Phase Relation between a Flux and the Voltage It Induces:

The difference in phase between a flux and the voltage it induces, when both are considered with respect to a coil and when both are considered with respect to an inductor, should not be overlooked. An inductor is one of the two active sides of each turn of a coil. Its length is equal to the length of that portion of the coil side which actually cuts flux.

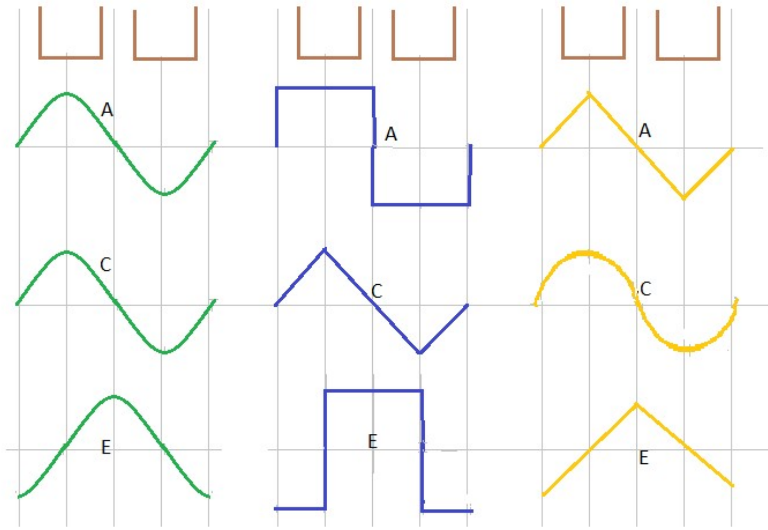
Assume that the two active sides of a coil is 180^o electrical apart. Under this condition when the coil is directly over a pole and contains a maximum flux, its two active sides are midway between the poles and are in zero fields. They are cutting no flux and voltage induced in them are zero. When the coil moves forward 90^o electrical , the flux through it is zero, but the inductors are now directly under the centers of opposite poles and are in the strongest part of the fields. The voltages induced in the two coil sides have maximum vales and are in the same direction around the coil. The effective voltage in the coil is always equal to the vector difference between the effective voltages induced in its two sides.

It follows that, while a voltage in a coil is in time quadrature with respect to the flux through it, the voltages in the inductors and the intensity of the fields or the flux densities at the inductors are in time phase. The inductors move at uniform speed across the field, and the voltage induced in them must at every instant be proportional to the strength of the field in which they are moving at the instant in question. The voltage is equal to the flux density multiplied by the length of the inductor and the component of its velocity at right angles to the field.

Shape of Flux and Voltage Waves When the Coil Sides are 180^o Electrical Apart:

If the sides of the coil are 180^o electrical apart and the distribution of the air-gap flux which they cut is a sine function of the distance measured from a point midway between the poles, the variation of flux through the coil with respect to time is a cosine function and the voltage induced in the coil is sinusoidal. If the flux has any other distribution, the time variation of the flux through the coil has a wave form different from that of the space distribution of the flux. The wave form of the voltage induced in the coil however is the same as the waveform of the space distribution of the flux in the air gap, since the voltages in the two coil sides are opposite and at each instant proportional to the strength of the fields in which they move.

Figure (a) shows curves of the space variation of flux density in the air gap, the corresponding time curves of flux through a coil, and the voltage induced in a coil with sides 180^o electrical apart, by a sine, a rectangular and a triangular space distribution of the air-gap flux density. In curve C and E of fig. (a) , the position of the coil is considered to be fixed by the position of its centre with respect to the centers of the poles. Note that the voltage curves are displaced by 90^o electrical with respect to the flux density curves and that the voltage curve in each has the same shape as its corresponding curve of flux density. This is always the case for a full pitch coil and for each phase of a concentrated full pitch winding.



A Air gap Flux density; C Flux through the Coil; E Voltage in the coil

Fig, (a)

If the non-sinusoidal flux density curve is split into its fundamental and harmonic components, the effective flux produced by each component may be calculated by $= 4.44Nf\phi_m$. the frequency of the qth harmonics is q times the frequency of the fundamental. Let the space distribution of flux, i.e., the flux density, in the air gap of a synchronous generator be

$$B = B_1 sin\ \alpha + B_3 sin\ 3\alpha + \ B_5 sin\ 5\alpha$$

Where B's represent the maximum flux densities for the fundamental and harmonics components and α is the angular distance in electrical degrees measured around the air gap from the reference point midway between the poles. The flux per pole equals

$$K\int_{o}^{\pi} Bd\alpha = 2K\left(B_1 + \frac{B_3}{3} + \frac{B_5}{5}\right) = \ \phi_{m1} + \phi_{m3} + \phi_{m5}$$

K is a constant involving the inductor length and the choice of units $\phi_{m1}, \phi_{m3}, \phi_{m5}$ are the maximum values of the flux linking a full pitch coil on the armature due to the fundamental, third harmonics and fifth harmonic components of the flux density curve. The voltage is given by :

$$e = -N\frac{d\emptyset}{dt}$$

Then the qth harmonics in the voltage wave has a effective value of $E_q \ = 4.44qfN\phi_{mq}$

Calculations of the Voltage Induced in a Coil When the Coil Sides are not 180^o Electrical Apart:

If the coils sided are not 180^o electrical apart, the voltage in them are not in phase at every instant when considered around the coil. The effective voltage in the coil however is still equal to the vector difference of the effective voltages induced in its active sides.

If the distribution of the flux density is not sinusoidal but its distribution in the air gap is known in terms of a fundamental and a series of harmonics, the fundamental and the harmonics voltages in the coil can be found by taking the vector differences of the effective voltages induced in the sides of the coil by the fundamental and each of the harmonics separately. Figure (b) gives a distribution of flux density which contains a fundamental and third and fifth harmonics. Inspection of the curve makes it clear that any change ρ in the angular distance between the two sides of a coil corresponds to a change in phase between the voltages in the two sides of ρ for the fundamental and qρ for the qth harmonic.

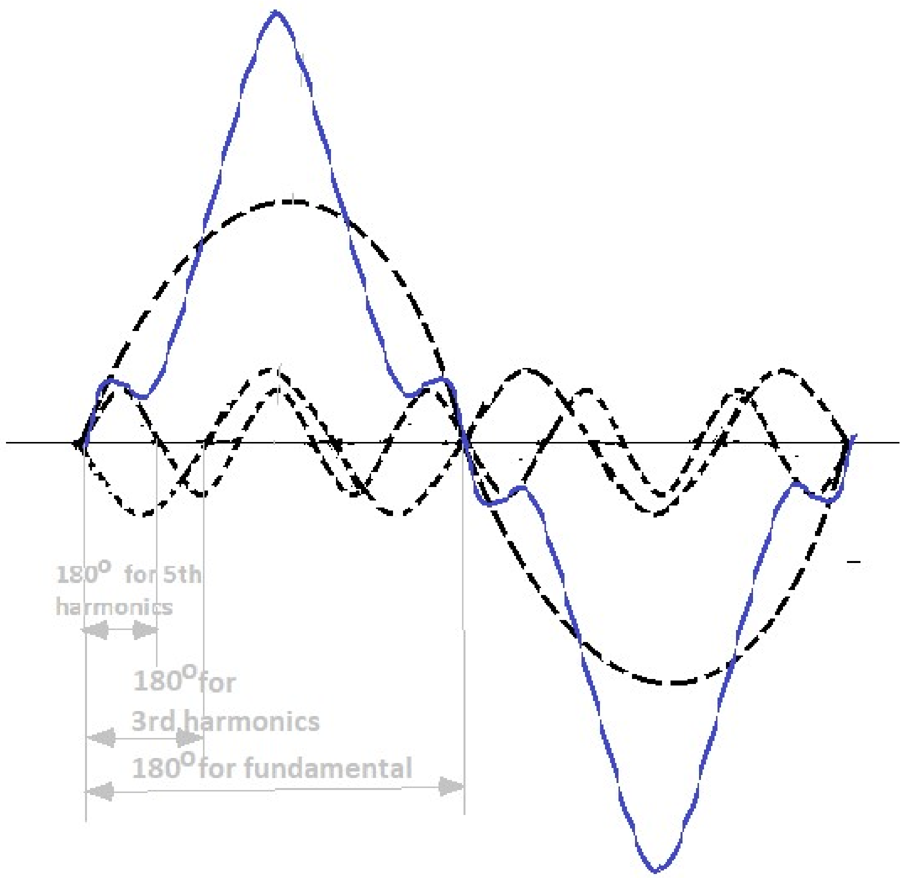


Fig. (b)

Let the space distribution of flux, i.e., the flux density, in the air gap of a synchronous generator measured from a point midway between the poles be:

$$B = B_1 sin\,\alpha + B_3 sin\,3\alpha + \ B_5 sin\,5\alpha$$

Where B's represent the maximum flux densities for the fundamental and harmonics and α is the angular distance in electrical radians measured around the gap from the reference point midway between the poles. If the inductors of the coil are 160° electrical apart, the fundamental of the voltages in the two inductors are 20° out of phase opposition; the third harmonics are 3x20° = 60° and the fifth harmonics are 5x20°=100° . the vectors for the fundamental and the harmonics are shown in fig.(c). in the figure R's are the resultant voltages; 1 and 2 are the voltages in coil side 1 and 2

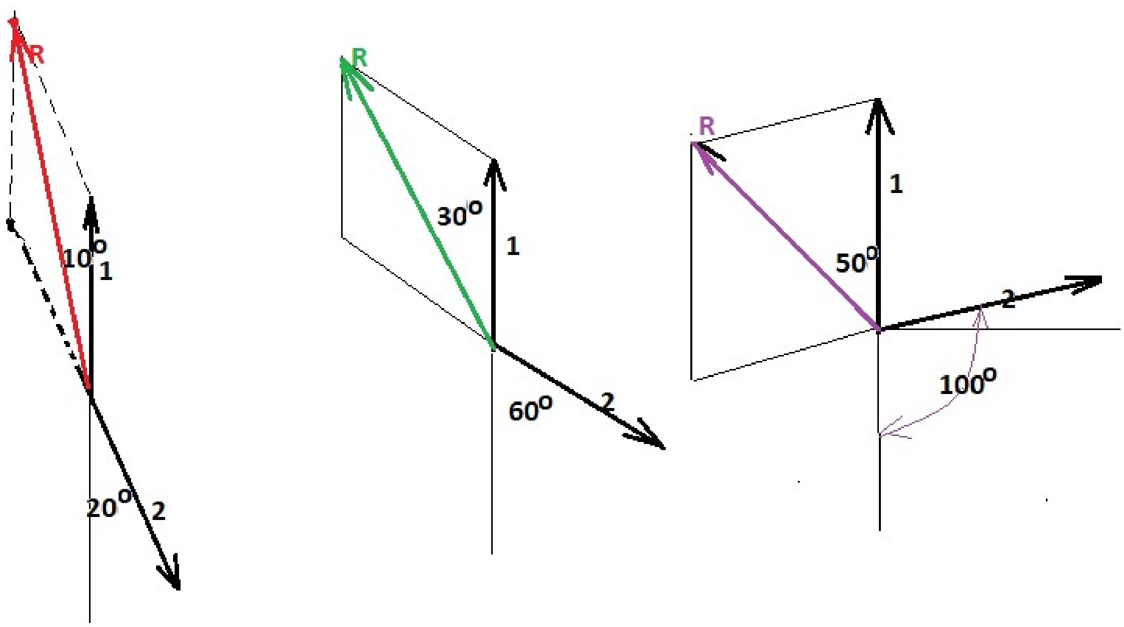
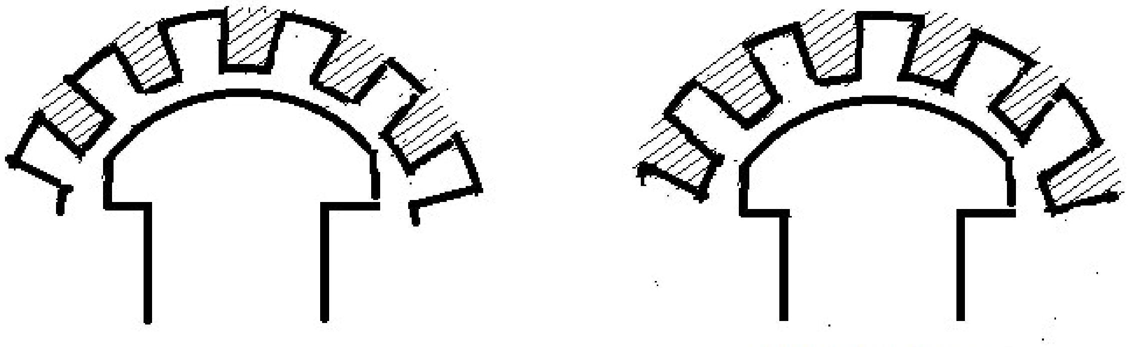


Fig. (c)

If the coil contains N turns and moves with a speed of v m per sec. and the length of the inductors are L meters, the instantaneous voltage in volt induced in the coil referred to the voltage in inductor 1 is $e = 2LvN[B_1 \cos 10^o \sin(\alpha + 10^o) + B_3 \cos 30^o \sin(3\alpha + 30^o) + B_5 \cos 50^o \sin(5\alpha + 50^o)]$

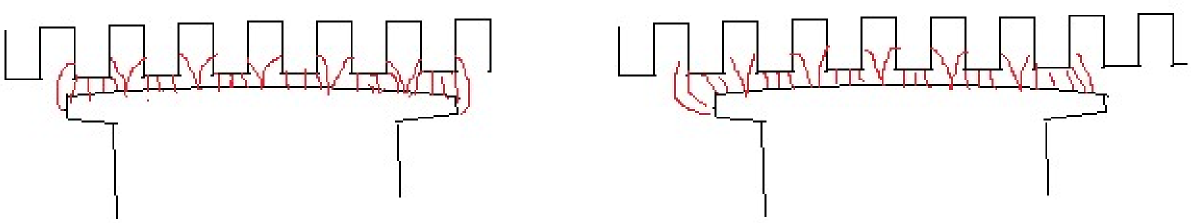
The rms value of this voltage is equal to the square root of one-half the sum of the squares of the maximum values of the fundamental and harmonics. when rms voltages are desired, it is easer to make use of the pitch factors.

Harmonics Due to Slots:



Minimum reluctance

The above figure shows two position of armature core relative to a pole. The reluctance of the magnetic circuit is less for the position shown on the right. The movement of the slots across the pole therefore causes the pole flux to pulsate with a frequency of 2nf, where n is the number of slots per pole and f is the normal frequency of the machine. It the effective width of the pole is equal to any integer times the width of a slot plus a tooth, there is no variation in the reluctance of the magnetic circuit as a whole, because of the relative movement of the slots and poles. The effective width of the pole is slightly greater than the actual width on account of the fringing of the flux at the pole edges.



Because of the slots entering and leaving the polar region, there is a periodic variation in the fringing of the flux at the pole tips which causes an oscillation of the field axis with respect to the axis of the poles.

For the relative position of the slots and the field poles shown in the left figure, the fringing is greater at the right side of the pole than at the left. The figure on the right shows the condition after a relative movement of the pole with respect to the slots equal approximately to the width of a slot. In this case fringing is greater at the left side than at the right. The change in the fringing produces an oscillation of the axis of the flux with respect to that of the poles at frequency which equals 2nf.

The slots also cause ripples in the flux wave which move across the pole faces. The movement of these ripples cannot cause any harmonics in the armature voltage since there is no relative movement between them and the armature inductors. Either the variation in the magnitude of the field or the position of its axis with respect to the poles may induce tooth harmonics in the armature voltage. The voltage induced by either the oscillation of the axis of the variation in the field strength is of the form

$$e = k(\sin 2n\omega t)\sin \omega t = \, ^1\!/_2 \; k[\cos(2n-1)\omega t - \cos(2n+1)\omega t]$$

Either effect may therefore produce harmonics of two different frequencies in the armature winding of the orders (2n-1) and (2n+1).

Effects of Sub-harmonics:

It is well known that nonlinear loads will produce harmonics by drawing currents that are not necessarily sinusoidal. In effect, inductive loads will produce harmonics that are multiples of the fundamental (e.g., 100Hz, 150Hz, 200Hz, etc.) In a similar manner, when a circuit involving a resistor, capacitor and inductor is connected in series, voltages and currents with frequencies below the fundamental will be created. These are called sub-harmonic frequencies; they will be denoted f_{er}.

The sub-harmonic frequencies in an RLC circuit will be damped out through the resistance of the circuit and will coexist in the system without presenting major problems. However, during faults or switching events, the subharmonic currents are amplified and excited so that the resistance of the circuit can reach a point that is not enough to damp the sub-harmonic frequencies. As a result, these frequencies can cause voltage and current amplification. After decades of research, there are various phenomena that can be attributed to sub-harmonics. Subharmonics can create induction generator effects, torsional interactions, torque amplification, subsynchronous resonance, and transformer saturation. For the interaction between wind generation and series compensated lines, research that will help determine the real cause is still underway; however, the evidence records seem to point towards an induction generator effect phenomena.

Induction Generator Effect

When a series capacitor is used to cancel a portion of the system reactance, the system will always end up with a natural frequency which is less than the system frequency and is referred to as the subharmonic [2]. This sub-harmonic frequency is defined by as:

$$f_{er} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \;; \quad \therefore \; f_{er} = f_o\sqrt{\frac{1}{(2\pi f_o L)(2\pi f_o C)}} = f_o\sqrt{\frac{1}{(\omega L)(\omega C)}} = f_o\sqrt{\frac{X_c}{X_L}}$$

Where X_c is the reactance of the series capacitor ; $X_L$is the inductive reactance of the system (i.e, total impedance of the generator, transformer, transmission line and load) and $f_o$is the system frequency.

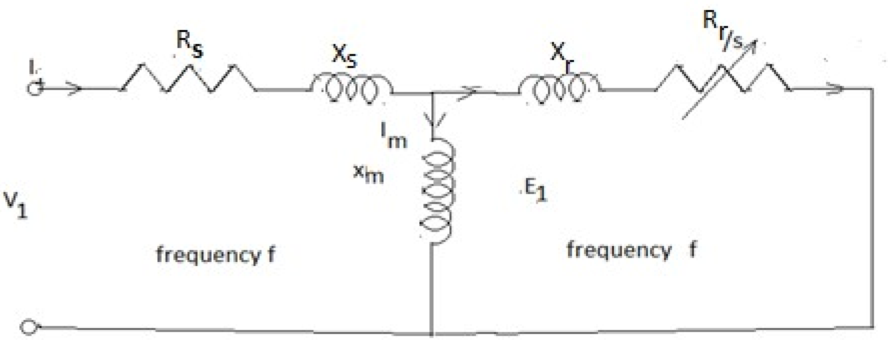
The generator armature sub-harmonic currents produce magnetic fields with frequency f_{er} which are induced in the generator rotor. As a result, the induce currents in the rotor produces a rotor frequency defined as:

$$f_r = f_o - f_{er}$$

In consequence, this new added rotor frequency will result in sub-synchronous armature voltages which may enhance the sub-synchronous currents and may cause generator self–excitation. Because the rotor is turning faster than the sub-harmonic armature currents with frequency f_{er}, a slip is created simulating an induction machine and the slip is defined by s:

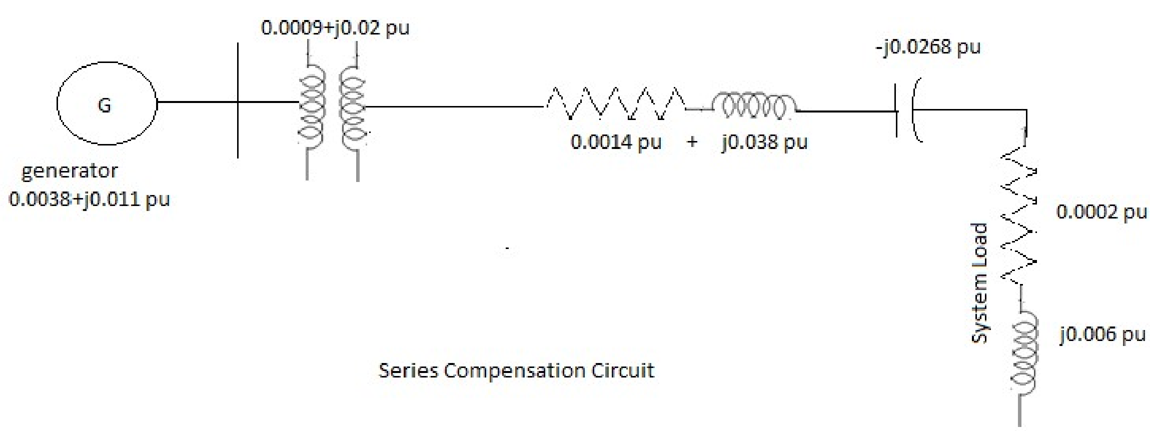
$$slip\ (s) = \frac{f_{er} - f_o}{f_{er}}$$

the slip will always be negative, which will result in a negative rotor resistance R' as described in Figure 1.



Induction generator effect circuit.

The rotor resistance is $R' = {}^{R_r}/_{s}$; Since the slip (s) will always be negative, the damping effect of R' will always be negative. If the series compensation is very high, the slip (s) will be very small and therefore R' will be a negative large number. If the addition of the damping resistance R' and the resistance of the system is negative, voltage and current growing oscillations may build up to very dangerous high values. In order to emphasize this concept better, let take a look at the following example. The following figure shows a series compensation circuit through a couple 525kV transmission line



Series Compensation Circuit

the natural frequency of this circuit is

$$f_{er} = f_o\sqrt{\frac{X_c}{X_L}} = 60\ \times \sqrt{\frac{0.0268}{0.011 + 0.02 + 0.038 + 0.006}} = 60 \times \sqrt{\frac{0.0268}{0.075}} = 35.87\ Hz\ \approx 36\ Hz$$

$$\therefore slip\ (s) = \frac{36 - 60}{36} = \ -0.66$$

The damping with a rotor resistance of 0.02 p.u: $R' = \frac{R_r}{s} = \ \frac{0.0038}{-0.066} = -0.00575\ pu$

The total resistance of the network of line, transformer and load is

$$R_{network} = 0.0009 + 0.0014 + 0.0002 = 0.0025\text{pu}$$

The total effective resistance is:

$$R_{eff} = R_{network} + R' = -0.00575 + 0.0025 = -0.00325\ pu$$

Since the net resistance is negative for this series compensated circuit, we can expect increasing subharmonic oscillation.