where S_t, I_t are the number of susceptible and infected individual at time t and b, γ, β are positive parameters such that $\beta < 1,\ b + \gamma < 1,\ N = S_t + I_t$.

Find all fixed points of the model. Determine the stability criteria of the endemic fixed point. 5

6. Define hyperbolic equilibrium point of the system $\dot{x} = f(x),\ x \in \mathbb{R}^2$. 1

B. Sc. Mathematics (Hons.) Examination, 2022

(3rd Year, 2nd Semester)

Bio Mathematics

Paper – DSE-4B

Time : Two hours Full Marks : 40

Use a separate Answer scripts for each part.

The figures in the margin indicate full marks.

(Symbols/Notations have their usual meanigns)

Part – I (Marks : 24)

Answer any *Three* questions. **8×3=24**

1. Derive the explicit solution of a single species model of the form $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$, where the symbols and parameters have their usual meanings. Also explain the result graphically for different values of K.

2. Deduce classical Lotka-Volterra Model of Prey-perdator interaction stating its underlying assumptions. Show that all solutions of the model are periodic and structurally unstable.

3. Prey-Predator interaction is taken in the form :

$$\frac{dx}{dt} = xg(x) - yp(x)$$
$$\frac{dy}{dt} = yq(x),$$

where the symbols have their usual meanings. Determine the steady states and discuss their qualitative behavior.

4. Write down the growth equations of two competing populations for common rersource. Determine the steady states and discuss the stability properties of the interior equilibrium point.

5. Give the mathematical formulation of the deterministic model of general epidemic and hence discuss the Kermack-McKendrick threshold theorem.

Part – II (Marks : 16)

Answer Q.No. 6 and any *Three* questions from the rest.

1. Determine the nature of the system $\dot{X} = AX$ with $A = \begin{pmatrix} 1 & -\alpha \\ 4 & -3 \end{pmatrix}$, $X = (x, y)^T$, $\alpha \in R$. Draw the phase portrait. 5

2. a) Define Lyapunov stable equilibrium point of the system $\dot{x} = f(x),\ x \in R^2$.

   b) Define Lyapunov function for the system $\dot{x} = f(x),\ x \in R^2$.

   Construct a suitable Lyapunov function to determine the nature of the equilibrium point (0, 0) of the system

$$\dot{x} = -y^3$$
$$\dot{y} = x^3.$$

1+4

3. Determine the nature of the equilibrium point(s) of the system

$$\dot{x} = \mu x + x^2$$
$$\dot{y} = -y\,(\mu \in R)$$

Draw the phase portrait and bifurcation diagram. 5

4. a) Consider the first order difference equation $x_{n+1} = f(x_n)$ with $\bar{x}$ as a fixed point. Determine nature of the fixed point $\bar{x}$ if $f'(\bar{x}) = \pm 1$. Justify your answer.

   b) Find period of the point $x_0 = 0$ for the difference equation $x_{n+1} = \dfrac{(2 - x_n)(3x_n + 1)}{2}$.

   c) Using Schwarzian derivative, determine nature at $x = -2$ of the map $f(x) = x^2 + 3x$ on [–3 3].

2+1+2

5. Consider the discrete SI-type epidemic model

$$S_{t+1} = S_t - \frac{\beta S_t I_t}{N} + b(N - S_t)$$

$$I_{t+1} = I_t(1 - \gamma - b) + \frac{\beta S_t I_t}{N},$$