

B. SC. MATHEMATICS EXAMINATION, 2022

(3rd Year, 2nd Semester, Supplementary)

MECHANICS

PAPER – UG /CORE/TH/14

Time : Two hours

Full Marks : 40

The figures in the margin indicate full marks.

(Symbols/Notations have their usual meanings)

Group - A

Any two questions

2X7=14

1. Two forces act, one along the line $y = 0, z = 0$ and the other along the line $x = 0, z = c$. As the forces vary, show that the surface generated by the axis of their equivalent wrench is $(x^2 + y^2)z = cy^2$
2. The distance from the cusp of the centroid of the area of the cardioid $r = a(1 + \cos \theta)$, when the density at any point varies as the square of the distance from the cusp is $\frac{6}{5}a$.
3. A uniform rod of length $2l$, is attached by a smooth ring at the ends to a parabolic wire, fixed with its axis vertical and vertex downwards, and of latus rectum $4a$. Show that the angle θ which the rod makes with the horizon in a slanting position of equilibrium is given by $\cos^2 \theta = \frac{2a}{l}$, and that, if these positions exist, they are stable.

Group – B

Any two questions

2X7=14

4. A particle is moving under the influence of an attractive force $\frac{2m}{y^3}$ towards the axis of x . Show that, if it is projected from the point $(0,1)$ with component velocities U and V parallel to the axes of x and y , it

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will not strike the axis of x unless $2 > V^2$, and that in this case the distance of the point of impact from the origin is $\frac{U}{\sqrt{2-V}}$.

5. A particle is projected horizontally from the lowest point of a rough sphere of a radius 'a' and comes to rest at the lowest point after describing an arc less than a quadrant, show that the velocity of projection must be $\sin \alpha \sqrt{2ga \frac{(1+\mu^2)}{(1-2\mu^2)}}$ where μ is the coefficient of friction and $a\alpha$ is the arc through which the particle moves.
6. An insect crawls at a constant rate u along the spoke of a cart wheel, of radius a , the cart moving with a velocity v . Find the acceleration along and perpendicular to the spoke.

Group – C

Any two questions

2X6=12

7. Show that the moment of inertia of a right solid cone where height is 'h' and radius of whose base is 'a' is $\frac{3Ma^2}{20} \frac{6h^2+a^2}{h^2+a^2}$ about a slant side and $\frac{3M}{80} (h^2 + a^2)$ about a line through the centre of gravity of the cone and perpendicular to its axis.
8. A bent lever whose arms are of lengths 'a' and 'b', the angle between them being '45°', makes small oscillation in its own plane about the fulcrum. Show that the length of the corresponding simple equivalent pendulum is $\frac{2}{3} \frac{a^3+b^3}{\sqrt{a^4+\sqrt{2}a^2b^2+b^4}}$.
9. Define compound pendulum. Show that the centre of suspension and centre of oscillation are convertible.