B. Sc. Mathematics (Hons.) Examination, 2022

(3rd Year, 2nd Semester)

Mechanics

Paper – Core-14

Time : Two hours Full Marks : 40

The figures in the margin indicate full marks.

(Symbols/Notations have their usual meanings)

Group – A

Any *two* questions : **2×6=12**

1. Two forces P and Q act along the straight lines whose equations are $y = x\tan\alpha$, $z = c$ and $y = -x\tan\alpha$, $z = -c$ respectively. Show that their central axis is $y = x\dfrac{P-Q}{P+Q}\tan\alpha$, $\dfrac{z}{c} = \dfrac{P^2-Q^2}{P^2+2PQ\cos 2\alpha+Q^2}$. Also prove that this line is a generator of the surface $\left(x^2+y^2\right)z\sin 2\alpha = 2cxy$ for all values of P and Q.

2. The distance from the cusp of the centroid of the area of the cardioid $r = a\left(1+\cos\theta\right)$, when the density at any point varies as the nth power of the distance from the cusp is $\dfrac{(n+2)(2n+5)}{(n+3)(n+4)}a$.

3. A uniform rod of length $2l$, is attached by a smooth ring at the ends to a parabolic wire, fixed with its axis vertical

and vertex downwards, and of latus rectum $4a$. Show that the angle θ which the rod makes with the horizon in a slanting position of equilibrium is given by $\cos^2\theta = \frac{2a}{l}$, and that, if these positions exist, they are stable.

Group – B

Any *two* questions : **2×7=14**

4. A particle is moving under the influence of an attractive force $\frac{m\mu}{y^3}$ towards the axis of x. Show that, if it is projected from the point $(0, k)$ with component velocities U and V parallel to the axes of x and y, it will not strike the axis of x unless $\mu > V^2K^2$, and that in this case the distance of the point of impact from the origin is $\frac{UK^2}{\sqrt{\mu} - VK}$.

5. A particle is projected horizontally from the lowest point of a rough sphere of a radius '1' unit and comes to rest at the lowest point after describing an arc $\frac{\pi}{6}$, show that the velocity of projection must be $\sqrt{\frac{g\left(1+\mu^2\right)}{2\left(1-2\mu^2\right)}}$ where μ is the coefficient of friction.

6. A particle moves in a nearly circular orbit of radius a under a central force $\mu r^3\left(3+2r^2\right)$, show that apsidal angle is $\pi\sqrt{\frac{3+2a^2}{18+16a^2}}$.

Group – C

Any *two* questions : **2×7=14**

7. Show that moment of inertia of a regular hexagon about any line through its centre in the the plane of the hexagon is $\frac{5Mc^2}{24}$, where c is the length of each side.

8. A thin rod of length $2a$ can turn freely about one end O which is fixed. It is started from the position in which it hangs vertically with such a angular velocity just sufficient to carry it to the highest point. If the density of the rod is proportional to the nth power of the distance from O along the rod, then prove that the time taken by the rod to describe an angle $\theta(<\pi)$ with the downwards vertical through O is $\sqrt{\frac{2a(n+2)}{g(n+3)}}\log\tan\left(\frac{\pi}{4}+\frac{\theta}{4}\right)$.

9. A bent lever whose arms are of lengths a and b, the angle between them being 'α', makes small oscillation in its own plane about the fulcrum. Show that the length of the corresponding simple equivalent pendulum is $\frac{2}{3}\frac{a^3+b^3}{\sqrt{a^4+2a^2b^3\cos\alpha+b^4}}$.