Ref. No. Ex/SC/PHY/UG/DSE/TH/01/3A/2022

B. Sc. Physics Examination, 2022

(3rd Year, 2nd Semester)

Advance Dynamics

Time : Two hours | Full Marks : 40

Answer any **four** questions

1. (a) A two dimensional linear system is described by

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x}\,, \quad \text{where} \quad \mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{and} \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

Analyze the equation and classify fixed points depending on $\Delta = \det$ (A) and $\tau =$ trace (A). (6)

(b) Find the fixed points of the nonlinear system,

$$\frac{dx}{dt} = -x + x^3 \quad \text{and} \quad \frac{dy}{dt} = 2y.$$

Use linearization to classify the fixed points. Hence show the phase portrait. (4)

2. (a) Write down equation of motion as 2-dim dynamical system for a particle in a double well potential.

$V(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4$. Find the fixed points and the phase portrait. (5)

(b) Equation of motion of a simple pendulum may be written as,

$$\frac{d\theta}{dt} = v, \quad \frac{dv}{dt} = -\sin(\theta).$$

Find the fixed points and the phase portrait for $-1.5\pi < \theta < 1.5\pi$. (5)

3. (a) Analyze and plot the bifurcation diagram for the one dimensional system,

$$\frac{dx}{dt} = rx - x^3.$$ (4)

(b) Analyze and identify the bifurcation for the two dimensional system,

$$\frac{dx}{dt} = \mu - x^2 \quad \text{and} \quad \frac{dy}{dt} = -y\,.$$ (6)

4. (a) Argue why limit cycles cannot exist in a linear system. (2)

(b) Identify the bifurcation(s) and the critical values μ_c of parameter μ for the system,

$$\frac{dr}{dt} = \mu r + r^3 - r^5 \quad \text{and} \quad \frac{d\theta}{dt} = -1\,.$$

Show flow lines around the fixed points before and after the bifurcation(s). (8)

5. (a) A two-dimensional velocity field is given as $\vec{V} = 2y\,\hat{i} + 3x\,\hat{j}$ in m/s.
   (i) Check whether the flow is irrotational and incompressible.
   (ii) At $t = 0$, a particle of fluid passes through a location $(1m, 2m)$.
   Find its velocity, local acceleration and convective acceleration at that location.
   (b) Define streamlines, pathlines and streaklines. [(2+5)+3]

6. (a) Briefly explain the Lagrangian and Eulerian approach of describing fluid flows.
   (b) Derive continuity equation in differential form for a fluid flow in three dimensions. [4+6]