## Synchronous Machine Reactance:

### I. Positive Sequence Reactance: $X_d$; $X_q$; $X'_d$; $X'_q$; $X''_d$ and $X''_q$

The magnitudes of different positive sequence reactance depend upon:

i) The angular position of the field structure with respect to the armature m.m.f. wave

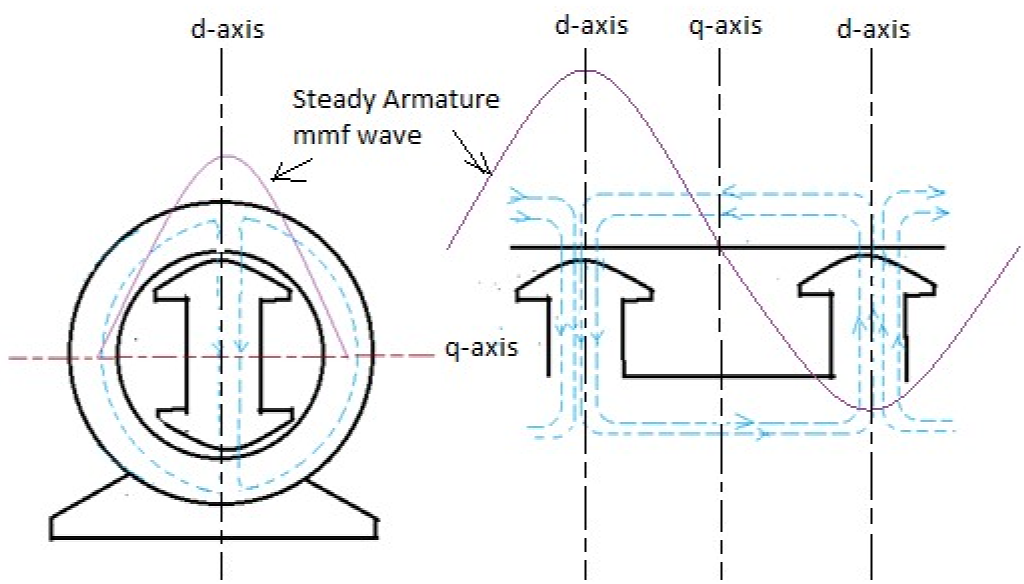
ii) Whether the armature currents are steady or are varying.

### A. Direct axis Synchronous Reactance $X_d$

If the peak of steady armature m.m.f. wave is in line with the direct axis, then the reactance offered to armature flux is minimum. For given armature current the armature flux has the maximum value. Under this condition armature flux linkage per ampere in the armature winding is called direct axis synchronous inductance $L_d$

$$L_d = \frac{armature\ flux \times armature\ no.of\ turns/phase}{armature\ phase\ current}$$

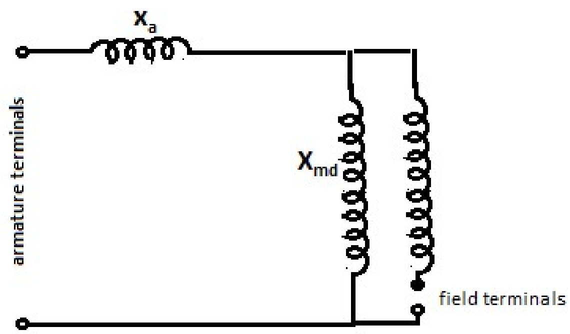
$$X_d = 2\pi f L_d$$




The d-axis equivalent circuit for a synchronous machine without damper bars is given by:

$$X_d = x_a + X_{md}$$

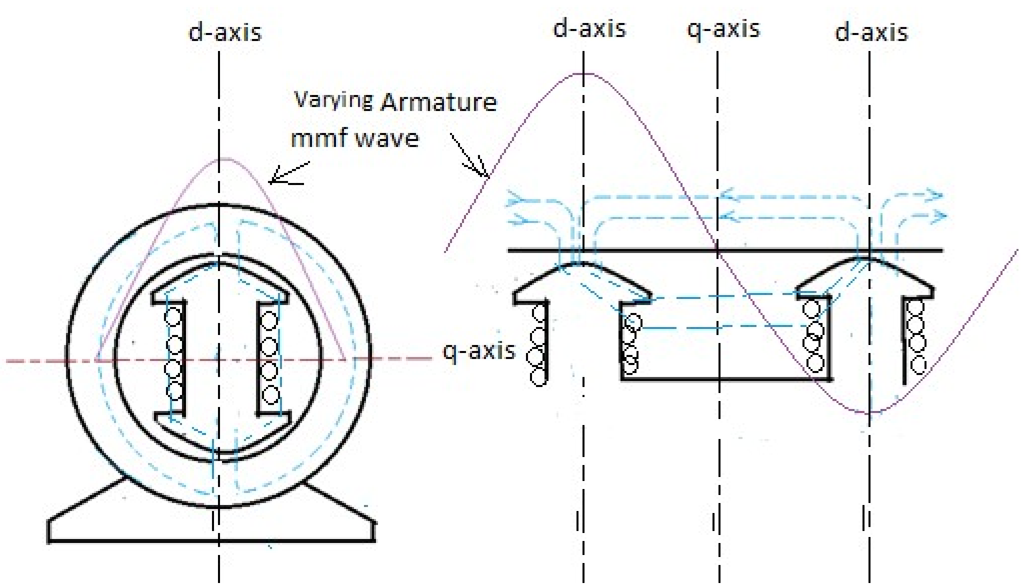
Under steady state the armature current have no effect on the field current, so field circuit is treated as open. At steady state, the damper bars also have no effect. So, $X_d$ for machine with damper bars is also same. $X_d$ is obtained from O.C.C. and S.C.C.

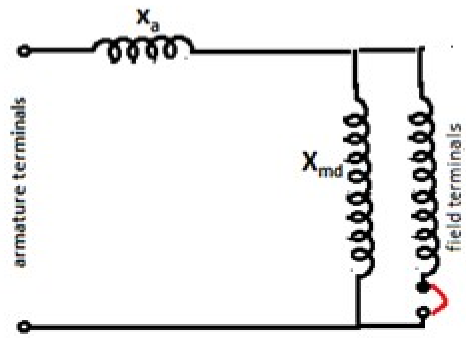



## B. Direct Axis Transient Reactance $X'_d$

For defining $X'_d$, it is assumed that the rotor only has the field winding (i.e. damper bar current is zero). Now if the peak of the suddenly applied armature mmf wave coincides with the direct axis, tends to force the flux lines through the rotor core. Since flux constancy has to be maintained, a current flows through the field winding to keep the flux same. In view of this the flux lines due to varying armature mmf wave can't pass through the field body, so the flux path is as shown in figure. Since the path is largely through air, reluctance is high and armature flux is less. Under this condition the armature flux linkage per armature ampere is call direct axis transient inductance $L'_d$ and direct axis transient reactance $X'_d = 2\pi f L'_d$ . The figures reveal that the armature flux linkage is much less compared to steady state condition so, $X'_d < X_d$. The flux path and the equivalent circuit is shown below. From equivalent circuit it follows that :

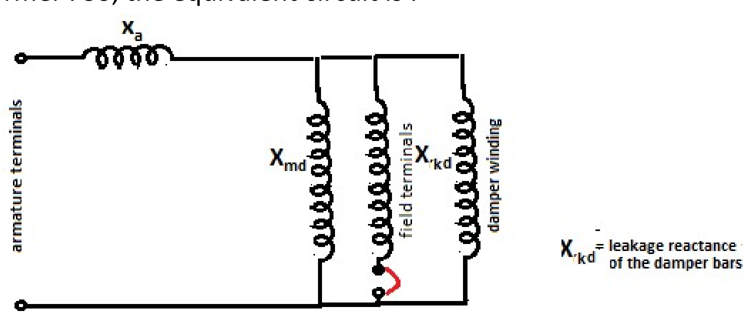
$$X'_d = x_a + \frac{1}{\frac{1}{X_f} + \frac{1}{X_{md}}}$$



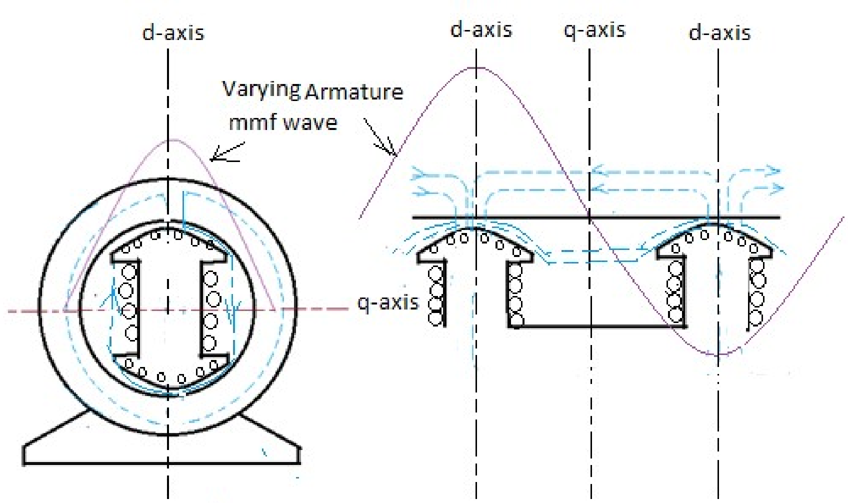



## C. Direct Axis Sub-transient Reactance $X''_d$

For sudden disturbances in the armature circuit, a change in armature current is associated with a corresponding change in the field current so as to, maintain constancy of pre-disturbance field flux linkage as per the constant flux linkage theorem. So, the field circuit is closed. As the damper bars are present on the pole face, due to sudden change in armature current, e.m.f is induced in the damper bars and currents flows. So the damper bars behaves like short circuited secondary of a transformer . So, the equivalent circuit is :




$$X''_d = x_a + \frac{1}{\frac{1}{X_f} + \frac{1}{X_{md}} + \frac{1}{X_{kd}}}$$



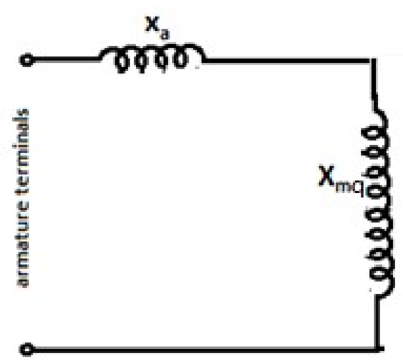
Since the damper bars are near the air gap, the flux lines are forced to follow the leakage path mainly in the air. The armature flux linkage per armature ampere under this conditions, are called direct axis sub-transient reactance $L''_d$ and $X''_d = 2\pi f L''_d$ . Comparing the flux paths it is seen that flux linkage is less during sub-transient condition compared to transient condition; consequently $X''_d < X'_d$ . even if the damper bars are absent, the effect of saturation and additional rotor circuit provided by pole-bolts, field collars etc. causes $X''_d$ to be less than $X'_d$.

## D. Quadrature axis Synchronous Reactance $X_q$

If the peak of steady armature m.m.f. wave is in line with the quadrature axis, then the reluctance offered to armature flux by the large air gap is maximum. For given armature current the armature flux has the minimum value. Under this condition armature flux linkage per ampere in the armature winding is called quadrature axis synchronous inductance $L_q$

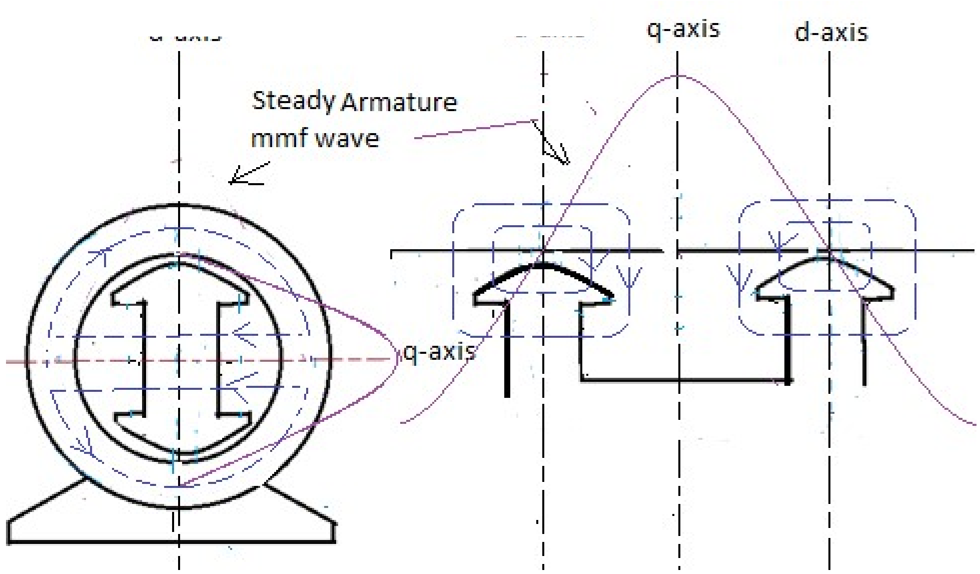
$$L_q = \frac{armature\ flux \times armature\ no.of\ turns/phase}{armature\ phase\ current}$$

$$X_q = 2\pi f L_q$$




There is no field winding in the quadrature axis. So the equivalent circuit is as shown in the figure and

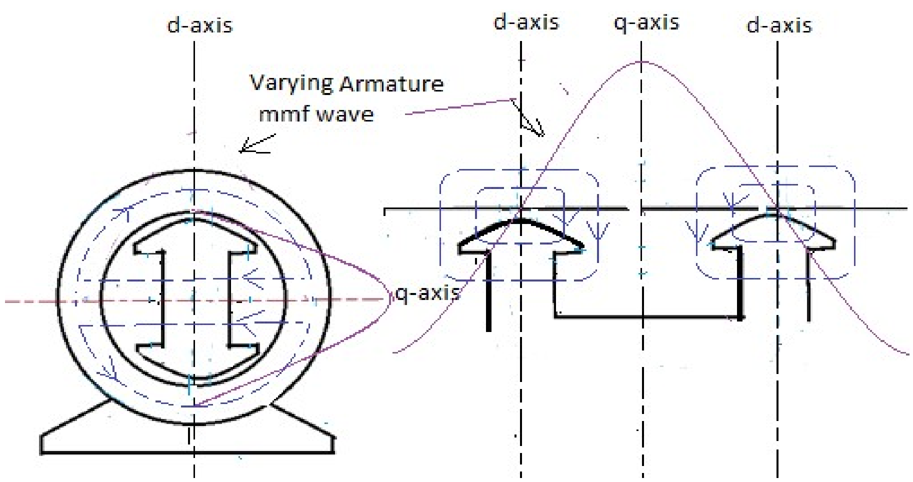
$$X_q = x_a + X_{mq}$$

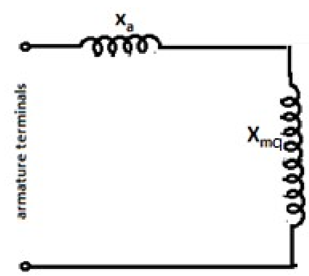


## E. Quadrature Axis Transient Reactance $X'_q$

For defining $X'_q$, it is assumed that the rotor only has the field winding (i.e. damper bar current is zero). Now if the peak of the suddenly applied armature mmf wave coincides with the quadrature axis, the flux path is as shown in figure. The flux path is perpendicular to the field winding, so there is no linkage with the field winding. Inspection reveals that the flux path is identical with the flux path for quadrature axis reactance. So, $X'_q = X_q$. Further the reluctance offered to the flux is less than the reluctance offered for direct axis transient reactance. So, $X'_q > X'_d$. The flux path and the equivalent circuit is shown below. From equivalent circuit it follows that :

$$X'_q = x_a + X_{mq}$$

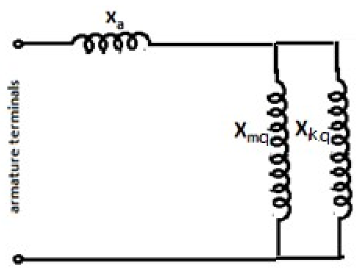


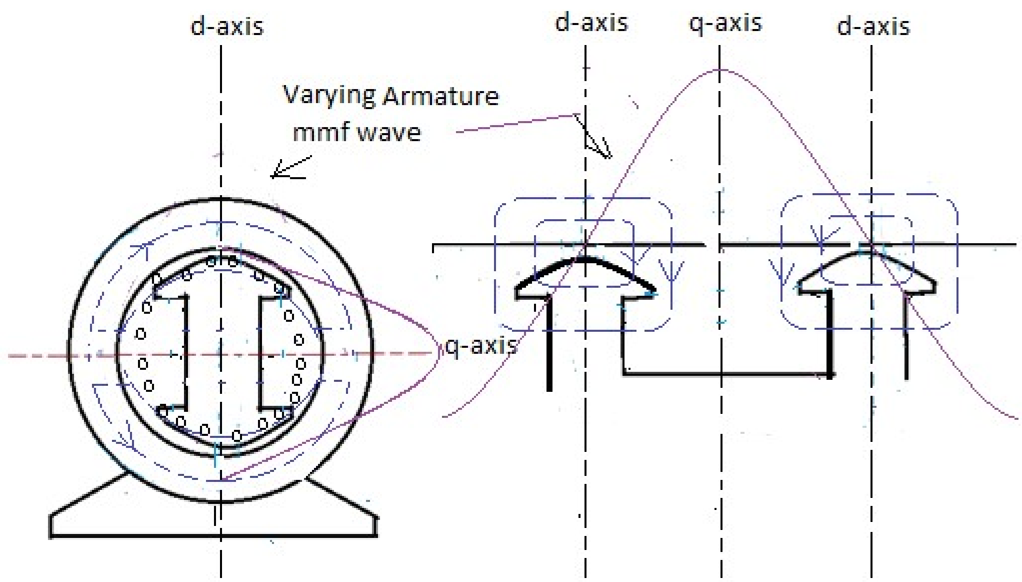

## F. Quadrature Axis Sub-Transient Reactance $X''_q$

If the armature current is suddenly applied such that the armature mmf peak is coincident with thqe quadrature axis, the damper bars in the quadrature axis forece the flux lines to follow the leakage flux , as shown in the figure. The flux linkage to the q-axis damper bars should remain zero before and after the sudden appearance of the armature flux. Under this condition the armature flux inkage per unit armature ampere is called q-axis sub-transient inductance $L''_q$ and $X''_q = 2\pi f L''_q$. As the reluctance is high $X''_q < X'_q$

It is also evident that for $X''_d$ the leakage flux path consist of a part of pole face and q-axis air space and for $X''_q$ the leakage flux path consist of a part of the q-axis air space and whole of pole face iron. In view of this for same armature mmf armature leakage flux is more in the second case. So, $X''_q$ is slightly higher than $X''_d$




$$X_q'' = x_a + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{kq}}}$$




$$X_d > X_d' > X_d''; \quad X_q = X_q'; \quad X_q' \gg X_q'' \ and\ X_d'' < X_q''$$

$$X_d = x_a + X_{md}; \quad X_d' = x_a + \frac{1}{\frac{1}{X_f} + \frac{1}{X_{md}}}; \quad X_d'' = x_a + \frac{1}{\frac{1}{X_f} + \frac{1}{X_{md}} + \frac{1}{X_{kd}}}$$

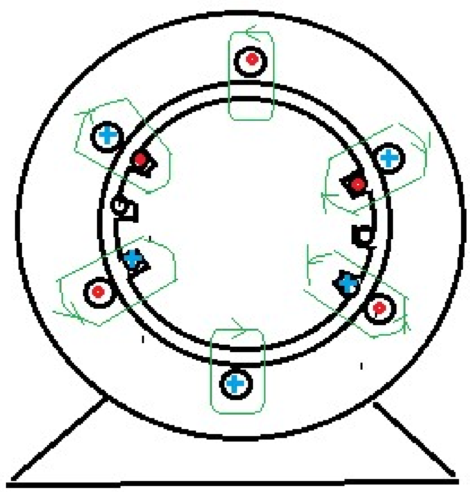
$$X_q = x_a + X_{mq} = X_q'; \quad X_q'' = x_a + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{kq}}}$$

## II. Negative Sequence Reactance:

For defining $X_2$ , the unexcited field winding is closed and is assumed on the rotor. The direction of rotation of the rotor and the armature mmf are opposite. So, the armature mmf is travelling at twice the synchronous speed w.r.t. the rotor. So the rotating mmf is aligned with the d-axis and after a few moments $\left(\frac{30}{PN_s}\ sec\right)$ it gets aligned with the q-axis. Since the flux linkage with rotor are zero, it should remain zero. So, the flux path is mainly through air, similar to the flux path for $X_d''$ and $X_q''$. The armature flux linkage per armature ampere under this condition is called negative sequence inductance $L_2$and $X_2 = 2\pi f L_2$. Since the armature mmf experiences d-axis reluctance corresponding to $X''_d$ and q-axis reluctance corresponding to $X''_q$ , $X_2$ is taken as the arithmetic mean of $X''_d$ and $X''_q$. $\therefore X_2 = (X''_d + X''_q)/2$

## III. Zero Sequence Reactance

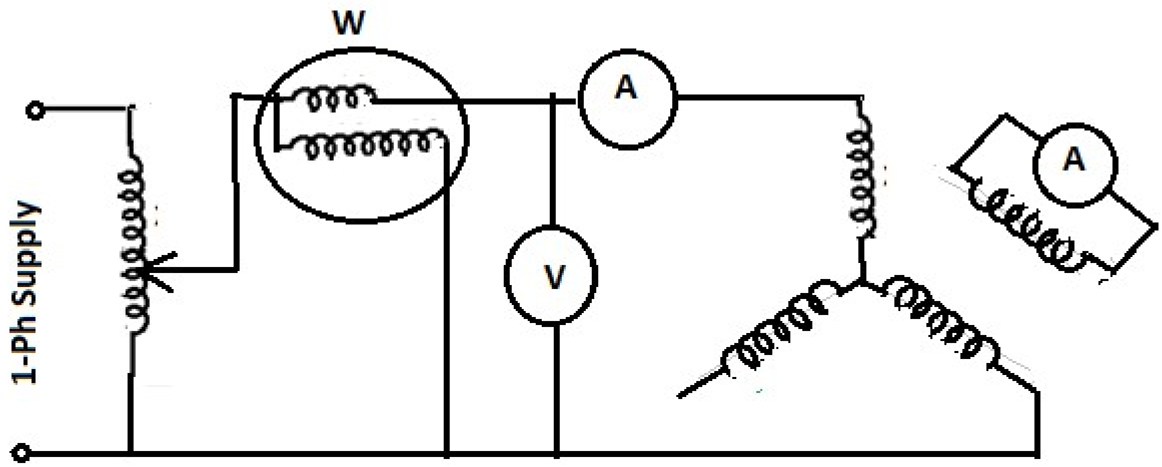
If the three phase armature winding is connected in series and a single phase supply is given to this series combination, then the armature leakage flux is produced as shown in the figure. The armature leakage flux is stationary and pulsating in nature. This pulsating armature mmf is opposed by currents induced in the closed circuits on the field structure, so air gap flux is very small. The armature flux linkage per unit armature ampere under this condition is called zero sequence inductance $L_o$ and $X_o = 2\pi f L_o$. The zero sequence reactance is smallest os all the synchronous machine reactances.



| Typical values of Synchronous Machine Constants | | | |
|---|---|---|---|
| **Machine Constant** | **Turbo-alternator** | **Hydro-generator** | **Salient Pole Motors** |
| $X_d$ | 1.20 | 1.10 | 1.20 |
| $X_q$ | 1.17 | 0.70 | 0.90 |
| $X'_d$ | 0.30 | 0.35 | 0.46 |
| $X'_q$ | 0.30 | 0.70 | 0.90 |
| $X''_d$ | 0.15 | 0.25 | 0.30 |
| $X''_q$ | 0.17 | 0.35 | 0.40 |
| $X_2$ | 0.15 | 0.25 | 0.35 |
| $X_o$ | 0.02 | 0.10 | 0.15 |

## Measurement of Reactance

### A. Measurement of $X''_d$ and $X''_q$




Two of the phases are connected in series and a single phase voltage is applied. The rotor is at rest. The voltage is adjusted to pass sufficient current in the series connected windings. The rotor position is adjusted to get maximum deflection the field ammeter. The induced current in short circuited field winding is maximum only when the field winding axis is along the direction of resultant armature mmf. Under this condition d-axis sub-transient impedance $Z''_d$ is measured. If V, $I_{max}$ and W are the voltmeter, ammeter and wattmeter readings, then

$$Z''_d = \frac{V}{2I_{max}}; \quad \cos\theta = \frac{P}{VI_{max}} \quad and \quad \sin\theta = \sqrt{1-\left(\frac{P}{V\,I_{max}}\right)^2}$$

$$\therefore \; d - axis\ subtransient\ reactance = X''_d = Z''_d \sin\theta$$

$$\therefore X''_d = \frac{V}{2I_{max}}\sqrt{1-\left(\frac{P}{V\,I_{max}}\right)^2} = \frac{1}{2{I_{max}}^2}\sqrt{(V\,I_{max})^2 - P^2}$$

Now if the shaft is rotated by hand through half a pole-pitch( i.e. 90° electrical) then the peak of the armature mmf will coincide with q-axis. At this position field circuit ammeter reading should be minimum. Under this condition the instrument readings give:

$$Z''_q = \frac{V}{2I_{min}}; \quad \cos\theta = \frac{P}{VI_{min}} \quad and \quad \sin\theta = \sqrt{1-\left(\frac{P}{V\,I_{min}}\right)^2}$$

$$\therefore \; q - axis\ subtransient\ reactance = X''_q = Z''_q \sin\theta$$

$$\therefore X_q'' = \frac{\mathrm{V}}{2\mathrm{I}_{\min}}\sqrt{1-\left(\frac{P}{V\,I_{min}}\right)^2} = \frac{1}{2{\mathrm{I}_{\min}}^2}\sqrt{(V\,I_{min})^2 - P^2}$$

If we neglect the resistance, the relations simplifies to

$$X_d'' = \frac{\mathrm{V}}{2\mathrm{I}_{\max}} \quad \text{and} \quad X_q'' = \frac{\mathrm{V}}{2\mathrm{I}_{\min}}$$

## B. Measurement of Negative Sequence Reactance:

i. **Method I:** $X_d''$ and $X_q''$ are measured and $X_2 = (X_d'' + X_q'')/2$

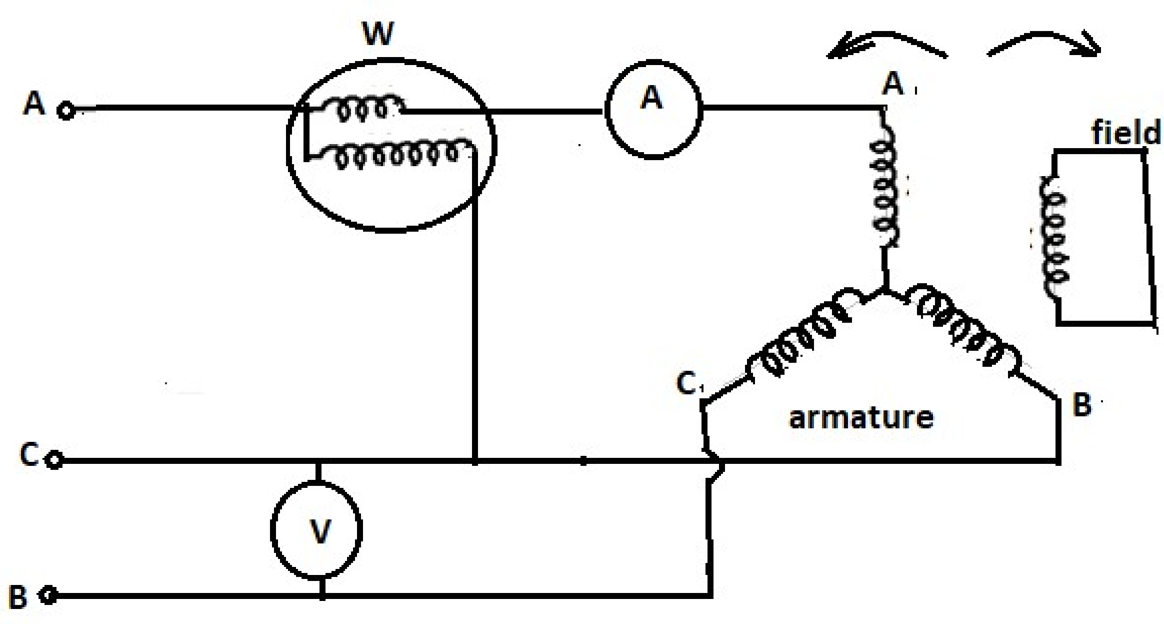
ii. **Method** II: The field winding is short circuited through an ammeter and the rotor is rotated at synchronous speed. A balanced negative sequence voltage is applied to the armature, so that the armature mmf rotates in a direction opposite to the rotor rotation. The applied voltage is adjusted and the meter readings are recorded. The negative sequence impedance is given by:

$$Z_2 = \frac{V}{\sqrt{3}I}$$

And negative sequence reactance is :

$$X_2 = Z_2 \sin\theta = \frac{V}{\sqrt{3}I}\sqrt{1-\left(\frac{P}{\sqrt{3}V\,I}\right)^2} = \frac{1}{3I^2}\sqrt{3V^2I^2 - P^2}$$

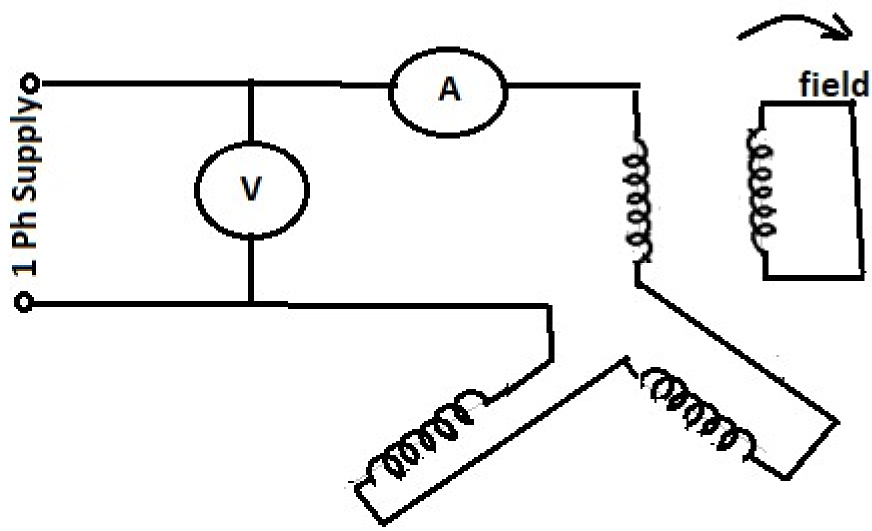
$$\text{if resistance is neglected } X_2 = \frac{V}{\sqrt{3}I}$$



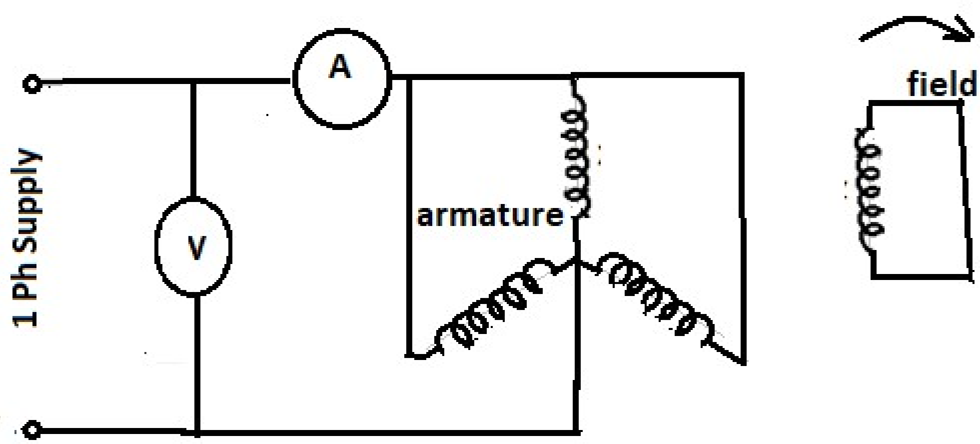
## C. Measurement of Zero Sequence Reactance:

The three phase windings are connected in series and a single phase voltage is applied. The rotor is shorted and is rotated at synchronous speed. Under this condition:

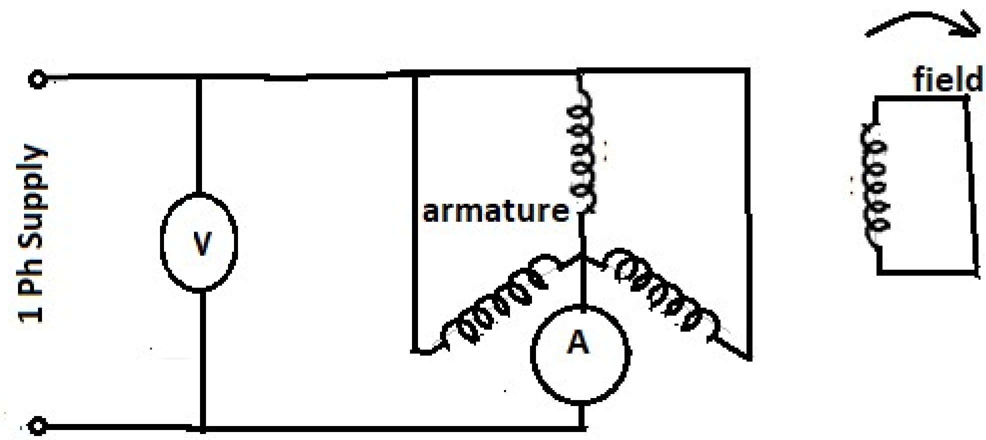
$$X_o = \frac{per\ phase\ voltage,\ V/3}{I} = \frac{V}{3I}\ \Omega$$




Only if the three line and the neutral terminals are available then the windings are connected in parallel.




$$X_o = \frac{V}{I/3} = \frac{3V}{I}\ \Omega$$




$$X_o = \frac{V}{I}\ \Omega$$