B. Sc. Mathematics (Hons.) Examination, 2022

(3rd Year, 2nd Semester)

Differential Geometry

Paper – DSE-3C

Time : Two hours　　　　Full Marks : 40

Answer any *four* of the followings

1. a) Let $\vec{r}(t)$ be a regular curve in space. Prove that its torsion $\tau = \dfrac{\left[\dot{\vec{r}} \;\; \ddot{\vec{r}} \;\; \dddot{\vec{r}}\right]}{\left|\dot{\vec{r}} \times \ddot{\vec{r}}\right|^2}$, where $\dot{\vec{r}} = \dfrac{d\vec{r}}{dt}$.

   b) Find the curvature and torsion of the circular helix $\vec{r} = \left(a\cos\dfrac{s}{c},\ a\sin\dfrac{s}{c},\ \dfrac{bs}{c}\right)$; $c = \sqrt{a^2+b^2}$, and s is arc length. 6+4

2. a) Define Bertrand mates. Prove that for Bertrand mates the relation between curvature k and torsion τ satisfies the relation $ak + b\tau = 1$, where a, b are constants.

   b) Find the surface area bounded by the parametric curves $u = 0$ to $u = \dfrac{\pi}{4}$ and $v = 0$ to $v = \dfrac{\pi}{2}$ for the surface given by
   $\vec{r}(u,v) = (a\sin u\cos v,\ a\sin u\sin v,\ a\cos u)$
   where a is constant. 6+4

3. a) Find the Gaussian curvature of the surface $\vec{r}(u,v) = (u\cos v,\ u\sin v,\ cv)$, c being a constant. Is it developable? Justify.

   b) If the components of a contravariant vector in (x^i) coordinate system are (3, 4), find its components in $\left(\bar{x}^i\right)$ coordinate system, where $\bar{x}^1 = 7x^1 - 5x^2$ and $\bar{x}^2 = -5x^1 + 4x^2$. 6+4

4. a) If A^i and B^i are two non-null vectors such that $g_{ij}U^iU^j = g_{ij}V^iV^j$, where $U^i = A^i + B^i$ and $V^i = A^i - B^i$, then find the angle between A^i and B^i.

   b) Calculate the Christoffel symbols of the second kind in cylindrical coordinate system.

   c) Find the Riemann curvature tensor for the metric $ds^2 = \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2$ and give the geometrical interpretation. 3+4+3

5. a) Compute the first and second fundamental forms for a patch $\vec{r}$ on a surface of revolution given by $\vec{r}(u,v) = (f(u)\cos v, f(u)\sin v,\ g(u))$, where f and g are smooth functions and $f(u) > 0$.

   b) Find the arc length for a curve $\vec{r}(t) = \left(e^t\cos t,\ e^t\sin t,\ 0\right)$ from $t = 0$. 6+4

6. State and prove fundamental theorem of space curve. 10