+4

2. a) Give the geometrical interpretation of the shearing strain components e_{xy}, e_{yz}, e_{zx}.

   b) The displacement components in an elastic solid are given by $u_x=a\left(x+2y+3z\right)$, $u_y=a\left(-2x+y\right)$, $u_z=a\left(x+4y+2z\right)$, where a is a small quantity. Find the infinitesimal strain tensor and the volumetric strain. 6+4

[Turn over