BACHELOR OF SCIENCE EXAMINATION, 2022

(1st Year, 2nd Semester)

MATHEMATICS - I

PAPER – GE-2

Time : Two hours Full Marks : 40

Use separate answer script for each Part.

Symbols and notaions have their usual meanings.

Part – I (Marks : 16)

Answer ***any four*** questions: 4×4=16

1. Prove that the pairs of lines joining the origin to the point of intersection of the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by the line $lx+my+n=0$ are perpendicular to each other if $\frac{a^2+b^2}{l^2+m^2}=\frac{a^2b^2}{n^2}$.

2. Show that the locus of the point (r,θ) having equation $r^2-ra\cos 2\theta\sec\theta-2a^2=0$ consists of a straight line and a circle.

3. Find the pole of the focal chord of the parabola $y^2=4ax$ passing through $(9a,6a)$.

4. A sphere of constant radius *k* passes through the origin *O* and meets the axis in *A*, *B*, *C*. Prove that the locus of the centroid of triangle ABC is the sphere $9\left(x^2+y^2+z^2\right)=4k^2$.

5. Find the equation of the circle cutting $x^2+y^2+2x-9=0$, $x^2+y^2-8x-9=0$ orthogonally and touching $y=x+4$.

Part – II (Marks : 12)

Answer ***any three*** questions: 3×4=12

1. a) If $y=e^{a\sin^{-1}x}$ then show that
$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-\left(n^2+a^2\right)y_n=0$$

b) Find the values of *a* and *b* such that
$$\lim_{x\to 0}\frac{a\sin 2x-b\sin x}{x^3}=1$$

c) Expand sin*x* in powers of *x* in infinite series. State the condition under which the expansion is valid.

d) If $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$

Show that both partial derivatives exist at (0, 0) but *f* is not continuous at (0, 0).

e) If $u=xf\left(\dfrac{y}{x}\right)+g\left(\dfrac{y}{x}\right)$, then find the value of $x^2\dfrac{\partial^2u}{\partial x^2}+2xy\dfrac{\partial^2u}{\partial x\partial y}+y^2\dfrac{\partial^2u}{\partial y^2}$.

Part – III (Marks : 12)

Answer ***any three*** questions: 3×4=12

1. State and prove the Fundamental Theorem of Integral Calculus. 4

2. Prove that $\frac{1}{2}<\int_0^1\frac{dx}{\sqrt{4-x^2+x^3}}<\frac{\pi}{6}$ 4

3. Show that

a) $B(m,n)=B(n,m)$

b) $B(m,n)=2\int_0^{\frac{\pi}{2}}(\sin\theta)^{2m-1}(\cos\theta)^{2n-1}\,d\theta$ 1+3

4. a) Test the convergence of the integral $\int_0^1\frac{dx}{x^{2/3}}$

b) Evaluate, if possible, $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$ 2+2

5. Evaluate $\iint\left(x^2+y^2\right)dxdy$ over the region enclosed by the triangle having its vertices at (0,0), (1,0), (1,1). 4