Ex/SC/MATH/UG/CORE/TH/13/2022 (S)

B. Sc. Mathematics Examination, 2022

(3rd Year, 2nd Semester, Supplementary)

Mechanics (Honours)

Paper – UG /Core/TH/13

(Group Theory II)

Time : Two hours | Full Marks : 40

All questions carry equal marks

Answer any **four** questions

1. (a) Let G be a group. Define the group of automorphisms of G and inner automorphisms on G. Show that the set of all inner automorphisms of G is a normal subgroup of the group of automorphisms of G.

   (b) Let G be a finite group with the identity e. Let $T : G \longrightarrow G$ be an automorphism such that $T(x) = e$ implies $x = e$. If T^2 is the identity map on G, then show that G is abelian.

2. (a) Let G be a group of order 75 and H be a subgroup of G of order 25. Show that H is a normal subgroup of G.

   (b) Let G be a group of order $2n$ such that half of the elements of G are of order 2 and other half form a subgroup H of order n. Prove that H is of odd order and is an abelian subgroup of G.

3. (a) Let G be a group of order 24. Then show that G has a normal subgroup of even order.

   (b) Define a simple group. Show that any group of order 56 is not simple.

4. (a) Let G be a group and $x \in G$. Define the conjugacy class, C_x of x and the normalizer, $N(x)$ of x. Prove that $N(x)$ is a subgroup of G and the index of $N(x)$ in G is equal to the number of elements of C_x.

   (b) Let G be a non-commutative group of order 125 and $Z(G)$ be the center of G. Prove that $|Z(G)| = 5$.

5. (a) Define Sylow subgroups of a finite group. Prove that a Sylow subgroup H of a finite group G is normal if and only if G has only one subgroup of order $|H|$.

   (b) Show that any group of order 391 is cyclic.

6. (a) Let G be a group of order n (where n is a natural number) and p be a prime factor of n. Then show that G has an element of order p.

   (b) Find all non-isomorphic abelian groups of order 360.