Ex/SC/MATH/PG/DSE/01A/2022

M. Sc. Mathematics Examination, 2022

(1st Year, 2nd Semester)

Advanced Topology

Paper – DSE-01A

Time : Two Hours Full Marks : 40

Answer Q. No. **1** and ***any three*** questions from the rest.

1. a) Give an example of a topological space which is locally compact but not Hausdorff.

   b) Answer with reasons whether the uncountable product of the real number space with usual topology is metrizable. 2+2

2. a) Prove that a filter $\mathcal{F}^*$ is an ultrafilter in X if and only if any subset A of X which intersects every member of $\mathcal{F}^*$ belongs to $\mathcal{F}^*$.

   b) Prove that a function $f : X \to Y$ is continuous at $x \in X$ if and only if for every net (s_n) in X eonverging to x, the net $(f(s_n))$ converges to $f(x)$.

   c) Show that if a maximal net (s_n) has a cluster point x_0 then (s_n) converges to x_0. 6+4+2

3. a) Prove that among all Hausdorf compactifications of a locally compact, non-compact Tychonoff space, the one point compactification is the smallest and Stone-Cech compactification is the largest.

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   b) If (X,τ) is locally compact and $f:(X,\tau)\to(Y,\sigma)$ is open, continuous and onto then show that Y is also locally compact. 8+4

4. a) Show that every open σ-locally finite cover of a topological space has a locally finite refinement.

   b) Give an example of a paracompact space which is not compact.

   c) Prove that a Hausdorff paracompact space is regular. 5+2+5

5. a) State and prove Uryshon's metrization theorem.

   b) Show that in a compact metrizable space X, every metric is a B-metric (bounded metric). 8+4

6. a) Prove that in the set of all paths $P(X)$ of all paths in a topological space X having end points x_0 and x_1 in X, the path homotopy relation is an equivalence relation.

   b) When a topological space is said to be contractible? Prove that a topological space X is contractible if and only if X is homotopically equivalent to $\{x\}$ for some $x \in X$. 8+4