B. Sc. Mathematics Examination, 2022

(3rd Year, 2nd Semester, Supplementary)

Biomathematics

Paper – DSE-4B

Time : Two hours Full Marks : 40

Use a separate Answer scripts for each part.

The figures in the margin indicate full marks.

(Symbols/Notations have their usual meanigns)

Use separate Answer scripts for each part.

PART – I (Marks: 24)

Answer any *Three* questions.

1. Explain single species growth model of the form

$$\frac{dN}{dt} = f(N)$$

And discuss the stability of its critical value. Evaluate the asymptotic behavior of the model when $f(N) = -\lambda N \log\left(\frac{N}{\theta}\right)$, λ and θ are positive parameters.

5+3

2. Explain functional response and numerical response in a prey-predator interactions. Describe different Holling type functional responses in mathematical form and draw the response curve. 8

3. Prey-Predator interaction is taken in the form:

$$\frac{dx}{dt} = ax - yp(x)$$
$$\frac{dy}{dt} = y\{Kp(x) - m\},$$

where the symbols have their usual meanings. Determine the steady states and discuss their qualitative behaviour of the model. 8

4. Write down the growth equations of two mutualistic populations. Determine the steady states and discuss the stability properties of the interior equilibrium point. 8

5. Write down a prey-predator model where the prey population has its own intrinsic mechanism and predator's functional response is Holling type-II form and discuss the local stability analysis of the model. 8

PART – II (Marks: 16)

Answer any *Two* questions.

6. a) Consider the first order difference equation $x_{n+1} = f(x_n)$ with $\bar{x}$ as a fixed point. Determine nature of the fixed point $\bar{x}$ if $|f'(\bar{x})| > 1$. Justify your answer.

   b) Consider a predator-prey system

$$x_{n+1} = (p+1)x_n - rx_n^2 - bx_n y_n,$$

$$y_{n+1} = cx_n y_n + (1-d)y_n,$$

where x_n and y_n represent, respectively, the prey and predator species at the nth generation. The parameters $r, b, c \in \mathbb{R}^+$ with $0 < d < 1$.

Find all fixed points of this system. Determine the stability criteria of the interior fixed point. 3+5

7. a) Define positive semi-definite function.

   b) Determine the nature of the fixed point(s) of the system $\dot{x} = \mu x + x^3$, $\mu \in \mathbb{R}$.

   Draw the phase portrait and bifurcation diagram. 2+6

8. a) If $A = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$ then show that $e^{At} = \begin{pmatrix} e^{\lambda t} & te^{\lambda t} \\ 0 & e^{\lambda t} \end{pmatrix}$.

   b) Determine the nature of the system $\dot{X} = AX$ with $A = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}$, $X = (x, y)^T$, $\alpha, \beta \in \mathbb{R}$. Draw the phase portrait. 3+5