1 Open circuit and short circuit test data of of a $150 \mathrm{MW}, 13 \mathrm{kV}, 0.85 \mathrm{pf}, 50 \mathrm{~Hz}$ synchronous generator is given by :
Open Circuit Test

| $\mathrm{I}_{\mathrm{f}}(\mathrm{A})$ | 200 | 450 | 600 | 850 | 1200 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~V}_{\text {oc }}(\mathrm{L}-\mathrm{L})(\mathrm{kV})$ | 4 | 8.7 | 10.8 | 13.3 | 15 |

Short Circuit Test
$\mathrm{I}_{\mathrm{f}}=750 \mathrm{~A} \quad \mathrm{I}_{\mathrm{sc}}=8000 \mathrm{~A}$
I) Determine unsaturated synchronous reactance
II) Determine adjusted synchronous reactance. What is its pu value?
III) Determine short circuit ratio of the machine

Solution:
i) $\quad$ For $\mathrm{V}_{\text {oc }}=13 \mathrm{kV}$ on the air gap line, $\mathrm{I}_{\mathrm{sc}}=7000 \mathrm{~A}$

$$
X_{s}(\text { unsaturated })=\frac{13 \times 10^{3}}{\sqrt{3} \times 7000}=1.072 \Omega
$$

ii) For Voc $=13 \mathrm{kV}$ on $\mathrm{OCC}, \mathrm{Isc}=8600 \mathrm{~A}$

$$
\begin{gathered}
X_{s}(\text { adjusted })=\frac{13 \times 10^{3}}{\sqrt{3} \times 8600}=0.873 \Omega \\
I_{a}(\text { rated })=\frac{150 \times 10^{6}}{\sqrt{3} \times 13 \times 10^{3} \times 0.85}=7837 \mathrm{~A} \\
Z_{\text {base }}=\frac{13 \times 10^{3}}{\sqrt{3} \times 7837}=0.958 \Omega
\end{gathered}
$$

$$
X_{s}(\text { adjusted }) p u=\frac{0.873}{0.958}=0.911 p u
$$

iii) Short Circuit Ratio $(\mathrm{SCR})=\frac{I_{f} \text { required to produce rated voltage }}{I_{f} \text { required to produce rated current }}$

$$
S C R=\frac{o f \prime^{\prime}}{o \prime^{\prime \prime \prime}}=\frac{806.25}{734.7}=1.09
$$



As SCC is linear $\frac{o^{\prime} c}{o f^{\prime \prime \prime}}=\frac{o^{\prime} c^{\prime}}{o f^{\prime}}$

$$
\begin{gathered}
\therefore \frac{o f^{\prime}}{o f^{\prime \prime \prime}}=\frac{o^{\prime} c^{\prime}}{o^{\prime} c} \\
\frac{o f^{\prime}}{o f^{\prime \prime \prime}}=\frac{8600}{I_{a}(\text { rated })}=\frac{V_{\text {rated }}}{V_{\text {rated }}} \times \frac{8600}{I_{a}(\text { rated })} \\
=\frac{V_{\text {rated }}}{\sqrt{3} \times I_{a}(\text { rated })} \times \frac{\sqrt{3} \times 8600}{V_{\text {rated }}} \\
=Z_{\text {base }} \times \frac{1}{X_{S(\text { adjusted })}}=\frac{1}{X_{s(\text { ad justed }) p u}} \\
\therefore S C R=\frac{1}{0.911}=1.09
\end{gathered}
$$

2 A $6600 \mathrm{~V}, 1200 \mathrm{kVA}$ alternator has a reactance of $25 \%$ and is delivering full load at 0.8 pf lagging. It is connected to constant frequency bus-bar. If the steam supply is gradually increased, calculate:
I) At what output will the power factor become unity
II) The maximum load which it can supply without dropping out of synchronism and the corresponding power factor.

Solutions:
i) $\quad V_{p h}=\frac{6600}{\sqrt{3}}=3810 \mathrm{~V} ; \quad I_{1}=\frac{1200 \times 10^{3}}{\sqrt{3} \times 6600}=105 \mathrm{~A}$

$$
Z_{\text {base }}=\frac{3810}{105}=36.28 \Omega ; \quad \mathrm{X}_{\mathrm{s}}=0.25 \times 36.28=9.07 \Omega
$$

p.f.is $0.8 \therefore$ active component of current $I_{1 R}=105 \times 0.8=84 \mathrm{~A}$ and reactive component of current $I_{1 x}=105 \times 0.6=63 \mathrm{~A}$

$$
E_{o}=V_{p h}+j I_{1} \times(0.8-0.6 j) \times X_{s}
$$



$$
\begin{aligned}
= & 3810+j 105 \times(0.8-0.6 j) \times 9.07 \\
= & 3810+j 761.88+571.41=4447.15 \angle 9.8^{\circ} V \\
& \text { Excitation is constant } \therefore \text { for } p . f .
\end{aligned}
$$

$=1.0$ the tip of $E_{o}$ must be $A 1$

$$
\begin{gathered}
\therefore I_{2} X_{s}=\sqrt{\left(E_{o}^{2}-V_{p h}^{2}\right)} \\
=\sqrt{\left(4447.15^{2}-3810^{2}\right)}=2293.4 \mathrm{~V} \\
\therefore I_{2}=252.8 \mathrm{~A}
\end{gathered}
$$

$$
\therefore P_{\text {out }}=\sqrt{3} \times 6600 \times 252.8 \mathrm{~W}=2890 \mathrm{~kW}
$$

ii) For maximum power angle $=90^{\circ}$

$$
\begin{gathered}
I_{3} X_{s}=\sqrt{\left(E_{o}^{2}+V_{p h}{ }^{2}\right)} \\
=\sqrt{\left(4447.15^{2}+3810^{2}\right)}=5856 \mathrm{~V} \\
\therefore I_{3}=645.6 \mathrm{~A}
\end{gathered}
$$

let active component of current $I_{3 R}$ and reactive component $I_{3 X}$

$$
\therefore I_{3 R} X_{s}=4447.15 \mathrm{~V} \text { and } I_{3 X} X_{s}=3810 \mathrm{~V}
$$

$$
\therefore \quad I_{3 R}=290.3 \mathrm{~A} \text { and } I_{3 X}=420 \mathrm{~A}
$$

$\therefore \cos \varphi=\cos \left(\tan ^{-1} \frac{420}{490}\right)=0.759$ lead and $\varphi=40.58^{\circ}$ lead
$\therefore P_{\text {max }}=\sqrt{3} \times 6600 \times 645.6 \times 0.759 \mathrm{~W}=5601.57 \mathrm{~kW}$
3. A 3 phase $11 \mathrm{kV}, 10 \mathrm{MW}$, star connected synchronous generator has a synchronous impedance of $0.6+\mathrm{J} 10 \Omega / \mathrm{ph}$. if excitation is such that the open circuit voltage is 12 kV , determine:
i) maximum output of the generator
ii) current and p.f. at maximum output

Solution:

$$
\begin{gathered}
P_{\max } / p h=\frac{E_{f} V_{t}}{X_{s}} \\
E_{f}=\frac{12000}{\sqrt{3}}=6928 \mathrm{~V} V_{t}=\frac{11000}{\sqrt{3}}=6351 \mathrm{~V} X_{s}=10 \Omega \\
\therefore P_{\max } / p h=\frac{6928 \times 6351}{10}=4400 \mathrm{~kW} / \mathrm{ph} \\
\therefore P_{\max }=13200 \mathrm{~kW}=13.2 \mathrm{MW} \\
\text { ii) } I_{\max } X_{S}=\sqrt{\left(E_{f}^{2}+V_{t}^{2}\right)}=\sqrt{\left(6928^{2}+6351^{2}\right)}=9398.5 \mathrm{~V} \\
\therefore I_{\max }=\frac{I_{\max } X_{S}}{X_{s}}=\frac{9398.5}{10}=939.8 \mathrm{~A} \text { and } p . f .=\frac{E_{f}}{I_{\max } X_{S}}=\frac{6928}{9398.5}=0.737 \text { lead }
\end{gathered}
$$

4. The effective resistance of a 3 phase star connected 650 Hz 2200 V alternator is $0.5 \Omega / \mathrm{ph}$. On short circuit a field current of 40 A gives the full load current of 200A. An emf(L-L) of 1100 V is produced on open circuit with the same excitation. Determine synchronous impedance. Hence compute power angle and regulation at 0.8 lagging pf.
Solution:

$$
\begin{gathered}
Z_{S}=\frac{E_{f}(L-L)}{\sqrt{3} I}=\frac{1100}{\sqrt{3} \times 200}=3.18 \Omega \\
\therefore X_{S}=\sqrt{Z_{S}{ }^{2}-r_{a}{ }^{2}}=\sqrt{3.18^{2}-0.5^{2}}=3.14 \Omega \\
\mathrm{~V}_{\mathrm{t}}=\frac{2200}{\sqrt{3}}=1270 \mathrm{~V} \\
\mathrm{E}_{\mathrm{f}}=\mathrm{V}_{\mathrm{t}}+\mathrm{I}\left(\mathrm{r}_{\mathrm{a}}+\mathrm{jX} \mathrm{~S}_{\mathrm{s}}\right) \\
=1270+200(0.8-\mathrm{j} 0.6)(0.5+\mathrm{j} 3.14)=1726+\mathrm{j} 442=1782 \angle 14.4^{\mathrm{o}} \\
\therefore \text { power angle } \delta=14.4^{\mathrm{o}} \\
\text { regulation }=\frac{1782-1270}{1270} \times 100 \%=40.3 \%
\end{gathered}
$$

5. A 2000 kVA 11 kV 3 ph star connected alternator has a resistance of $0.3 \Omega / \mathrm{ph}$ and reactance of $5 \Omega / \mathrm{ph}$. It delivers full load current at 0.8 lagging p.f at normal voltage. Compute the terminal voltage for the same excitation and load at 0.8 leading p.f.

Solution;

$$
\begin{gathered}
V_{t}=\frac{11000}{\sqrt{3}}=6351 \mathrm{~V} \text { and } I=\frac{2000 \times 10^{3}}{\sqrt{3} \times 11000}=105 \mathrm{~A} \\
E_{f}=V_{t}+I\left(r_{a}+j X_{s}\right)=6351+105(.8-j .6)(.3+j 5) \\
\therefore E_{f}=6703.2 \angle 3.43^{o} \\
\therefore \text { for leadin } p . f . E_{f}=V_{t}+I(.8+j .6)\left(r_{a}+j X_{s}\right) \\
\therefore E_{f}=V_{t}+105(.8+j .6)(.3+j 5) \\
E_{f}=V_{t}-289.8+j 438.9 \\
\therefore E_{f}^{2}=\left(V_{t}-289.8\right)^{2}+438.9^{2} \\
V_{t} / p h=6978.6 \mathrm{~V} \text { and } V_{t}(L-L)=11776 \mathrm{~V}
\end{gathered}
$$

6. A 3 phase 2 pole $2000 \mathrm{kVA}, 6600 \mathrm{~V}, 3000 \mathrm{rpm}$ turbo alternator, has 60 armature slots with 4 conductors per slot and its effective resistance and leakage reactance are $0.1 \Omega$ and $2 \Omega$ per phase respectively. There are 10 rotor slots per pole with angular pitch of slots equal to $10^{\circ}$ and 20 conductors per slot. The open circuit characteristics are given by fig. 1. Determine regulation at full load 0.8 pf lagging.


$$
\begin{aligned}
& V_{t}=\frac{6600}{\sqrt{3}}=3816 \mathrm{~V} \\
& I=\frac{2000 \times 10^{3}}{\sqrt{3} \times 6.6 \times 10^{3}}=175 \mathrm{~A}
\end{aligned}
$$

$$
k_{w}=\frac{\sin \frac{q \gamma}{2}}{q \sin \frac{\gamma}{2}}=\frac{\sin \frac{10 \times 6^{o}}{2}}{10 \sin \frac{6^{o}}{2}}=\frac{\sin 30^{\circ}}{10 \times \sin 3^{0}}=0.955 \text { and } k_{p}=1.0
$$

$\therefore F_{a}=\frac{3}{2} \times \frac{4 \sqrt{2}}{\pi} \frac{40 \times 175}{2} 0.955 \times 1=9027.9 \mathrm{AT} /$ pole
For the field slot/pole $=10$ and conductor/slot $=20$

$$
\therefore N_{f}=\frac{10 \times 20}{2}=100 \text { and } k_{w f}=\frac{\sin \frac{10 \times 10}{2}}{10 \times \sin \frac{10}{2}}=0.879
$$

$\therefore$ for any field current $I_{f}, \quad F_{f}=\frac{4}{\pi} N_{f} I_{f} k_{w f} k_{p f}=\frac{4}{\pi} \times 100 \times 0.879 I_{f}=111.9 I_{f}$
$\therefore$ equivalent field amp.for $F_{a}=\frac{F_{a} A T / \text { pole }}{111.9}=\frac{9027.9}{111.9}=80.7 \mathrm{~A}$


$$
\begin{aligned}
& \alpha=90^{\circ}+36.87^{\circ}+3.8^{o}=130.7^{\circ} \\
& \bar{F}_{f}=\bar{F}_{r}-\bar{F}_{a} \\
& \quad \therefore\left|F_{f}\right|=\sqrt{{F_{r}}^{2}+F_{a}^{2}-2 F_{r} F_{a} \cos 130.7^{\circ}} \\
& \therefore \sqrt{215^{2}+80.7^{2}-2 \times 215 \times 80.7 \times \cos 130.7^{\circ}} \\
& \quad F_{f}=274.5 \mathrm{~A} \quad \therefore E_{f}=4500 \mathrm{~V} / \mathrm{ph} \\
&
\end{aligned}
$$

7. The full load torque angle of a synchronous motor at rated voltage and frequency is $30^{\circ}$. The motor resistance is negligible. How will the torque angle be affected by the following changes?
i) the load torque and terminal voltage remaining constant, the excitation and frequency are raised by $10 \%$
ii) the load power and terminal voltage remaining constant, the excitation and frequency are reduced by $10 \%$
iii) the load torque and excitation remaining constant, the terminal voltage and frequency are raised by $10 \%$
iv) the load power and excitation remaining constant, the terminal voltage and frequency are reduced by $10 \%$

Solution:
i) load torque is $=\frac{1}{\omega_{s}} \frac{E_{f} \times V_{t}}{X_{s}} \sin \delta$ is constant and terminal voltage $V_{t}$ is constant.

New excitation is $1.1 E_{f}$, new synchronous speed is $1.1 \omega_{s}$ and synchronous impedance is $1.1 X_{S}$

$$
\begin{aligned}
\therefore \frac{1}{\omega_{s}} \frac{E_{f} \times V_{t}}{X_{s}} \sin \delta & =\frac{1}{1.1 \omega_{s}} \frac{1.1 E_{f} \times V_{t}}{1.1 X_{s}} \sin \delta_{1} \quad \therefore \sin \delta \times 1.1=\sin \delta_{1} \\
\sin \delta_{1} & =1.1 \times \sin 30^{\circ}=0.55 \quad \therefore \delta_{1}=33.36^{\circ}
\end{aligned}
$$

ii) load power is $=\frac{E_{f} \times V_{t}}{X_{s}} \sin \delta$ is constant and terminal voltage $V_{t}$ is constant.

New excitation is $0.9 E_{f}$, new synchronous speed is $0.9 \omega_{s}$ and synchronous impedance is $0.9 X_{S}$

$$
\therefore \frac{E_{f} \times V_{t}}{X_{s}} \sin \delta=\frac{0.9 E_{f} \times V_{t}}{0.9 X_{s}} \sin \delta_{1} \quad \therefore \sin \delta=\sin \delta_{1} \quad \therefore \delta_{1}=30^{\circ}
$$

iii) load torque is $=\frac{1}{\omega_{s}} \frac{E_{f} \times V_{t}}{X_{s}} \sin \delta$ is constant and excitation $E_{f}$ is constant.

New terminal voltage is $1.1 V_{t}$, new synchronous speed is $1.1 \omega_{s}$ and synchronous impedance is $1.1 X_{S}$

$$
\begin{array}{r}
\therefore \frac{1}{\omega_{s}} \frac{E_{f} \times V_{t}}{X_{s}} \sin \delta=\frac{1}{1.1 \omega_{s}} \frac{E_{f} \times 1.1 V_{t}}{1.1 X_{s}} \sin \delta_{1} \quad \therefore \sin \delta \times 1.1=\sin \delta_{1} \\
\sin \delta_{1}=1.1 \times \sin 30^{\circ}=0.55 \quad \therefore \delta_{1}=33.36^{\circ}
\end{array}
$$

iv) load power is $=\frac{E_{f} \times V_{t}}{X_{s}} \sin \delta$ is constant and excitation $E_{f}$ is constant.

New terminal voltage is $0.9 V_{t}$, new synchronous speed is $0.9 \omega_{s}$ and synchronous impedance is $0.9 X_{s}$

$$
\begin{gathered}
\therefore \frac{E_{f} \times V_{t}}{X_{s}} \sin \delta=\frac{E_{f} \times 0.9 V_{t}}{0.9 X_{s}} \sin \delta_{1} \quad \therefore \sin \delta=\sin \delta_{1} \\
\therefore \delta_{1}=30^{\circ}
\end{gathered}
$$

8. A 1000 kVA 3 phase 11 kV star connected synchronous motor has negligible resistance and synchronous reactance of 35 ohm per phase.
I) what is the excitation emf of the motor if the power angle is $10^{\circ}$ and the motor takes rated current at a) lagging pf and b) leading pf . what is the mechanical power developed and the power factor .

rated current $I_{a}=\frac{1000 \times 10^{3}}{\sqrt{3} \times 11 \times 10^{3}}=52.5 \mathrm{~A}$

$$
V_{t}=\frac{11 \times 10^{3}}{\sqrt{3}}=6351 \mathrm{~V}
$$

$C D=I_{\mathrm{a}} \mathrm{X}_{\mathrm{s}} \cos \theta=\mathrm{E}_{\mathrm{f}} \sin 10^{\circ}$
or $52.5 \times 35 \cos \theta=0.1736 \mathrm{E}_{\mathrm{f}}$ or $1837.5 \cos \theta=0.1736 \mathrm{E}_{\mathrm{f}}$ or $\cos \theta=(0.1736 / 1837.5) \mathrm{E}_{\mathrm{f}}=.000095 \mathrm{E}_{\mathrm{f}}$
$A C=A B+B C=V_{t}+I_{\mathrm{a}} X_{s} \sin \theta=E_{f} \cos 10^{\circ}$ or $6351+1837.5 \sin \theta=0.9848 \mathrm{E}_{f}$ or $\sin \theta=0.0005359 \mathrm{E}_{\mathrm{f}}-3.51$ $\sin ^{2} \theta+\cos ^{2} \theta=1$
$\left(\frac{0.1736}{1837.5}\right)^{2} \mathrm{Ef}^{2}$

$$
+((0.9848 / 1837.5) \mathrm{Ef}-(6351 / 1837.5))^{2}=1
$$

$(0.1736)^{2} \mathrm{Ef}^{2}$

$$
+(0.9848 \mathrm{Ef}-6351)^{2}=1837.5^{2}
$$

$$
0.0301 E_{f}^{2}+.9698 E_{f}^{2}-12508 E_{f}+40335201
$$

$$
=3376406
$$

$$
E_{f}^{2}-12508 E_{f}+36958795=0
$$

$$
E_{f}=\frac{12508 \mp \sqrt{12508^{2}-4 \times 36958795}}{2}=\frac{12508 \mp \sqrt{8614885}}{2}
$$

$=\frac{12508 \mp 2935}{2}=7721$ or 4684

Excitation 7.72 kV for leading pf.


For leading pf. $\cos \theta=(0.1736 / 1837.5) \mathrm{E}_{\mathrm{f}}=(0.1736 / 1837.5) 7721$ $=0.7294$ lead
Power $=3 \times \frac{7721 \times 6351}{35} \sin 10=729.86 \mathrm{~kW}$
Excitation 4.68 kV for lagging pf.

For leading pf. $\cos \theta=(0.1736 / 1837.5) \mathrm{E}_{\mathrm{f}}=(0.1736 / 1837.5) 4684$ $=0.4225 \mathrm{lag}$

$$
\text { Power }=3 \times \frac{4684 \times 6351}{35} \sin 10=422.7 \mathrm{~kW}
$$

## From phasor for lagging pf:

$$
\begin{aligned}
& C D=I_{\mathrm{a}} \mathrm{X}_{\mathrm{s}} \cos \theta=\mathrm{E}_{\mathrm{f}} \sin 10^{\circ} \\
& \text { or } 52.5 \times 35 \cos \theta=0.1736 \mathrm{E}_{\mathrm{f}} \\
& \text { or } 1837.5 \cos \theta=0.1736 \mathrm{E}_{\mathrm{f}} \\
& \text { or } \cos \theta=(0.1736 / 1837.5) \mathrm{E}_{\mathrm{f}}=.000095 \mathrm{E}_{\mathrm{f}} \\
& A C=A B-B C=V_{t}-I_{\mathrm{I}} X_{5} \sin \theta=E_{f} \cos 10^{\circ} \\
& \text { or } 6351-1837.5 \sin \theta=0.9848 \mathrm{E}_{f} \\
& \text { or } \sin \theta=3.51-0.0005359 \mathrm{E}_{\mathrm{f}} \\
& \sin ^{2} \theta+\cos ^{2} \theta=1 \\
& \left(\frac{0.1736}{1837.5}\right)^{2} \mathrm{Ef}^{2}+((6351 / 1837.5)-(0.9848 / 1837.5) \mathrm{Ef})^{2}=1 \\
& (0.1736)^{2} \mathrm{Ef}^{2}+(6351-0.9848 \mathrm{Ef})^{2}=1837.5^{2} \\
& 0.0301 E_{f}^{2}+.9698 E_{f}^{2}-12508 E_{f}+40335201=3376406
\end{aligned}
$$

