B. Sc. Mathematics (Hons.) Examination, 2022

(3rd Year, 2nd Semester)

Mathematical Modelling

Paper – DSE-3B

Time : Two hours   Full Marks : 40

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Answer any *FIVE* questions.

1. Assuming that the emotion is measurable, propose a simple model for the love affair of Romeo and Juliet. Under what parametric conditions, the two lovers are said to be exactly opposte? Determine the stability condition of the equilibrium love levels under this situation.

    3+2+3

2. Suppose the growth of a population, P, is determined by the equation $\dfrac{dP}{dt} = \alpha P\left(1 - \dfrac{P}{A}\right)$, where α, A are positive constants and $P(0) = P_0$.

    a) Determine the size of the population at any time t.

    b) What would be the doubling time of this population?

    4+4

3. A particle is thrown vertically upward with initial velocity v_0 from the earth's surface. The height of the particle at time t is represented by the differential

equation $\dfrac{d^2z}{dt^2} = -\dfrac{gR^2}{(z+R)^2}$, with initial conditions $z(0)=0$

and $\dfrac{dz(0)}{dt} = v_0$. Here g is the gravitational acceleration and R is the radius of the earth. Nondimensionalize the above system with suitable transformations. 8

4. a) Derive the differential-difference equation for the queuing model $M/M/1/\infty/FCFS$.

   b) From the solution of the model, find the probability of exactly n customers in the queuing system. 4+4

5. a) Define the rate of a second order reaction for a single chemical reactant. What would be the concentration of the reactant at any time t ?

   b) A new drug was given in a single intravenous dose of 200 mg to an 80 kg male. After 6 hrs, the blood concentration of drug was found to be 1.5 mg/100 ml of blood. Assuming the apparent volume of distribution is 10% of the body weight, compute the total amount of drug in the body fluid after 6 hrs. What is the half-life of this drug? 4+4

6. a) Describe with proper assumptions, the linear dynamic models of arms race of two nations proposed by Richardson.

   b) What is the equilibrium defense level of your model

system?

   c) Find the conditions if two nations want to avoid instability in the arm's race. 3+2+3

7. What do you mean by optimum lot size of an inventory? Derive the optimum lot size and minumum total inventory cost in the usual notations when the lead time in zero, demand is uniform, production is instantaneous and there are no shortage. 1+7