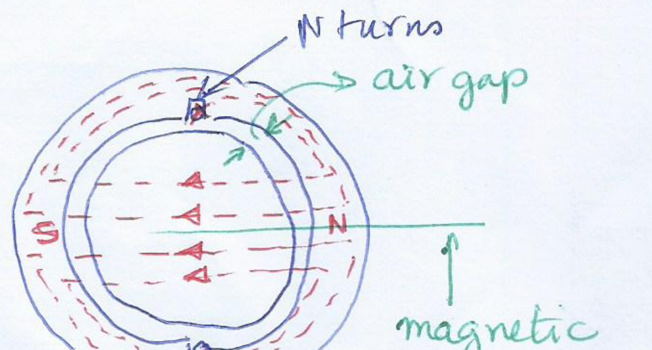
## Mmf produced by Distributed Winding.

The mmf produced by a winding depends on the winding arrangement and the winding current. Our object is to investigate the mmf produced by a winding distributed in the slots along the air gap periphery.

In order to make the analysis simplier the change in mmf over the slot portion is taken as stepped at the middle of the slot width. First let us consider a single coil.

### mmf. of a coil:



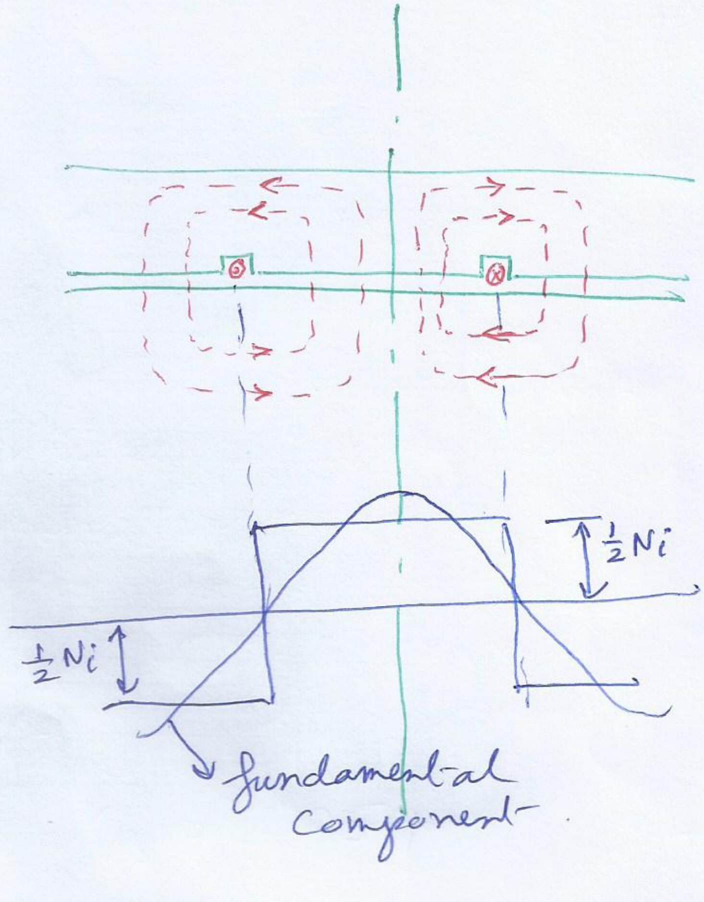




Fig. 5.

Let us consider a priliminary two pole cylindrical rotor machine. The stator consists of N turns carrying i amp. of current. In order to determine the coil mmf the following assumptions are made:

(i) Stator and rotor iron is infinitly permeable compare to air. So, it can be assumed that the reluctance offered to flux path is offered by the air gap.

(ii) The magnetic flux lines cross the air gap radially. This is permissible because air gap is very small.

According to Ampere's circuital law, the magnetic field strength H and the total current enclosed are given by

$$\oint H\,dl = \text{total current enclosed.}$$

So, we have

$$2gH = Ni$$

$$gH = \frac{1}{2} Ni$$

Thus the magnetic potential difference $gH$ across each air gap is $\frac{1}{2} Ni$. So, the variation of magnetic potential difference along the air gap periphery is rectangular

If the current is d.c. the magnitude a mmf wave does not vary with time and space. If the current is a.c., the amplitude of mmf wave varies with time but not with space, i.e., the air gap mmf will be time variant but space invariant (a pulsating field)

If the rectangular wave is represented by a Fourier series, then the peak of the fundamental sine component is given by

$$F_{1P} = \frac{4}{\pi} \frac{Ni}{2} \text{ ATs per pole.}$$

The axis of the peak mmf is always along the magnetic axis of the coil.

Now is 'i' is alternating, then $F_1$ varies will value of 'i'. It is maximum when 'i' is maximum, So,

$$F_{1pm} = \frac{4}{\pi} \frac{N\sqrt{2}I}{2} \text{ ATs per pole. for a 2 pole machine}$$

and for a P-pole machine

$$F_{1pm} = \frac{4}{\pi} \frac{N\sqrt{2}I}{P} \text{ ATs per pole.}$$

# M.m.f. of single-phase distributed winding.

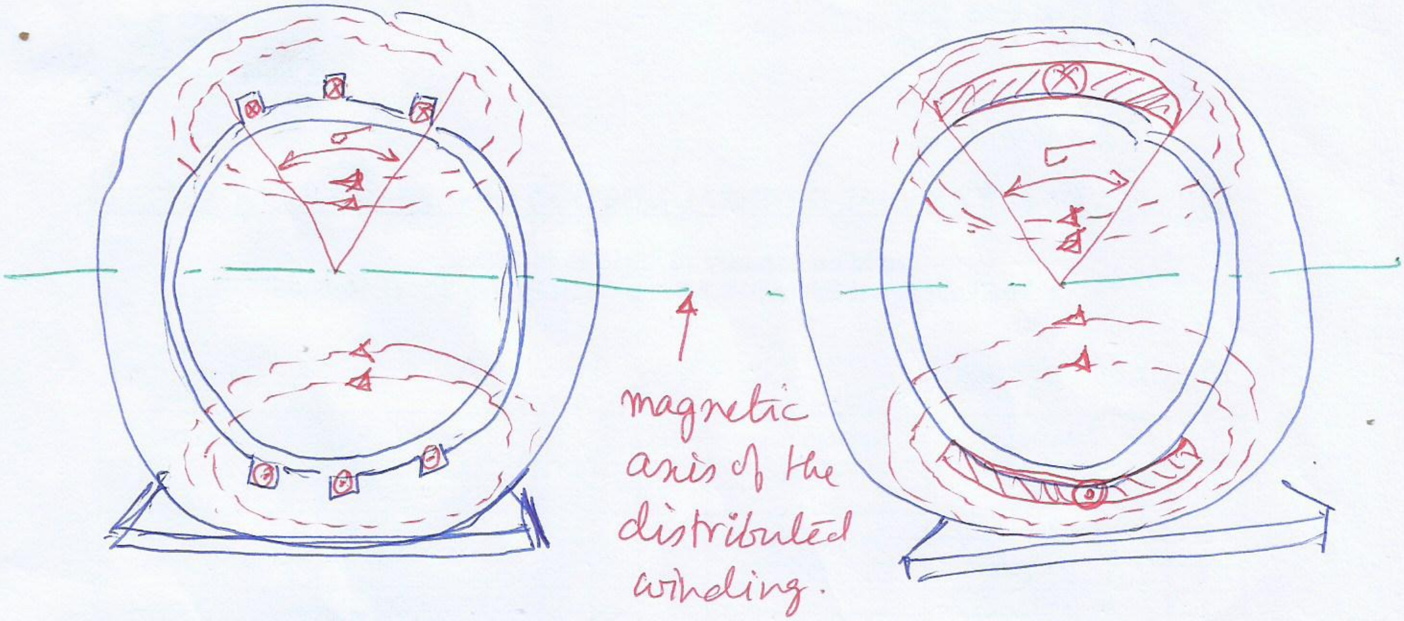



Fig 6.

Now, let us consider the same preliminary two pole machine, but with its stator turns distributed in slots as shown. This distributed winding can be considered as the combination of concentric windings and the mmf can be drawn as shown in fig 7.
It is observed that the effect of winding distribution has changed the ~~shape~~ of mmf wave. It is now a stepped wave.

Now, normally number of slots are large. So, we can neglect the steps and consider the mmf wave to be trapizoidal

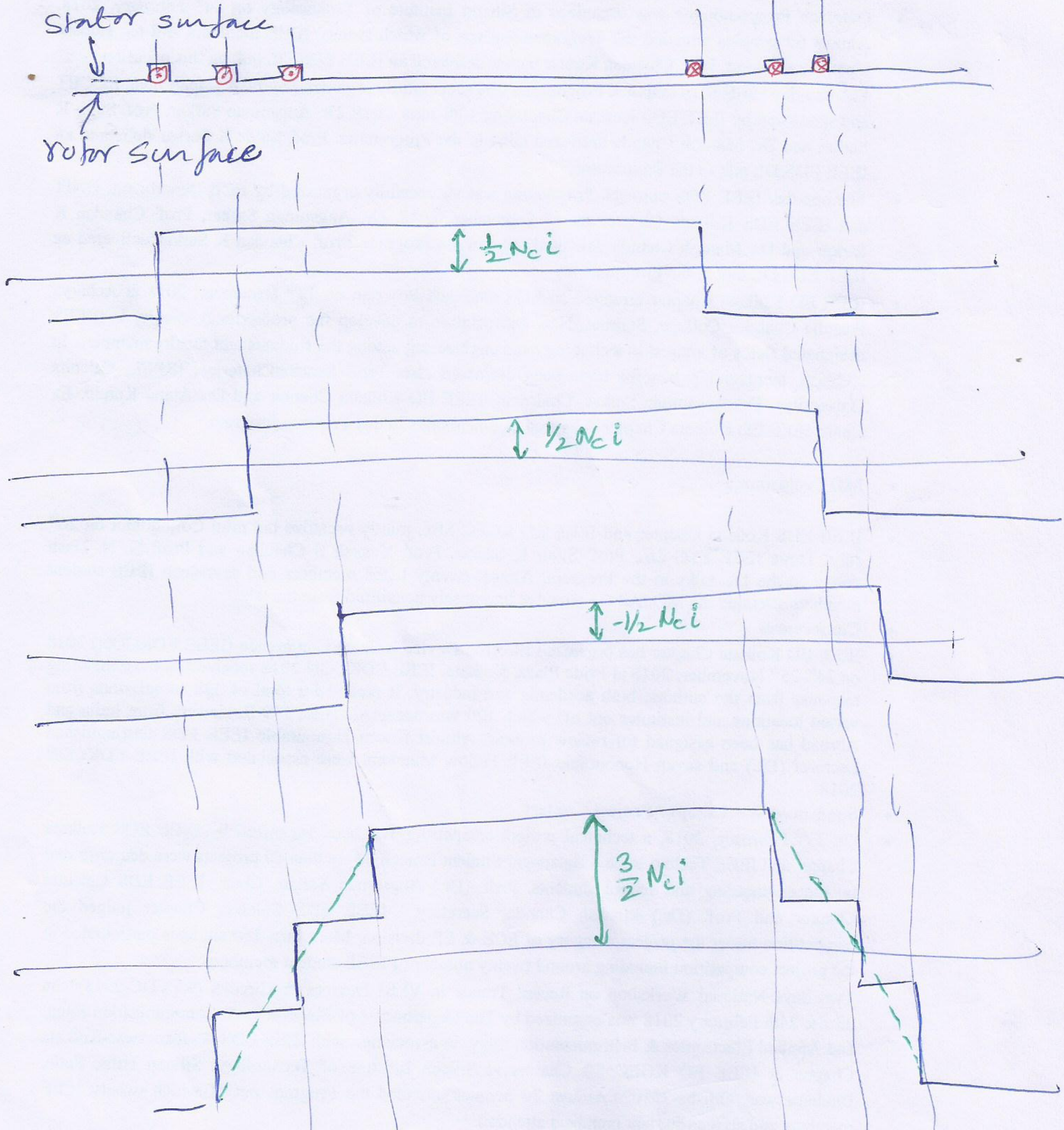

Stator surface
rotor surface
$\frac{1}{2} N_c i$
$\frac{1}{2} N_c i$
$-\frac{1}{2} N_c i$
$\frac{3}{2} N_c i$

Fig 7.

Actually the mmf distribution depends on (i) nature of slots (ii) nature of winding distribution and (iii) the current. If the winding is sinusoidally distributed then the equivalent current sheet is also sinusoidally distributed.

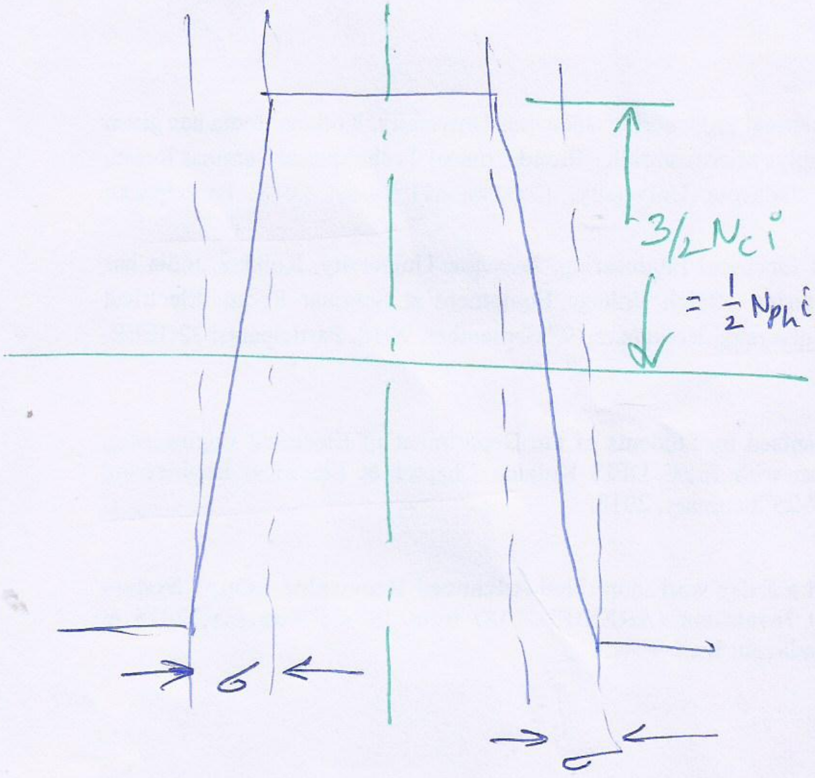



Fig 8

$$F_{1p} = \frac{4}{\pi}\left[\int_0^{\sigma/2} \frac{N_{ph} i}{\sigma}\,\theta \sin\theta \, d\theta + \int_{\sigma/2}^{\pi/2} \frac{N_{ph} i}{2} \sin\theta \, d\theta\right]$$

$$= \frac{4}{\pi}\,\frac{N_{ph} i}{2}\cdot\frac{\sin \sigma/2}{\sigma/2} \text{ AT per pole}$$

$$= \frac{4}{\pi}\cdot K_d \cdot \frac{N_{ph} i}{2} \text{ AT per pole}$$

So, the effect of distributing the turns in the various slots has resulted in the introduction of distribution factor $K_d$ in the expression. So, if we have a short pitched coil then

$$F_{1p} = \frac{4}{\pi} K_d K_p \frac{N_{ph} i}{2} \text{ AT per pole.}$$

If the current 'i' is alternating, then at the instant $i = 0$, $F_{1p} = 0$ and when $i = I_{max}$ $F_{1p}$ is also maximum.

$$F_{1pm} = \frac{4}{\pi} K_w \frac{N_{ph}\sqrt{2}I}{2} \text{ ATs/pole}$$

for a P pole machine

$$F_{1pm} = \frac{4}{\pi} K_w \frac{N_{ph}\sqrt{2}I}{P} \text{ ATs/pole.}$$

Axis of $F_{1p}$ is always along the magnetic axis of the winding.

At any point of time $t_1$, the current is say '$i_1$', the mmf waveform along the air gap periphery is sinusoidal with peak value $F_{p1}$ proportional to current '$i_1$' (we neglecting space harmonics) If the space angle '$\alpha$' along the air gap periphery is measured from the axis of the coil, the m.m.f $F_1$ at any space angle $\alpha$ can be written as

$$F_1 = F_{p1} \cos\alpha \quad \text{at } \omega t_1$$
$$= F_{p2} \cos\alpha \quad \text{at } \omega t_2$$
$$= F_{p3} \cos\alpha \quad \text{at } \omega t_3 \quad \ldots \text{so on.}$$

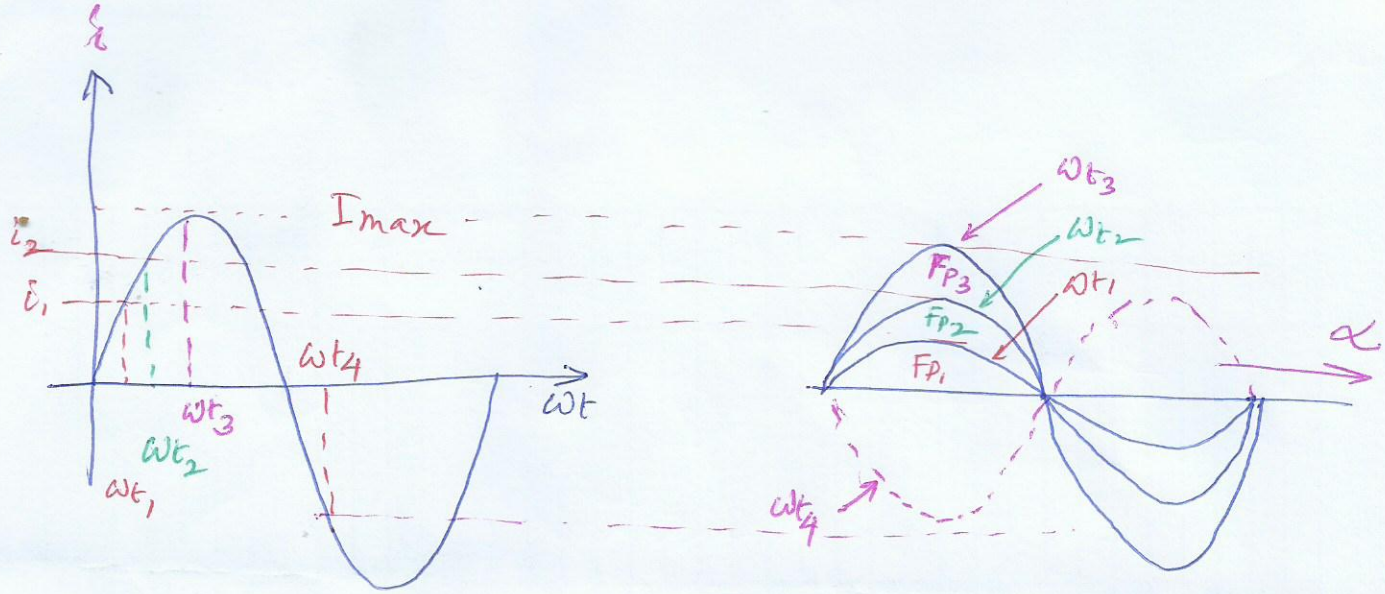



Fig a Mmf produced by a single phase winding

If we consider $i_1 = I_{max} \sin \omega t_1$, then corresponding value of $F_{P1} \propto$ (no. of turns) $(I_{max} \sin \omega t_1)$

$$F_{P1} = F_m \sin \omega t_1$$

$$F_{P2} = F_m \sin \omega t_2$$

$$F_{P3} = F_m \sin \omega t_3 \quad \ldots \text{ so on}$$

So, we get

$$F_1 = F_{P1} \cos\alpha = F_m \sin \omega t_1 \cos\alpha$$

$$F_2 = F_{P2} \cos\alpha = F_m \sin \omega t_2 \cos\alpha$$

$$F_3 = F_{P3} \cos\alpha = F_m \sin \omega t_3 \cos\alpha$$

So, we can write

$$F(\alpha, t) = F_m \sin \omega t \cos\alpha$$

# Rotating Magnetic Field.

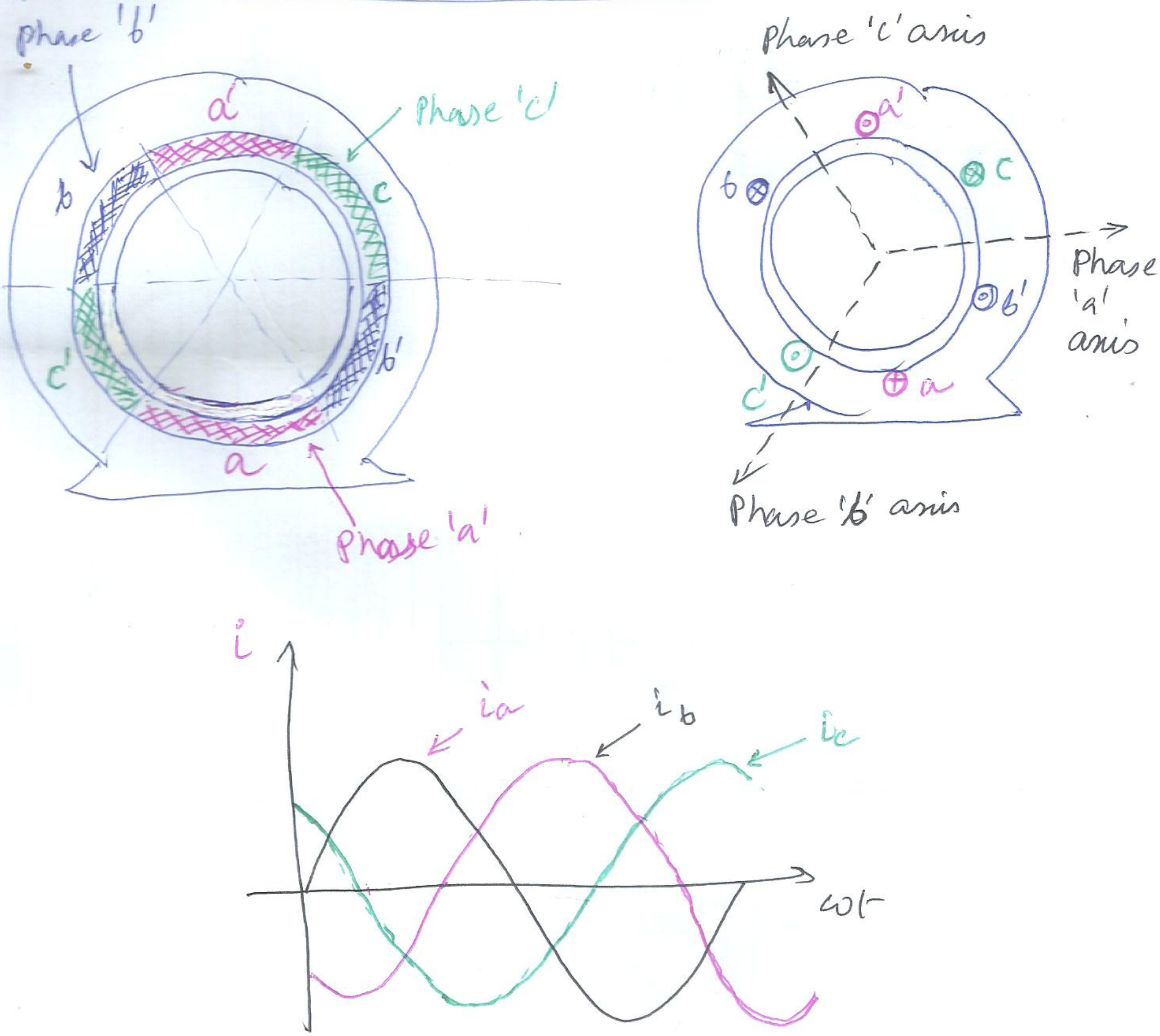



Fig 10 Three phase distributed winding

IF phase 'a' alone carries current, it creats a pulsating magnetic field directed along its axis. When phase a, b and c are carrying current they creats three pulsating fields $F_a$, $F_b$ and $F_c$ which combine together to give the resultant mmf $F_R$.

If the time origin is taken at the instant phase a current is zero and becoming positive then

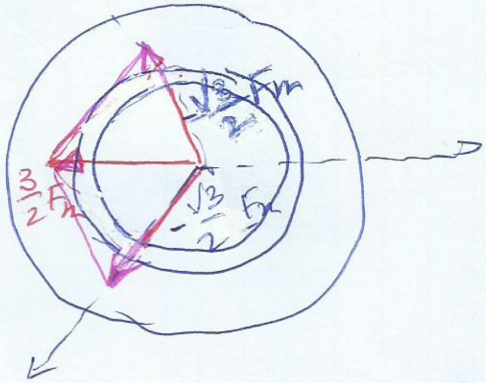
$$i_a = I_m \sin\omega t$$

$$i_b = I_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$i_c = I_m \sin\left(\omega t - \frac{4\pi}{3}\right)$$

at $\omega t = 0$ $\quad i_a = 0 \quad i_b = i_c = -\frac{\sqrt{3} I_m}{2}$

$F_a \propto i_a \quad F_b \propto i_b \quad F_c \propto i_c$




at $\omega t = 0$

$$\therefore F_a = F_m \sin\omega t$$

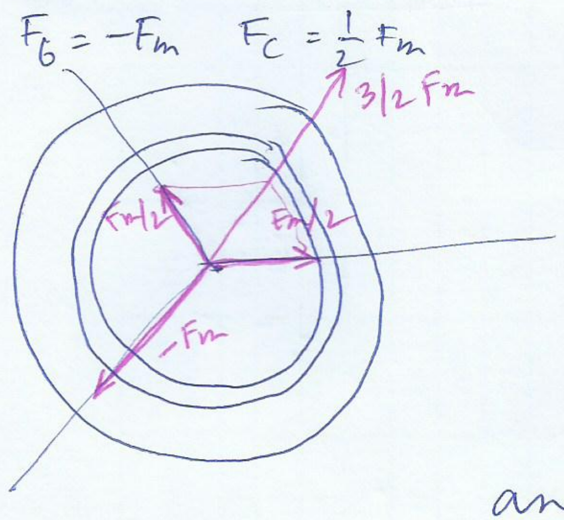
$$F_b = F_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

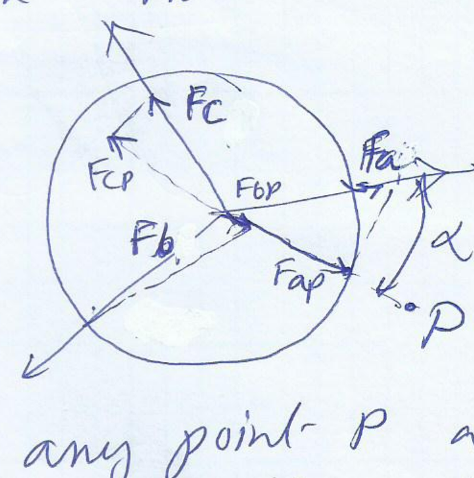
$$F_c = F_m \sin\left(\omega t - \frac{4\pi}{3}\right)$$

$F_m = N_{ph} I_m.$

at $\omega t = 30^\circ$ $\quad i_a = \frac{1}{2} F_m \quad \therefore F_a = \frac{1}{2} F_m$

$F_b = -F_m \quad F_c = \frac{1}{2} F_m$




At any point P at an angle 'α' with the axis of phase 'a' the resultant mmf will be the summation of the components of $F_a$, $F_b$ & $F_c$

Component of $F_a$ $= F_{ap} = F_a \cos\alpha = F_m \sin\omega t \cos\alpha$

Component of $F_C = F_{cp} = F_C \cos\left(\alpha - \frac{4\pi}{3}\right)$

$$= F_m \sin\left(\omega t - \frac{4\pi}{3}\right) \cos\left(\alpha - \frac{4\pi}{3}\right)$$

$$F_R(\alpha, t) = F_{ap} + F_{bp} + F_{cp}$$

$$= F_m \Big[ \sin\omega t \cos\alpha + \sin\left(\omega t - \frac{2\pi}{3}\right)\cos\left(\alpha - \frac{2\pi}{3}\right) + \sin\left(\omega t - \frac{4\pi}{3}\right)\cos\left(\alpha - \frac{4\pi}{3}\right)$$

$$\sin\omega t \cdot \cos\alpha = \frac{1}{2}\left[\sin(\omega t - \alpha) + \sin(\omega t + \alpha)\right]$$

$$\therefore F_R(\alpha, t) = \frac{F_m}{2}\Big[\sin(\omega t - \alpha) + \sin(\omega t + \alpha) + \sin(\omega t - \alpha) + \sin\left(\omega t + \alpha - \frac{4\pi}{3}\right) + \sin(\omega t - \alpha) + \sin\left(\omega t + \alpha - \frac{8\pi}{3}\right)\Big]$$

$$= \frac{3F_m}{2}\sin(\omega t - \alpha) + \frac{F_m}{2}\Big[\sin(\omega t + \alpha) + \sin\left(\omega t + \alpha - \frac{4\pi}{3}\right) + \sin\left(\omega t + \alpha - \frac{2\pi}{3}\right)\Big]$$

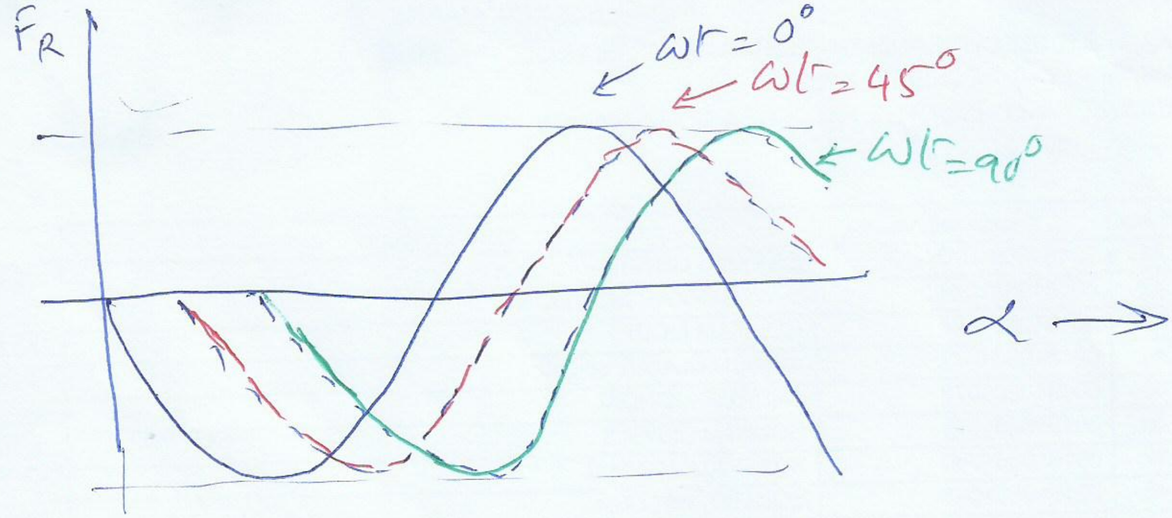
$$\left[\frac{8\pi}{3} = 2\pi + \frac{2\pi}{3} \Rightarrow \frac{2\pi}{3}\right]$$

$$= \frac{3}{2} F_m \sin(\omega t - \alpha)$$

at $\omega t = 0$ $\quad F_R(\alpha, t) = \frac{3}{2} F_m \sin(-\alpha)$

at $\omega t = 30^\circ$ $\quad F_R(\alpha, t) = \frac{3}{2} F_m \sin(30^\circ - \alpha)$

at $\omega t = 45^\circ$ $\quad F_R(\alpha, t) = \frac{3}{2} F_m \sin(45^\circ - \alpha)$




Thus it is seen that the resultant mmf wave is of constant amplit i.e $\frac{3}{2} F_m$ and is travelling in the air gap at a speed determined by the time angular frequency $\omega$

The individual phase mmf is not rotating but pulsating in nature. The combined action of the three mmfs results in constant amplitude rotating mmf. wave. In general rotating mmf is produced when any balanced polyphase winding is supplied from balanced polyphase supply.

Now $F_m$ is the mmf per pole corresponding to the maximum current $I_m$ and is given by

$$F_m = \frac{4\sqrt{2}}{\pi} K_w \frac{N_{ph} I}{P} \quad \text{ATs/pole}$$

$$\therefore F_R = \frac{3}{2} F_m$$

$$= \frac{3}{2} \frac{4\sqrt{2}}{\pi} K_w \frac{N_{ph} I}{P} \text{ ATs/pole.}$$

$$= 2.7 \, K_w \frac{N_{ph} I}{P} \text{ ATs/pole.}$$ for 3 phase machine.

for m - phase machine

$$F_R = m \frac{2\sqrt{2}}{\pi} K_w \frac{N_{ph} I}{P} \text{ ATs/pole.}$$

$$= 0.9 \frac{m K_w N_{ph} I}{P} \text{ ATs/pole.}$$

We have considered only the fundamental components. In practice there will be some space harmonics. The resultant wave will consists of

(i) Constant amplitude fundamental mmf wave rotating at synchronous speed.

(ii) Space harmonics of the order of 3, 9, 15, ... (triplen frequency) harmonics are absent

(iii) Constant amplitude fifth harmonic wave with amplitude equal to 1/5 of that of the fundamental rotates in a direction opposite to that of the fundamental at a speed of 1/5th of that of the

(iv) Constant amplitude seventh harmonic wave with amplitude equal to 1/7th that of the fundamental, rotates in the direction of the fundamental at a speed 1/7th that of the synchronous speed corresponding to the fundamental frequency.

In general

$$F_n = \frac{1}{n}\left[2.7\, K_{wn} \frac{N_{ph} I}{P}\right] \text{ ATs/pole.}$$

where $n$ = the order of the harmonics

$K_{wn} \Rightarrow$ the winding factor corresponding to the ~~the~~ $n^{th}$ harmonics

$n = 6K \pm 1$ where $K = 1, 2, 3$ - - -

Space harmonics of the order of $6K+1$ (7, 13, 19 - - -) have amplitude equal to $\frac{1}{6K+1} F_1$ and rotate at a speed $\frac{1}{6K+1}$ of the fundamental synchronous speed in the +ve direction.

Space harmonics of the order of $6K-1$ (5, 11, 17, - - -) have amplitude equal to $\frac{1}{6K-1} F_1$ and rotate at a speed equal to $\frac{1}{6K-1}$ times the fundamental synchronous speed in the -ve direction.