B. Sc. Mathematics (Hons.) Examination, 2022

(3rd Year, 2nd Semester)

Mathematical Physics and Relativity

Paper – DSE-4C

Time : Two hours Full Marks : 40

Answer any *Four* questions.

All questions carry equal marks.

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

1. a) Draw a diagram of the light cone and clearly distinguish the time-like, space-like and null vectors. Give their physical interpretations. 1+3+1

   b) If the space and time separation between two events in a S frame is L and and T respectively, then what will be the minimum and maximum space and time separation between these two events in another inertial frame S'? $2\frac{1}{2}+2\frac{1}{2}$

2. a) Show that Maxwell's equation of electrodynamics are invariant under Lorentz Transformation. 7

   b) If the position vectors of two points in 4D Minkowski space-time are $\left(\frac{1}{c},0,0,0\right)$ and

$\left(\frac{2}{c},2,1,1\right)$, then examine whether the points are causally connected or not. 3

3. Define 4-velocity, 4-acceleration and 4-momentum in Special Theory of Relativity. State the directions of four velocity and four acceleration with respect to world line. Are they orthogonal to each other? Find the magnitude of 4-velocity and an estimation of the magnitude of 4-acceleration. Clearly state which of the above vectors are time like? 3+1+1+2+2+1

4. Starting from Lorentz Transformation deduce the following:

   a) the principle of simultaneity,

   b) Newtonian limit,

   c) group property of the set of Lorentz Transformation,

   d) invariance of 4D volume element. 2+1+4+3

5. a) Deduce the expression for kinetic energy in STR. Hence formulate the energy-momentum conservation relation in STR. 3+2

   b) A spaceship plans to go to a star 5 light years away. The rocket accelerates quickly and then moves at a uniform speed. Calculate with what speed the rocket must move relative to the Earth if the spaceship is to

reach there in 1 year, as measured by a clock being at rest inside the rocket. 5

6. a) Deduce the generalized Lorentz Transformation for arbitrary relative motion between two inertial frames. 5

   b) Calculate the orientation of a rod of length l in an inertial frame that is moving with a velocity $\mu c(\mu<1)$ in a direction making an angle 2θ with the rod. 5