BACHELOR OF SCIENCE EXAMINATION, 2022
(DISCIPLINE SPECIFIC ELECTIVE)
Mathematics(Honours), Semester VI
DSE 4: Combinatorics & Graph Theory

Time : 2 hours Full Marks : 40

(Symbols have usual meanings, if not mentioned otherwise)

PART–I (20 marks)

Attempt any ***two*** questions from this part $10 \times 2 = 20$

1. (a) Show that every self-complementary graph has $4n$ or $4n+1$ points where n is a positive integer.

   (b) If a graph G has p points and $\delta(G) \geq (p-1)/2$, then prove that G is connected. $5+5=10$

2. (a) Prove that a graph G is unicyclic if and only if $G-x$ is a tree for some edge x of G.

   (b) Prove: If every pair of points of a graph G are connected by a spanning path and $p \geq 4$, then G is 3-connected. $5+5=10$

3. (a) Prove that a graph G is eulerian if and only if G is connected and every block of G is eulerian.

   (b) Prove that every 5-connected planar graph has at least 12 points. Construct one example of such a plane graph. $5+5=10$

PART–II (20 marks)

Attempt **Question 4** and any ***two*** from the rest.

4. (a) What do mean by *boolean expression* and *boolean function*? Is the function

   $$T(w,x,y,z) = \sum(0,1,2,3,4,6,7,8,9,11,15)$$

   boolean? If the answer is negative then give your explaination supporting the answer, otherwise

   i. find all prime implicants and indicate which are essential; and

   ii. find a minimal expression for T and determine whether it is unique.

   (b) Show that a lattice is distributive if and only if for any elements a, b, c in the lattice

   $$(a \vee b) \wedge c \leq a \vee (b \wedge c). \qquad 6+4=10$$

5. Find the particular solution of the recurrence relation $a_r - 3a_{r-1} + 2a_{r-2} = 2^r + 3$. 5

6. Let a_r denote the number of ways to divide r identical balls into four distinct urns so that each urn has an odd number of balls that is larger than or equal to three.

   (a) Determine the generating function $A(z)$.

   (b) Determine a closed-form expression for a_r. 5

7. Using Pigeonhole Principle, show that among $n+1$ integers less than or equal to $2n$ there are two of them which are relatively prime. 5