M. Sc. Mathematics Examination, 2022

(1st Year, 2nd Semester)

Design and Analysis of Algorithms

Paper – DSE-01C

Time : Two Hours Full Marks : 30

Use a separate answer script for each Part.

Symbols have usual meanings, if not mentioned otherwise

Part – I (15 Marks)

Answer **Q. No. 1 (compulsory)** and ***any one*** from the rest.

1. a) Can the following recurrence equation be solved using Master Theorem? If your answer is yes, solve it, but if your answer is no, give reasons?

$$T(n)=T\left(\frac{3n}{4}\right)+O(n)$$

b) Solve the following recurrence equation using the recursion tree method.

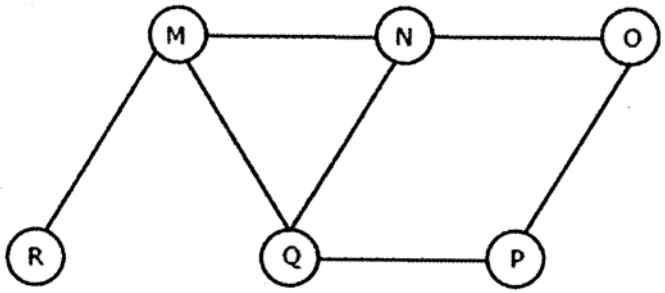
If n<=1, T(n)=1
else T(n)=T(n-1)+T(n-2)+O(1)

c) When can we say that an algorithm is randomized?

2+2+1=5

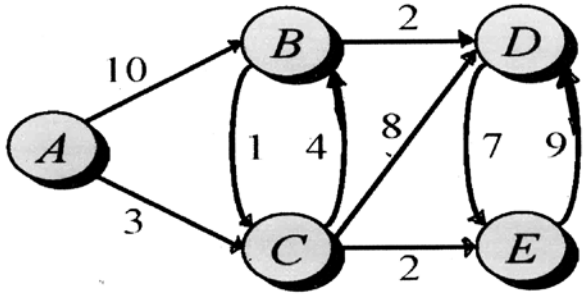
2. a) How is the following graph represented using the adjacency list method? If the Breadth First Search algorithm has been implemented using the queue

data structure, find one possible order of visiting the nodes of the following graph. What is the running time of this algorithm?



b) What are the differences between "*Dynamic Programming*" and "*Divide and Conquer*" approach? (2+3+2)+3=10

3. a) Write the one possible order of visiting nodes when the Dijktra's algorithm for finding the shortest path is applied on the following weighted directed graph with source node A. Give an example to show when this algorithm may fail.



b) Discuss how the coin changing problem is solved using the dynamic programming method. (4+2)+4=10

Part – II (15 Marks)

Attempt **Q. No. 1** and ***any one*** from the rest.

1. Consider the problem of implementing a *k*-bit binary counter that counts upward from 0.
   a) Write a pseudocode for incrementing the counter.
   b) Using aggregate analysis, determine the amortized cost per operation. 2+3=5
2. Consider a set of *n* points in the plane.
   a) Describe a divide-and-conquer algorithm, which runs in $O(n \lg n)$ time, to find the closest pair of points.
   b) Prove the correctness of your algorithm. 5+5=10
3. a) Show the red-black trees that result after successively inserting the keys 41, 38, 31, 12, 19, 8 into an initially empty red-black tree.
   b) Now show the red-black trees that result from the successive deletion of the keys in the order 8, 12, 19, 31, 38, 41. 5+5=10