B. Sc. Mathematics (Hons.) Examination, 2022

(3rd Year, 2nd Semester)

Combinatorics & Graph Theory

Paper – DSE-4A

Time : Two hours Full Marks : 40

(Symbols have usual meanings, if not mentioned otherwise)

Part – I (20 marks)

Attempt any *Two* questions from this part : 10×2=20

1. a) Show that every self-complementary graph has $4n$ or $4n+1$ points where n is a positive integer.

   b) Define n-cube. Find the number of points and lines in an n-cube. 5+5=10

2. a) Prove that a graph G is unicyclic if and only if $G-x$ is a tree for some edge x of G.

   b) Prove that there always exists a graph G such that $\kappa(G)=a$, $\lambda(G)=b$ and $\delta(G)=c$ where a, b, c are integers with $0<a\le b\le c$. 5+5=10

3. a) Prove that a planar graph can not be a 6-connected graph.

   b) Prove that every hamiltonian graph is 2-connected and every nonhamiltonian 2-connected graph has a theta subgraph. 5+5=10

[Turn over

Part – II (20 marks)

Attempt **Question 4** and **any *Two*** from the rest.

4. a) Given the function

$T(w,x,y,z)=\sum(1,2,3,5,13)+\sum_{\phi}(6,7,8,9,11,15)$:

i) Find a minimal sum-of-products expression.

ii) Find a minimal product-of-sums expression.

iii) Compare the expressions obtained in (a) and (b); if they do not represent identical functions, explain.

b) Let $(A,\vee,\wedge,')$ be a finite boolean algebra. Let b be any nonzero element in A, and $a_1,a_2,...,a_k$ be all the atoms of A such that $a_i \leq b$. Prove that $b=a_1 \vee a_2 \vee ... \vee a_k$ is the unique way to represent b as a join of atoms. 6+4=10

5. State the pigeonhole principle in an appropriate form to show the following. Suppose 50 chairs are arranged in a rectangular array of 5 rows and 10 columns. 41 students are seated on the chairs randomly one per chair. Using the above statement, show that some row contains at least 9 students, some column contains at least 5 students, some row contains at most 8 students and some column contains at most 4 students. 5

6. Solve the recurrence relation

$a_n-6a_{n-1}+8a_{n-1}=n4^n+3^n$. 5

7. Let a_r denote the number of ways to divide r identical balls into four distinct urns so that each urn has an odd number of balls that is larger than or equal to three.

a) Determine the generating function $A(z)$.

b) Determine a closed-form expression for a_r. 5