Bachelor of Arts Examination, 2022

(1st Year, 1st Semester)

HISTORY

Course – General Elective

[Mathematics]

Time : Two Hours | Full Marks : 30

Answer ***any five*** questions.

All questions carry equal marks.

1. a) Define equivalence relation with examples. Show that the intersection of two equivalence relations in a set is an equivalence relation on the set, but the union of two equivalence relations on a set is not necessarily an equivalence relation on the set.

   b) If R is a relation from a set A to a set B, define its inverse relation R^{-1}. Let R be a relation on a set A. Then show that R is a symmetric relation on A if and only if $R = R^{-1}$. 4+2

2. Prove that a monotone sequence of real numbers is convergent if and only if it is bounded.

   Let $e_n = \left(1+\frac{1}{n}\right)^n$ for $n \in N$. Show that the sequence $E = (e_n)$ is bounded and increasing and hence it is convergent. Find also the limit of this sequence. 3+3

[Turn over

3. a) State Rolle's theorem. Using this theorem prove the Cauchy mean value theorem. Show that the Lagrange's mean value theorem is a particular case of Cauchy mean value theorem.

b) Show that $x < \sin^{-1} x \leq \frac{x}{\sqrt{1-x^2}}$ if $0 \leq x < 1$. 4+2

4. State and prove Taylor's theorem with Lagrange's form of remainder. 6

5. a) If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(\theta h)$, $0 < \theta < 1$, find θ when $h = 1$ and $f(x) = (1-x)^{\frac{5}{2}}$.

b) Expand $\log(1+x)$ in an infinite series using Maclaurin's theorem. Deduce the condition of x for which the expansion is valid. 3+3

6. Evaluate the following limits:

i) $\lim_{x \to 0} \frac{(1+x)^{\frac{1}{x}} - e + \frac{ex}{2}}{x^2}$

ii) $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$ 4+2

7. a) State and prove Euler's theorem for two variables.

b) If $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$, then prove that

$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} \right)^2 u = -\frac{\sin u \cos 2u}{4\cos^3 u}$$ 3+3