Ref. No. Ex/SC/PHY/UG/DSE/TH/02/4A/2022

B. Sc. Physics Examination, 2022

(3rd Year, 2nd Semester)

Subject : Physics with Computer Simulations

Paper : DSE 4A1

(Subject Code : UG/SC/DSE/PHY/TH/04/A1)

Time : Two hours | Full Marks : 40

(20 marks for each group)

Prepare separate answerscripts for each group.

Group A

Answer any TWO questions.

$2 \times 10 = 20$

1. (a) SIR model is defined by the set of nonlinear equations,

$$\begin{cases} \frac{dS}{dt} = -\alpha\, S\, I, \\ \frac{dI}{dt} = \alpha\, S\, I - \beta\, I, \\ \frac{dR}{dt} = \beta\, I, \end{cases}$$

where $S(t)$, $I(t)$, $R(t)$ are the susceptible, infected and recovered populations at time t, and the total population $N = S(t) + I(t) + R(t)$. What is the significance of the parameters α and β? Determine the units of α and β.

(b) Find the integral form of $S(t)$ and $I(t)$.

(c) Find the approximate form of $\frac{dR}{dt}$, (Riccati equation) when $\alpha R(t)/\beta < 1$.

(d) (i) Write down the set of differential equations for the SEIQR model. (ii) What do you mean by the term "exposed population"? (iii) Explain the significance of additional parameters used in this model.

[2+2+2+(1+1+2)=10]

[Turn over

2. (a) Derive the expression for equation of motion for the oscillation of simple pendulum:

$$\dot{\theta} = \sqrt{2}\,\omega_0 \sqrt{\cos\theta - \cos\theta_c},$$

where the symbols have their usual meaning.

(b) Find the solution of the above differential equation, and obtain the expression for time period of oscillation in terms of complete elliptic integral of first kind.

(c) Using the binomial expansion of complete elliptic integral of first kind, show that time period of simple pendulum is always greater than that of the corresponding simple harmonic motion.

(d) Draw the variation of time period with energy for oscillation and rotation in the same plot.

[2+4+3+1=10]

3. While studying the motion of the lightest body for planar restricted circular three-body system in two-dimensional $x'y'$-plane of a rotated frame in normalized units, the equations of motion look like,

$$\ddot{x}'_3 - 2\dot{y}'_3 = x'_3 - \frac{\mu_1}{r'^3_1}(x'_3 + \mu_2) - \frac{\mu_2}{r'^3_2}(x'_3 - \mu_1),$$

$$\ddot{y}'_3 + 2\dot{x}'_3 = \left(1 - \frac{\mu_1}{r'^3_1} - \frac{\mu_2}{r'^3_2}\right) y'_3,$$

where $r'_1 = \sqrt{(x'_3 + \mu_2)^2 + y'^2_3}$, and $r'_2 = \sqrt{(x'_3 - \mu_1)^2 + y'^2_3}$.
The symbols bear their usual meaning.

(a) Express the equations of motion in terms of the gradient of pseudo potential,

$$\Phi(x'_3, y'_3) = \frac{1}{2}\left(x'^2_3 + y'^2_3\right) + \left(\frac{\mu_1}{r'_1} + \frac{\mu_2}{r'_2}\right).$$

(b) Obtain the expression of Jacobi constant (C'_5) in the rotated frame and normalized units.

(c) How do you mark the forbidden region?

(d) Express the equations of motion in terms of the gradient of gravitational potential and identify the Coriolis and centrifugal terms. [2+2+2+4=10]

Group B

Answer any TWO questions.

$2 \times 10 = 20$

1. (b) Suppose you have an array of size N which contains real numbers ranging from 0 to 1. Write a program/algorithm to get the probability distribution function in form of a histogram.

   (b) Write an algorithmic set up to generate Random numbers with range $[0 : 1]$ following a probability distribution function $p(x) = 2(1 - x)$. [5+5=10]

2. (a) Consider an one dimensional function $f(x) = e^{-|x-2|}$. Write an algorithmic set up to estimate the integration from $x = -6$ to $x = 6$. Explain the concept of Importance Sampling in this context.

   (b) It seems that others than sampling form Uniform distribution would be better in this case. Which distribution function would be better and how do you sample from it for this integration problem? [6+4=10]

3. (a) Discuss the basic properties of Two dimensional Classical Ising model. Can there be a phase transition in One dimension?

   (b) Write down the Metropolis Algorithm to simulate Ising model on the lattice given below mentioning how to generate the lattice and find the nearest neighbours. [4+6=10]