3. a) State a necessary and sufficient condition for a differential equation of first order and first degree to be exact.

b) Solve: $\left(x^3+xy^4\right)dx+2y^3dy=0$

c) Solve by method of undetermined coefficients

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}-\frac{1}{x^2}y=\log x \quad (x>0)$$ 1+4+5

Part – II (Marks : 20)

Answer ***all*** questions. 2×10=20

1. Solve the following differential equation near $x=0$ using Frobenius method

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x-1)y=0$$

Or

$$x(1-x)\frac{d^2y}{dx^2}+\left[c-(a+b+1)x\right]\frac{dy}{dx}-aby=0$$ 10

2. Find the general solution of the following linear system

$$\frac{dx}{dt}=\begin{pmatrix}7&0&4\\8&3&8\\-8&0&-5\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

Or

$$\frac{dx}{dt}=\begin{pmatrix}3&1&-1\\1&3&-1\\3&3&-1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$ 10

Ex/SC/MATH/UG/CORE/TH/04/2022

B. Sc. Mathematics (Hons.) Examination, 2022

(1st Year, 2nd Semester)

Differential Equation

Paper – Core-4

Time : Two hours Full Marks : 40

Use separate answer script for each Part.

Symbols and notaions have their usual meanings.

Part – I (Marks : 20)

Answer ***any two*** questions. 2×10=20

1. a) Define general solution of a differential equation.

b) Show that $\frac{1}{F(D)}e^{mx}V(x)=e^{mx}\frac{1}{F(D+m)}V(x)$.

c) Solve and find the singular solution of the differential equation

$$(px-y)(x-py)=2p\left(p=\frac{dy}{dx}\right).$$ 1+4+5

2. a) Define an orthogonal trajectory.

b) If $y=u$ is a solution of $\left(D^2+PD+Q\right)y=0$, then show that another solution $y=v$ is given by $v=u\int\frac{W(u,v)}{u^2}du$, where $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}-\frac{1}{x^2}y=\log x \ (x>0)$ is the Wronskian of u and v.

c) Solve: $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$ 1+4+5