Ref. No. Ex/SC/PHY/UG/CORE/TH/14/2022

B. Sc. Physics Examination, 2022

(3rd Year, 2nd Semester)

Subject : Statistical Mechanics

Time : Two hours Full Marks : 40

The figures in the margin indicate full marks.

Candidates are instructed to give their answers in their own words as far as practicable.

Answer any *four* questions: 4x10=40

1. (i) The probability density function (pdf) of the Poisson distribution reads as

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}.$$

Prove that $\text{Var}(ax+b) = a^2\lambda$, where a and b are some constants. Under what conditions the Binomial distribution tends toward Poisson distribution. [4+2]

(ii) In 1-D random walk problem, a drunk man wants to return his home from the bar. His steps are random. This means his steps are equally probable to the left or to the right. But there is a finite chance of return to home. Let us focus on a quantity which is net displacement to right $m = (n_1 - n_2)l$, where n_1 and n_2 are steps to the right and steps to the left, respectively, with total step $N = n_1 + n_2$. And, l is the each step length. Calculate the value of $\langle m \rangle$. What is the significance of this result? [3+1]

2. (i) Consider a monatomic ideal gas of N molecules, each of mass m enclosed in a container of volume V at temperature T. The molecules are non-interacting and indistinguishable. Show that the partition function of the system is

$$Z = \frac{{Z_1}^N}{N!}, \text{ where } Z_1 = V\left(\frac{2\pi m k_B T}{h^2}\right)^{3/2} \text{ is the partition function of a single molecule.}$$

Hence arrive at the expression of entropy S of the system, which is in agreement with the 'Sakur-Tetrode' equation. [4+3]

(ii) Consider a system of N particles. The energy of each particle can assume two distinct values, 0 and ϵ ($\epsilon > 0$). The total eneygy of the system is U. Prove that temperature of the system is

$$T = \frac{\epsilon}{k_B \ln\left(\frac{n_0 \epsilon}{U}\right)}, \text{ where } n_0 \text{ is the number of particles in the energy level 0.}$$

Can the temperature of the system be negative? [2+1]

3. (i) The 'Liouville's theorem' in statistical mechanics inquires the rate of change of density function $\rho(q,p,t)$ in Γ space, where all the symbols have their usual meanings. Write down the mathematical form of this theorem under equilibrium condition. Hence specify the value of ρ in case of micro-canonical ensemble. Also prove that, for a system of N molecules, the volume of each representative point in Γ space is h^{3N}, where h is Plank's constant. [2+1+2]

(ii) Consider a system whose energy is in the form of linear 'degree of freedom' rather than usual quadratic: $E = c \mid q \mid$, where q is any coordinate or momentum variable and c is a constant. By repeating the derivation of the equipartition theorem for this system, prove that the average energy $\langle E \rangle = k_B T$. Give an example of physical system whose energy can be in the form of linear 'degree of freedom'. [4+1]

4. Derive the expression of most probable distribution in Bose-Einstein statistics. For what type of particle is this statistics applicable? Five bosons are distributed in two compartments, the first having 3 cells and the second 4 cells. Find the thermodynamic probability for the macrostates $(5,0)$ and $(4,2)$. [5+1+(2+2)]

5. What is Fermi energy? Derive an expression of Fermi energy for electron gas. Calculate Fermi energy of $^{65}Cu_{29}$ (fcc lattice). The lattice constant for Cu is 0.3615 nm. [2+5+3]

6. Deduce the Planck's radiation formula from Bose-Einstein distribution function. Show that Planck's constant is related to Stefan's constant by the formula

$$h = \left(\frac{2\pi^5 k^4}{15\sigma c^2}\right)^{1/3}, \text{ consider } \int_0^\infty \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15}, \text{ where all the symbols have their usual meanings.}$$ [5+5]