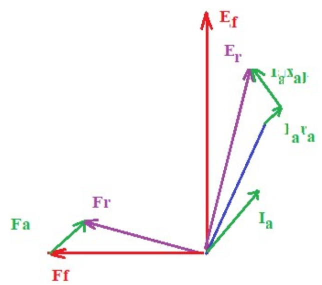
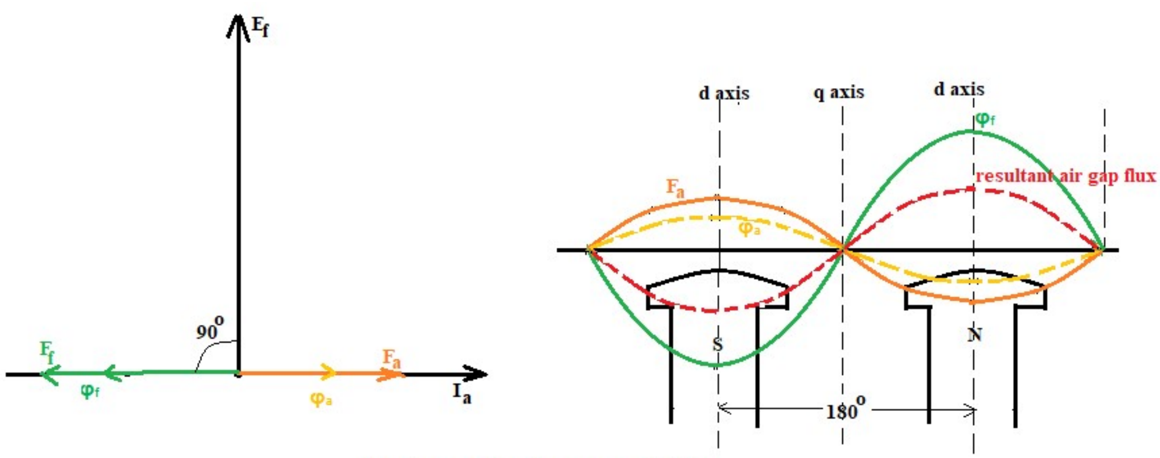
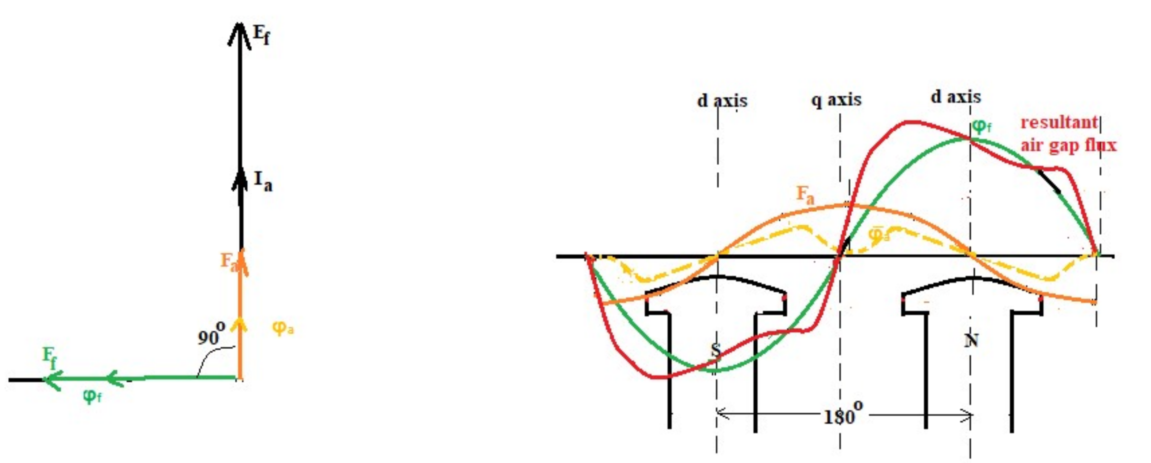


Phasor Diagram of Cylindrical Rotor Synchronous Machine

Two Reaction Theory:



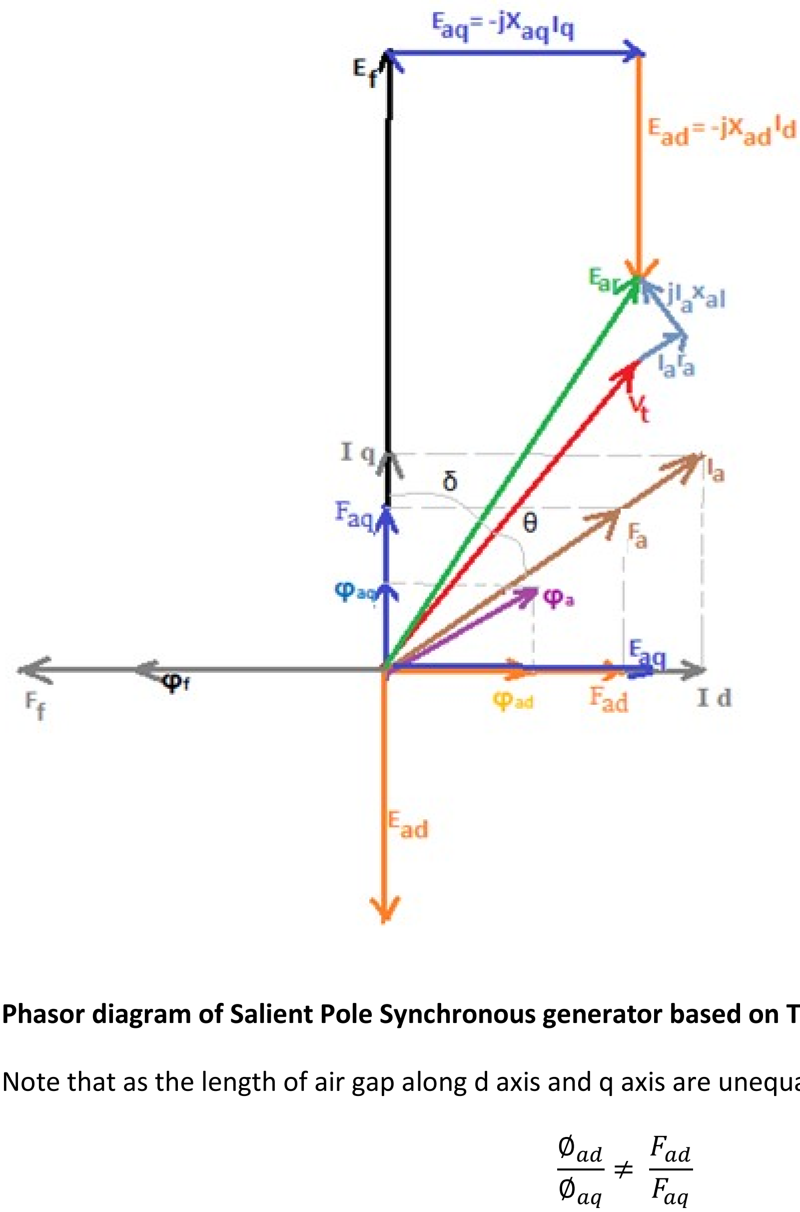
Armature reaction flux opposes field flux



Armature reaction flux is cross magnetising

The different values of reluctance along polar axis (d axis) and inter polar axis n(q axis) , make analysis of salient pole machine different from cylindrical rotor machine. Cylindrical rotor theory is not applicable for salient pole machine as the theory is based on constant air gap reluctance. The effect of salient pole is taken into account in the 'Two-Reaction Theory' proposed by Blondel.

According to two reaction theory the sinusoidal armature reaction m.m.f F_a is resolved into two sinusoidal components; F_{ad} along the d axis and F_{aq} along the q axis. The d axis component F_{ad} is magnetizing or demagnetizing depending on the power factor and produces only change in the field strength. Whereas the q axis component F_{aq} produces only distortion of the field flux wave. F_{ad} and F_{aq} are assumed to be produced by components of armature current along the d axis (I_d) and component along q axis (I_q) respectively.



Phasor diagram of Salient Pole Synchronous generator based on Two Reaction Theory

Note that as the length of air gap along d axis and q axis are unequal, the reluctances are different, so,

$$\frac{\emptyset_{ad}}{\emptyset_{aq}} \neq \frac{F_{ad}}{F_{aq}}$$

E_{ad} is induced by F_{ad} lagging it by 90^o and E_{aq} is induced by F_{aq} lagging it by 90^o. So,

$\overline{E_{ad}} = -jK_d\overline{F_{ad}} = -jCK_d\overline{I_d} = -jX_{ad}\overline{I_d}$ and $\overline{E_{aq}} = -jK_q\overline{F_{aq}} = -jCK_q\overline{I_q} = -jX_{aq}\overline{I_q}$

X_{ad} and X_{aq} are d axis and q axis magnetizing reactance (armature reaction reactance) of the synchronous machine. X_{ad} takes care of the effect of armature reaction along the d axis and X_{aq} takes care of the effect of armature reaction along the q axis.

The phasor sum of E_f, E_{ad} and E_{aq} gives the air gap voltage E_{ar}.

$$\overline{E_{ar}} = \overline{E_f} + \overline{E_{ad}} + \overline{E_{aq}}$$

Again $\overline{V_t} + \overline{I_a}r_a + j\overline{I_a}x_{al} = \overline{E_{ar}}$ and $\overline{I_d} + \overline{I_q} = \overline{I_a}$ $\therefore j\overline{I_a}x_{al} = j\overline{I_d}x_{al} + j\overline{I_q}x_{al}$

$$\therefore \overline{V_t} + \overline{I_a}r_a + j\overline{I_a}x_{al} = \overline{E_f} + \overline{E_{ad}} + \overline{E_{aq}};$$

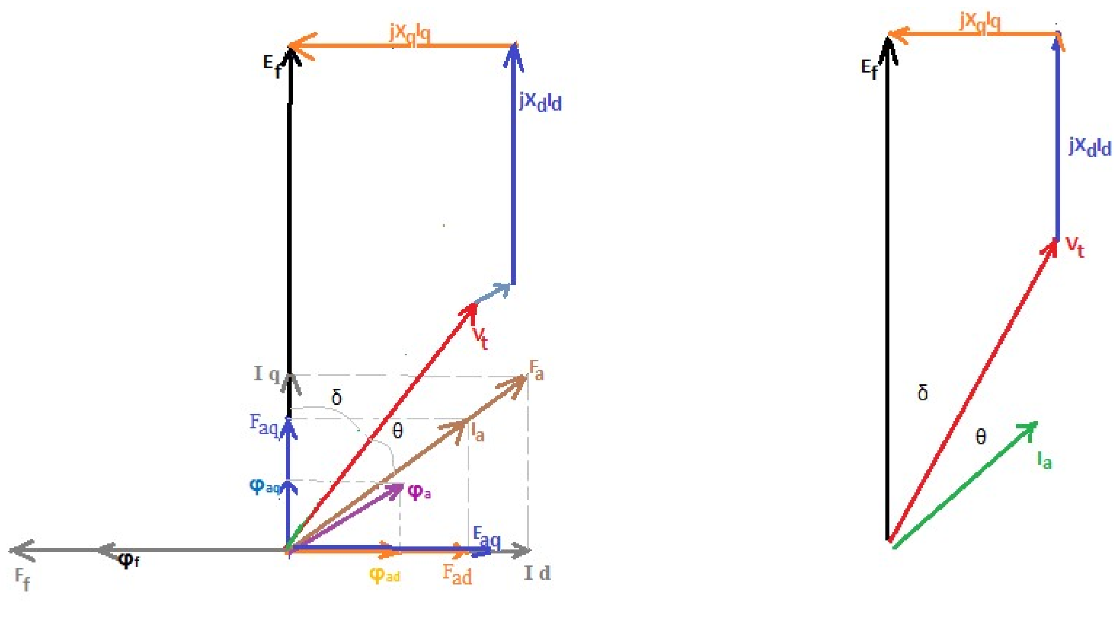
$$\therefore \overline{V_t} + \overline{I_a}r_a + j\overline{I_d}x_{al} + j\overline{I_q}x_{al} = \overline{E_f} - jX_{ad}\overline{I_d} - jX_{aq}\overline{I_q}$$

$$\therefore \overline{V_t} + \overline{I_a}r_a + j\overline{I_d(}x_{al} + X_{ad}) + j\overline{I_q(}x_{al} + X_{aq}) = \overline{E_f}$$

$$\overline{V_t} + \overline{I_a}r_a + j\overline{I_d(}X_d) + j\overline{I_q(}X_q) = \overline{E_f}$$

Where $X_d = x_{al} + X_{ad}$ is the direct axis synchronous reactance

$X_q = x_{al} + X_{aq}$ is the quadrature axis synchronous reactance



Phasor Diagram of Synchronous Generator

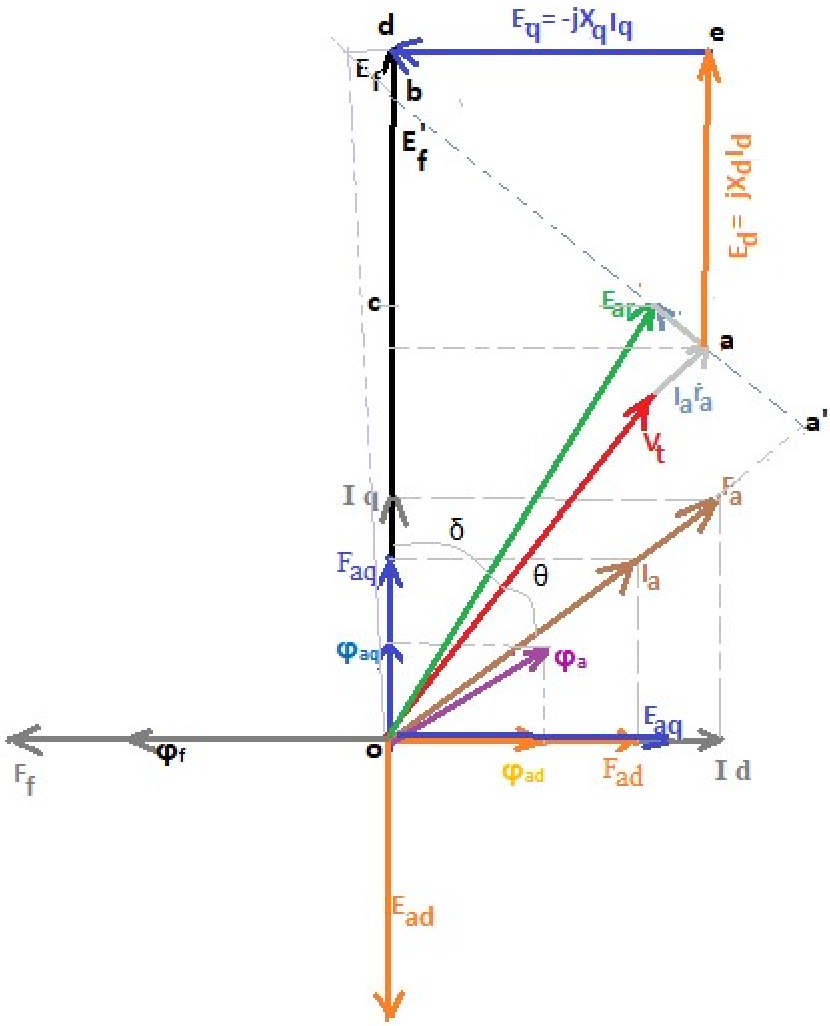
$I_d = I_a \sin(\delta+\theta)$ and $I_q = I_a \cos(\delta+\theta)$

However, note that it is not possible to draw the phasor diagram unless the angle between E_f and V_t (i.e. angle δ is known, because to draw the phasor diagram the armature current has to be resolved into two components.

For this purpose, V_t ; I_a and $I_a r_a$ phasors are drawn and 'ab' is drawn normal to $I_a r_a$. As it is 90^o away from I_a, it can be taken equivalent to a reactive voltage drop of $I_a X$. Draw 'ac, perpendicular to 'ob'. Oa'b and acb are right angled triangles and therefore , $\angle bac = \angle\delta + \theta$

$$\therefore ac = ab\ \cos(\delta + \theta) = XI_a \cos(\delta + \theta) = de$$

$$XI_q = X_q I_q \ \ \therefore X = X_q$$



Thus, $ab = X_q I_a$. if ob = E_f' then $\overline{E_f}' = \overline{V_t} + \overline{I_a} r_a + j\overline{I_a} X_q$

The phasors E_f' and E_f are are along the same line. So, angle between E_f' and V_t is δ. So, angle $(\delta + \theta)$ is now known and $I_d; I_q$ can be drawn.

Now $bd = cd - cb = X_d I_d - X_q I_a \sin(\delta + \theta) = X_d I_d - X_q I_d = (X_d - X_q) I_d$

$$\overline{E_f'} + \overline{bd} = \overline{E_f} = \overline{V_t} + \overline{I_a} r_a + j\overline{I_a} X_q + j\overline{I_d}(X_d - X_q)$$

Determination of X_d and X_q:

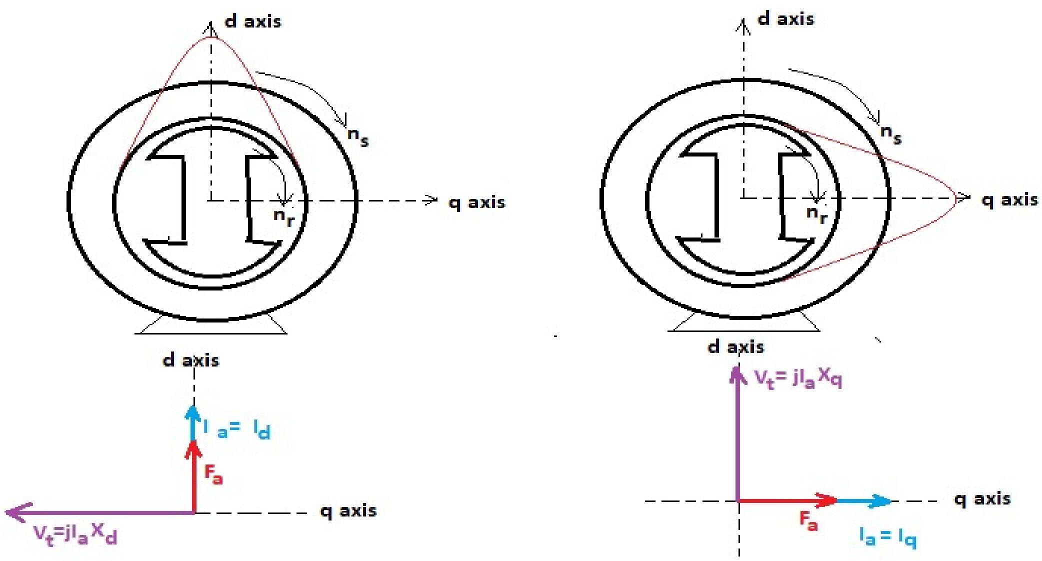
Determination of X_d

For short circuit condition V_t=0 therefore $I_q X_q$=0 $X_q \neq 0$ $\therefore$ $I_q = 0$ *and* $I_d = I_{sc}$ which leads to $E_f = I_d X_d$.

So $\frac{Open\ circuit\ voltage\ for\ a\ field\ current}{short\ circuit\ current\ for\ the\ same\ field\ current} = X_d$

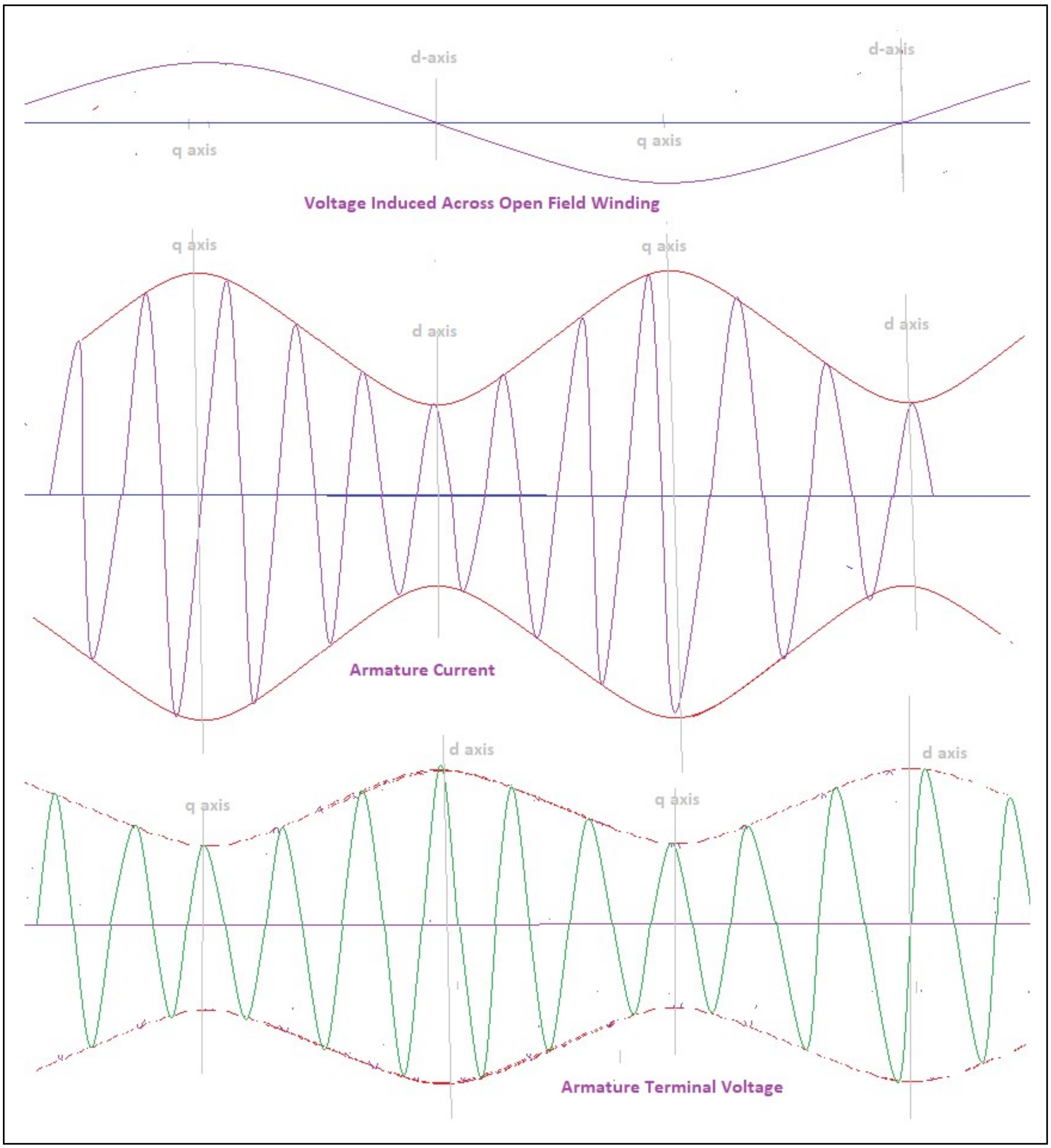
Slip Test:

The synchronous machine is driven by the prime mover at a speed slightly different from the slip speed. The field winding is kept open and a positive sequence balanced voltage of reduced magnitude (around 25% of the rated voltage) is applied to the machine. The relative speed between the rotor poles and the stator rotating magnetic field is the difference between the synchronous speed and the rotor speed, i.e. the slip speed. Small low frequency A.C voltages across the open field winding indicate that the field poles and the magnetic field are rotating in the same direction (essential for the test). If the field poles rotate in the opposite direction then negative sequence reactance would be measured.

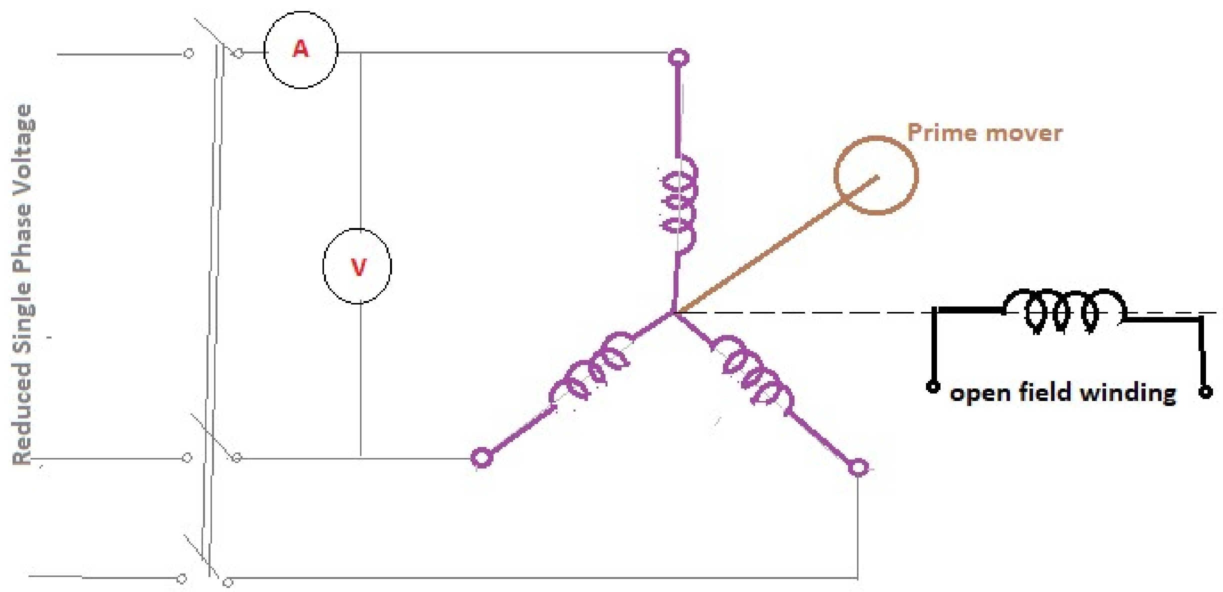


Phtsical concept of X_d and X_q

The rotor and the rotating magnetic field are rotating at different speed. At one instant when the peak of the armature m.m.f wave is directed along the pole axis (d-axis) the reluctance offered by the small air gap is minimum (refer to the above figure). At this instance the impressed terminal voltage per phase divided by the per phase armature current gives the direct axis synchronous reactance X_d. After one quarter of the slip cycle the peak of armature m.m.f. wave is coincident with the inter polar axis (q-axis), the reluctance offered by the large air gap is maximum (refer to the above figure). At this instance the impressed terminal voltage per phase divided by the per phase armature current gives the quadrature axis synchronous reactance X_q.



Oscillograms of armature current; armature terminal voltage and e.m.f induced in the open field winding is shown in the above diagram. A much larger slip has been considered for convenience. In practice the slip is very small. When the armature m.m.f. is along the direct axis, the armature flux linkage of the open field winding is maximum; i.e. the rate of change of flux $\frac{d\varphi_a}{dt}$ is zero. So, the induced voltage is zero, so the D-axis can be located on the oscillogram. When the armature m.m.f. wave is along the quadrature axis, the armature flux linkage of field winding is zero,i.e. $\frac{d\varphi_a}{dt}$ is maximum. Thus the Q-axis can also be located on the oscillogram.



Slip Test Connection Diagram

If oscillogram is not available, then an ammeter and a voltmeter is used as shown in the above figure. The prime mover speed is adjusted until the ammeter and voltmeter pointers swing slowly between maximum and minimum values. The maximum and minimum readings of ammeter and voltmeter are recorded.

Since applied voltage is constant, the air gap flux should be constant. When crest of the m.m.f. wave is coincident with the D-axis, the air gar reluctance is minimum, so the magnetizing current required for the establishment of constant flux is minimum. So, armature reading is minimum, so corresponding drop in the armature circuit is minimum. The armature terminal voltage is applied voltage minus armature voltage drop. So, armature supply voltage is maximum.

$$\therefore\ X_d = \frac{Maximum\ armature\ terminal\ voltage}{minimum\ armature\ current}$$

By a similar thought process;

$X_q = \frac{Minimum\ armature\ terminal\ voltage}{maximum\ armature\ current}$

The swing of the ammeter pointer is wide, but the swing of the voltmeter pointer is small, because the impedance voltage drop in the leads and connecting wire is small. Since low voltage is applied, the values measured are unsaturated values.

While performing the experiment the slip should be small; otherwise

- Current in the damper windings will introduce large error in the measurement.
- The pointers swing at high speed, making reading noting difficult.

The applied voltage should be small, because:

- it is difficult to maintain a small slip, so that reluctance torque developed by the machine is small.
- The swing in meter reading due to impedance drops can be measured.

For larger applied voltage the reluctance torque developed due to saliency, tries to bring the rotor into synchronisim with the rotating magnetic field and it is difficult to note the small change in meter readings.

However, the inertia of the moving systems of the meters also introduces error in the reading.

So, the advantages of oscillographic method are:

- Elimination of inertia effect of voltmeter and ammeter
- The possibility of large slip speed, which allows higher terminal voltage to be applied.

In practice, there may be error in reading the oscillgram and voltmeter ammeter readings are not very reliable due to effect of inertia of the moving parts. In view of this short comings the slip test is done only to determine X_q/X_d ratio.

$\frac{X_q}{X_d} = \frac{Minimum\ armature\ terminal\ voltage}{maximum\ armature\ current} \times \frac{minimum\ armature\ current}{Maximum\ armature\ terminal\ voltage}$

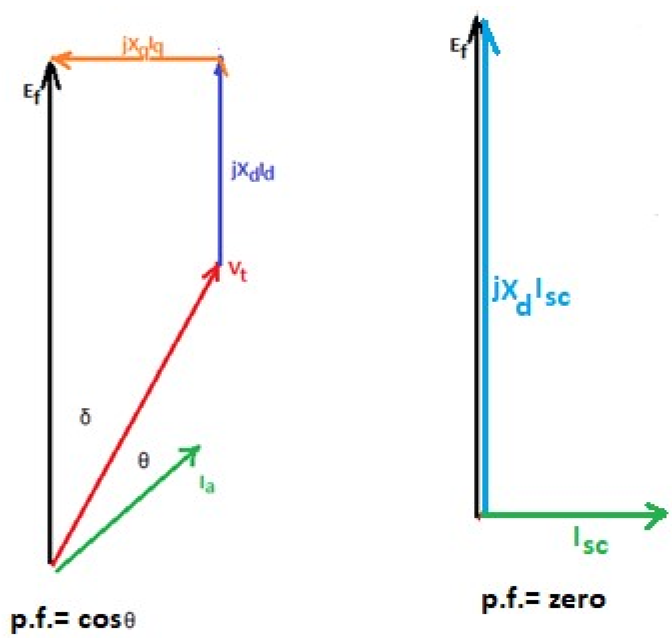
X_d is determined from O.C test and S.C. test, X_q can be determined as follows:

$$X_q = \frac{X_q}{X_d} \times X_d\ from\ O.C.and\ S.C.test$$

X_d from O.C and S.C. Test

O.C test is performed at synchronous speed and the O.C.C is the plot between the open circuit phase voltage and the field current.

S.C test is preferably performed at synchronous speed, but can also be performed at a speed slightly different from the synchronous speed. S.C.C is the plot between the short circuit current and the field current.



Phasor Diagram of Salient pole Synchronous Generator

The above figures represent salient pole synchronous generator approximate phasor diagram, where armature resistance ra has been neglected.

For any power factor $\cos\theta$

$$I_d = I_a \sin(\delta+\theta);\ \ I_q = I_a \cos(\delta+\theta);\ \ E_f = V_t \cos\delta + I_d X_d\ ;\ V_t \sin\delta = I_q X_q$$

Under short circuit condition p.f. is almost equal to zero as $r_a \ll X_d, X_q$ and thus it can be assumed that $r_a \approx 0$

Thus:

$V_t \sin\delta = I_q X_q = 0$, but X_q cannot be zero, so $I_q = 0\ \ \therefore\ \ I_d = I_{sc}\ \ and\ \ E_f = I_d X_d$

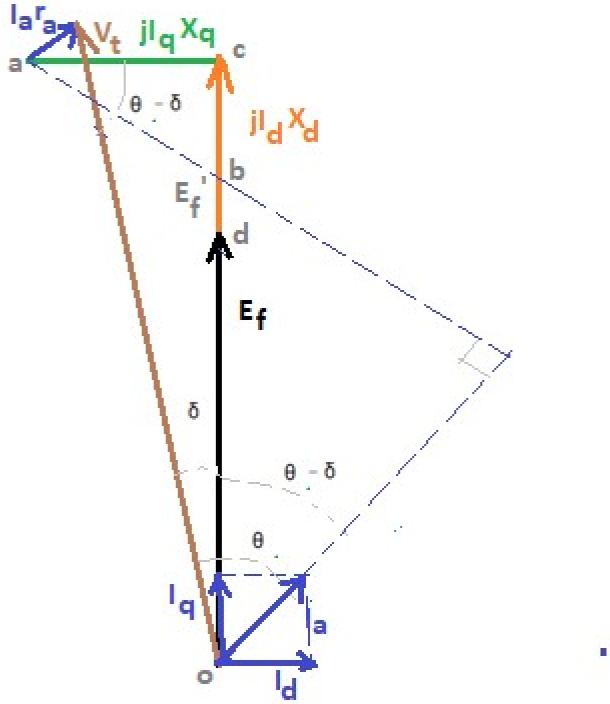
$$\therefore\ X_d = \frac{Open\ circuit\ voltage\ for\ a\ field\ current}{short\ circuit\ currrent\ for\ the\ same\ field\ current}$$

Type equation here.

Salient Pole Synchronous Motor Phasor Diagram:

The motor voltage equations can be obtained from the generator voltage equation by replacing Ia by – Ia. Therefore the voltage equation for a salient pole synchronous motor is,

$$\overline{V_t} = \overline{E_f} + \overline{I_a} r_a + j\overline{I_d} X_d + j\overline{I_q} X_q$$



Salient pole Synchronous Motor Phasor Diagram

In order to compute **E_f**, 'ab' is drawn perpendicular to current Ia. So, 'ab' can be considered as an reactance drop = **I_aX**. From the above figure

$$\angle abc = 90^o - (\theta - \delta) \;\; and \; \angle bac = (\theta - \delta)$$

$$\therefore ac = ab \; \cos(\theta - \delta) = XI_a cos(\theta - \delta) = XI_q$$

$$\therefore XI_q = X_q I_q \;\; and \; \therefore X = X_q$$

So, angular position of **E_f** is along $ob = \widetilde{E_f'} = \overline{V_t} - \overline{I_a} r_a - j\overline{I_a} X_q$. So the angle $\theta - \delta$ is known and I_d and I_q are calculated and $\overline{V_t} = \overline{E_f} + j\overline{I_d}(X_d - X_q) + j\overline{I_a} X_q + \overline{I_a} r_a$

Note that the term $\overline{I_d}(X_d - X_q)$ appear due to saliency and reduces to zero for cylindrical rotor synchronous machines(as $X_d = X_q$). In salient pole machine $\mathbf{X_d}$ is approximately 60% larger than $\mathbf{X_q}$. However in cylindrical rotor synchronous machine $\mathbf{X_d}$ and $\mathbf{X_q}$ may differ slightly due to the effect of field winding slots in q-axis.

Problem 1:

A salient pole synchronous generator has the following per unit parameters: $\mathbf{X_d}$ =1.2; $\mathbf{X_q}$= 0.8 ; $\mathbf{r_a}$=0.025

Compute the excitation voltage on a per unit basis, when the generator is delivering rated kVA at rated voltage and at a power factor (a) o.8 lagging and (b) 0.8 leading

Solution:

(a) $\overline{V_t} = 1.00 + j0.00 \quad and \quad \overline{I_a} = 1.00\angle - 36.9^0 = 0.80 - j0.60$

$jI_aX_q = j(0.80 - j0.60)0.80$ = 0.48+j0.64 and $\overline{I_a}r_a = 0.020 - j0.015$

$$\widetilde{E_f'} = \overline{V_t} + \overline{I_a}r_a + j\overline{I_a}X_q = 1.00 + j0.00 + 0.48 + \text{j}0.64 + 0.020 - j0.015$$
$$= 1.50 + j0.625 = 1.625\angle 22.6^o$$
$$\therefore \delta = 22.6^o \; and \; \angle\delta + \theta = 22.6^o + 36.9^0 = 59.5^0$$
$$\therefore I_d = 1.00 \sin 59.5^0 = 0.861 \quad and \quad I_q = 1.00 \cos 59.5^0 = 0.507$$
$$E_f = E_f' + I_d(X_d - X_q) = 1.625 + 0.861 \times 0.4 = 1.9694$$
$$\overline{E_f} = 1.9694\angle 22.6^o$$

(b) $\overline{V_t} = 1.00 + j0.00 \quad and \quad \overline{I_a} = 1.00\angle 36.9^0 = 0.80 + j0.60$

$jI_aX_q = j(0.80 + j0.60)0.80$ =- 0.48+j0.64 and $\overline{I_a}r_a = 0.020 + j0.015$

$$\widetilde{E_f'} = \overline{V_t} + \overline{I_a}r_a + j\overline{I_a}X_q = 1.00 + j0.00 - 0.48 + \text{j}0.64 + 0.020 + j0.015$$
$$= 0.54 + j0.655 = 0.849\angle 50.50^o$$
$$\therefore \delta = 50.50^o \; and \; \angle\delta - \theta = 50.50^o - 36.9^0 = 13.60^0$$
$$\therefore I_d = 1.00 \sin 13.60^{0^0} = 0.235 \quad and \quad I_q = 1.00 \cos 13.60^{0^0} = 0.9719$$
$$E_f = E_f' + I_d(X_d - X_q) = 0.849 + 0.235 \times 0.4 = 0.943$$
$$\overline{E_f} = 0.943\angle 50.50^o$$

Problem 2:

For an over excited Synchronous motor, prove that:

$$\tan\delta = \frac{I_a(X_q \cos\theta + r_a \sin\theta)}{V_t + I_a(X_q \sin\theta - r_a\cos\theta)}$$
$$X_q = \frac{V_t \sin\delta - I_a r_a \sin(\delta + \theta)}{I_a \cos(\delta + \theta)}$$

Solution:

$$ab = ac + cd = I_d r_a + I_q X_q$$

$$\sin\delta = \frac{ab}{oa} = \frac{ac + ad}{oa} = \frac{I_d r_a + I_q X_q}{V_t}$$

$$or \quad V_t \sin\delta = I_d r_a + I_q X_q$$

Again, $I_d = I_a \sin(\delta + \theta) \quad and\ I_q = I_a \cos(\delta + \theta)$

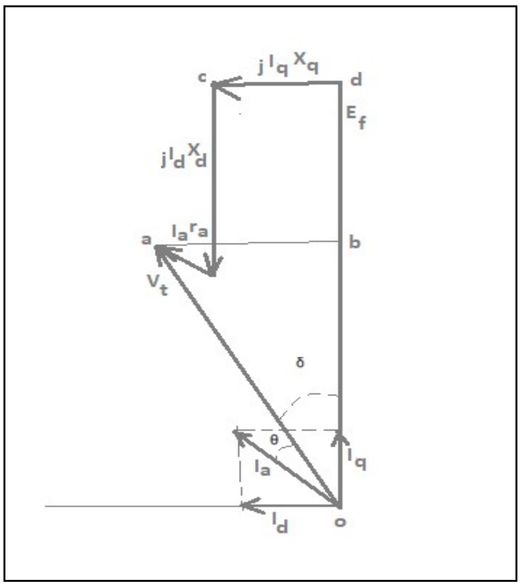
$$\therefore \ V_t \sin\delta = I_d r_a + I_q X_q = I_a r_a \sin(\delta + \theta) + I_a \mathrm{X_q} \cos(\delta + \theta)$$

$$= \ I_a r_a (\sin\delta\ cos\theta + cos\delta\ sin\theta)$$

$$+ I_a \mathrm{X_q} (\cos\delta\ cos\theta - sin\delta\ sin\theta)$$

$$sin\delta\left(V_t - I_a r_a cos\theta + I_a X_q sin\theta\right) = cos\delta\left(I_a r_a sin\theta + I_a X_q cos\theta\right)$$

$$\therefore tan\delta = \frac{sin\delta}{cos\delta}$$



$$= \frac{\left(I_a r_a sin\theta + I_a X_q cos\theta\right)}{\left(V_t - I_a r_a cos\theta + I_a X_q sin\theta\right)}$$

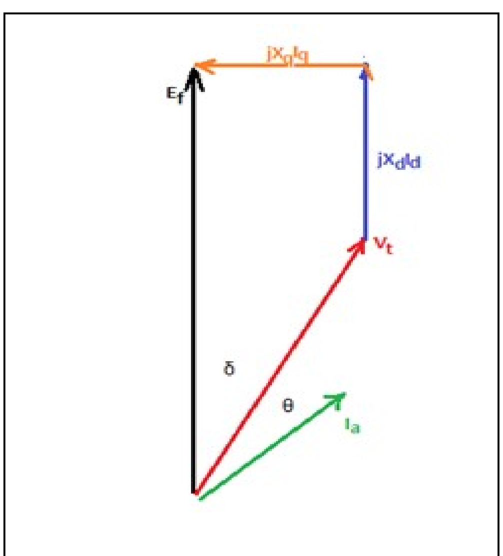
$$= \frac{I_a\left(r_a sin\theta + X_q cos\theta\right)}{\left(V_t - I_a (r_a cos\theta + X_q sin\theta)\right)}$$

$$V_t \sin\delta = I_d r_a + I_q X_q$$

$$\therefore \ X_q = \frac{V_t \sin\delta - I_d r_a}{I_q}$$

$$X_q = \frac{V_t \sin\delta - I_a \sin(\delta + \theta)\ r_a}{I_a \cos(\delta + \theta)}$$

Power Angle Characteristics of a Salient Pole Synchronous Machine:



From the diagram per phase power developed is:

$$P = I_d V_d + I_q V_q = \ I_d V_t sin\delta + I_q V_t\ cos\delta$$

$$V_t sin\delta = ab = dc = I_q X_q \ \therefore \ I_q = \frac{V_t sin\delta}{X_q}$$

$$V_t\ cos\delta = oa = od - ad = od - bc$$

$$= E_f - I_d X_d$$

$$\therefore I_d = \frac{E_f - V_t\ cos\delta}{X_d}$$

$$\therefore P = I_d V_t sin\delta + I_q V_t\, cos\delta$$

$$= \frac{E_f - V_t\, cos\delta}{X_d} V_t sin\delta + \frac{V_t sin\delta}{X_q} V_t\, cos\delta$$

$$= \frac{E_f V_t sin\delta}{X_d} + V_t^{\,2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) sin\delta\, cos\delta$$

$$\therefore P = \frac{E_f V_t sin\delta}{X_d} + \frac{V_t^{\,2}}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) sin2\delta$$

So, power expression has two components. The first term is called electromagnetic power, because its existence depends on the presence of field excitation. This is similar to the power expression of a cylindrical rotor machine.

The second term is called reluctance power. Reluctance power exists even when the field excitation is zero. In a salient pole machine the reluctance along the direct axis and the quadrature axis are different and the armature reaction has a tendency to get oriented along the low reluctance path i.e. along the direct axis and the reluctance power is developed. Since $\frac{V_t^{\,2}}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) sin2\delta$ because of the difference in reluctance, it is called reluctance power and $\frac{1}{\omega_s}\frac{V_t^{\,2}}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) sin2\delta$ is called reluctance torque. So, a salient pole motor connected to the infinite bus will run as a reluctance motor when its field current is reduced to zero.

For maximum power $\frac{dP}{d\delta} = 0$

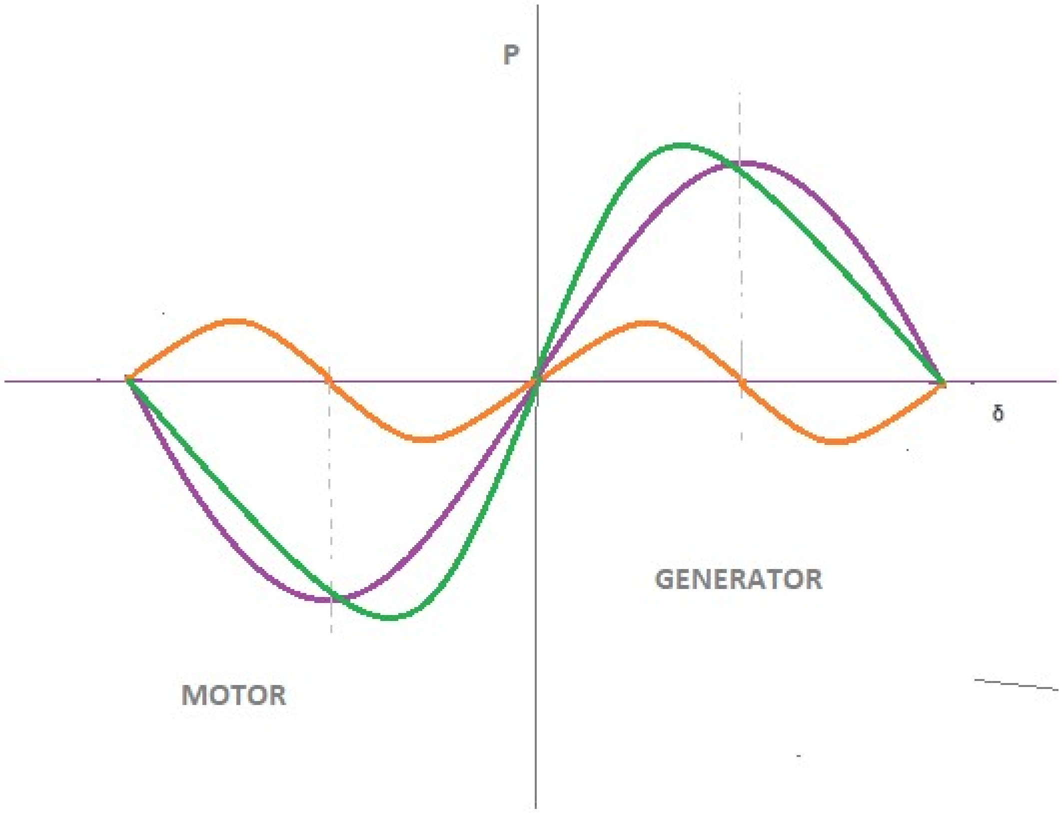
$$\therefore \frac{dP}{d\delta} = \frac{E_f V_t cos\delta}{X_d} + V_t^{\,2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) cos2\delta \ = 0$$

$$\therefore cos\delta = -\frac{E_f X_q}{4V_t\left(X_d - X_q\right)} \pm \sqrt{\frac{1}{2} + \left[\frac{E_f X_q}{4V_t\left(X_d - X_q\right)}\right]^2}$$

Synchronizing Power and Synchronizing Torque:

The variation of synchronous power associated with small change in load angle δ, is called synchronizing power. Synchronizing power coefficient P_{sy}, given by:

$$P_{sy} = \frac{dP}{d\delta} = \frac{E_f V_t}{X_s}\cos\delta + V_t^{\,2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) cos2\delta$$



Power Angle Characteristics of Salient Pole Machine

So for small change in load angle synchronizing power is given by :

$$P_s = \frac{dP}{d\delta}\Delta\delta = \frac{E_f V_t}{X_s}\cos\delta\ \Delta\delta + {V_t}^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)cos2\delta\ \Delta\delta$$

Synchronizing torque is given by $T_s = \frac{mP_s}{\omega_s}$; $or, T_s = \frac{1}{\omega_s} m\ \frac{dP}{d\delta}\Delta\delta;$

$$or\ T_s = m\frac{1}{\omega_s}\left(\frac{E_f V_t}{X_s}\cos\delta\ \Delta\delta + {V_t}^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)cos2\delta\ \Delta\delta\right)$$