B. Sc. Mathematics Examination, 2022

(3rd Year, 2nd Semester, Supplementary)

MATHEMATICAL PHYSICS AND RELATIVITY

PAPER – DSE-4C

Time : Two hours   Full Marks : 40

The figures in the margin indicate full marks.

(Symbols/Notations have their usual meanigns)

Answer any *Four* questions.   4×10=40

1. a) What is proper time? Find the relation between co-ordinate time and proper time. Is proper time Lorentz invariant? Justify.   2+2+1

   b) At what speed v_0 will the Galżlean and the Lorentz expression for x differ by 1%?   5

2. a) Derive the relativistic energy-momentum relation.   4

   b) The space and time co-ordinates of two events as measured in a fram S are as follows :

   Event 1 : $x_1 = x_0$, $y_1 = z_1 = 0$, $t_1 = \dfrac{x_0}{c}$

   Event 2 : $x_2 = 2x_0$, $y_2 = z_2 = 0$, $t_2 = \dfrac{x_0}{2c}$

   There exists a frame S' in which these two events occur at the same time. Find the relative velocity of S' with respect to S. What is the time at which both

events occur in S'?   4+2

3. a) Suppose S and S' are two inertial frame of references where S'-frame is moving relative to S-frame with constant velocity v. Does the clock of an observer in S' frame go slow relative to the clock of an observer in S-frame? Justify your answer.   6

b) If u and v are two velocities in the same direction and V be their resultant velocity, given by

$$\tanh^{-1}\frac{V}{c} = \tanh^{-1}\frac{u}{c} + \tanh^{-1}\frac{v}{c}$$

then deduce the law of composition of velocity.   4

4. Derive an expression for K.E. in STR. Deduce from it the expression for K.E. in classical mechanics. Hence deduce the energy-mass relation in relativistic.   4+2+4

5. a) Deduce the transformation laws for the momentum components.   3

b) A rod is of length l_0 in its rest frame. In another inertial frame S it is oriented in a direction of the unit vector $\vec{e}$ and is moving with a velocity $\vec{v}$. Show that the length of the rod in the 2nd frame is

$$l = \frac{l_0\sqrt{c^2 - v^2}}{\sqrt{c^2 - v^2 \sin^2\theta}}$$

where $v = |\vec{v}|$ and θ is the angle between $\vec{e}$ and $\vec{v}$.

7

6. a) A circular ring moves parallel to its plane relative to an inertial frame S. Determine the shape of the ring relative to S-frame.   4

b) Show that $\Box = \dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2} - \nabla^2$ is invariant under Lorentz transformation.   6