Part – II (Statistics-II)

Attempt ***any two*** questions.

Each question carries Ten (10) marks: 2×10=20

1. Define the following terms with examples:

    a) Simple and Composite Statistical Hypotheses

    b) Probability of Type I and Type II Errors

    c) Critical Region.

    d) Power of Test 3+3+2+2

2. Let a random sample of size 15 be drawn from a uniform $(0, \theta)$ population, where $\theta > 0$ is an unknown parameter. Find a maximum likelihood statistic based on the sample above. Also find the probability density function of this statistic.

3. Define a Most Powerful (MP) size α test. State and prove how will you find an MP size α test for testing the following pair of simple hypotheses:

    $H_0 : \theta = \theta_0$

    vs

    $H_1 : \theta = \theta_1$

    where θ is a real unknown parameter and $\theta_0 \neq \theta_1 \in \mathbb{R}$.

Ex/SC/UG/GE/STAT/02/2022

Bachelor of Science Examination, 2022

(2nd Year, 2nd Semester)

Paper – GE 4

Statistics - II

Time : Two hours Full Marks : 40

Use separate answer script for each Part.

Symbols and notaions have their usual meanings.

Part – I

Answer ***any four*** questions. 5×4

All questions carry equal marks.

1. a) Define the term 'consistency' of the estimators with example. If $\hat{\theta}_n$ is an unbiased estimate of θ_n with variance σ_n^2 and $\theta_n \to \theta$ and $\sigma_n \to 0$ as $n \to \infty$, then prove that $\hat{\theta}_n$ is a consistent estimate of θ.

    b) Define unbiased estimate of a population parameter. Show that, if $x_1, x_2, ..., x_n$ are the random sample of size n from a population with variance σ^2, then $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$ is an unbiased estimate of σ^2 where $\overline{x}$ is the sample mean.

2. a) Prove that for Cauchy's distribution, not sample mean but sample median is a consistent estimator of the population mean.

[Turn over

b) A random sample $(x_1, x_2, x_3, x_4, x_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ.

i) $t_1 = \frac{1}{5}(x_1 + x_2 + x_3 + x_4 + x_5)$

ii) $t_2 = \frac{x_1 + x_2}{2} + x_3$

iii) $t_3 = \frac{2x_1 + x_2 + \lambda x_3}{3}$

where λ is such that t_3 is an unbiased estimator. Find λ. State giving reasons, the estimator which is best among t_1, t_2 and t_3.

3. If T_1 is a minimum variance unbiased estimator for θ and T_2 is any other unbiased estimator of θ with efficiency e, then prove that the correlation coefficient between T_1 and T_2 is given by $\rho = \sqrt{e}$.

4. Define the term 'sufficiency' of an estimator with illustration.

State Factorization theorem (Neymann).

Let $x_1, x_2, ..., x_n$ be a random sample from a uniform population on $[0, \theta]$. Find a sufficient estimator for θ using this theorem.

5. a) Consider the Logistic trend curve: $U = \frac{K}{1 + e^{a+bt}}$

What are the main properties of this curve? Estimate the parameters by the method of three selected points.

b) Given the three selected points U_1, U_2 and U_3 corresponding to $t_1 = 2$, $t_2 = 30$ and $t_3 = 58$ as follows:

$t_1 = 2$, $U_1 = 55{\cdot}8$

$t_2 = 30$, $U_2 = 138{\cdot}6$

$t_3 = 58$, $U_3 = 151{\cdot}8$

Fit the Logistic curve by the method of selected points.

6. a) Define the following index numbers and discuss their merits and demerits:

i) Laspeyre's Index Number,

ii) Paasche's Index Number,

iii) Fisher's ideal Index Numbers.

b) Given the data

	Commodities	
	A	B
p_0	1	1
q_0	10	5
p_1	2	X
q_1	5	2

where p and q respectively stand for price and quantity and subscripts stand for time period. Find X, if the ratio between Laspeyre's (L) and Paaschi's (P) index numbers is $\frac{L}{P} = \frac{28}{27}$.