

**APPLICATION OF CROSS TEAGER-KAISER ALGORITHM  
IN THE CLASSIFICATION OF ECG SIGNALS**

Thesis submitted in partial fulfilment of the requirements for the degree of  
**MASTER OF ELECTRICAL ENGINEERING**

By

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I hereby recommend that the thesis prepared under my supervision and guidance by **Subhabrata Naskar** entitled “**APPLICATION OF CROSS TEAGER-KAISER ALGORITHM IN CLASSIFICATION OF ECG SIGNALS**” be accepted in partial fulfilment of the requirements for award of the degree of “**MASTER OF ELECTRICAL ENGINEERING**” at **JADAVPUR UNIVERSITY**.

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I hereby declare that this thesis contains literature survey and original research work by the undersigned candidate, as part of his **Master of Electrical Engineering** studies.

All information in this document have been obtained and presented in accordance with academic rules and ethical conduct.

I also declare that, as required by these rules and conduct, I have fully cited and referenced all materials and results that are not original to this work.

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## List of Acronyms

<u>Acronyms</u>	<u>Full-Form</u>
CVD	Cardiovascular Disease
ECG	Electrocardiogram
CTKEO	Cross Teager Kaiser Energy Operator
TKEO	Teager Kaiser Energy Operator
PSD	Power Spectral Density
DWT	Discrete Wavelet Transform
PCA	Principle Component Analysis
ANN	Artificial Neural Network
HOS	High Order Spectral
SVM	Support Vector Machine
AAMI	Association for the Advancement of Medical Instrumentation
ADC	Analog Digital Converter
SVEB	Supraventricular ectopic beat
VEB	Ventricular ectopic beat
FIR	Finite Impulse Response
SRM	Structural Risk Minimization
RBF	Radial Basis Function
EEG	Electroencephalogram
EMG	Electromyogram

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# CHAPTER 1

## INTRODUCTION

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### **1.1 Importance of Automated tools in Bioelectric signal Processing**

In the modern days, a large human population all over the world suffers from various forms of cardiovascular diseases (CVD) which is a class of diseases that involve the heart and the associated blood vessels. It was reported that over 17.5 million people died, in 2012 with some form of cardiovascular disease [8]. The common causes of CVD are high blood pressure, high blood cholesterol, poor balanced diet, smoking and abnormal glucose levels, among others. Hence the early detection, prevention and treatment of cardiovascular diseases have important bearing on the health of many people. In this backdrop, the biomedical signal processing technique has emerged as an active area in research work in the last decade. The process of identifying and classifying different forms of CVD can be very troublesome for a human being. So far as the interpretation of electrocardiograms (ECG) is concerned, there is a possibility of human error during the ECG records analysis which is further aggravated by the presence of noise. Therefore, an alternative is to use computational techniques for automatic classification. The main objective of this thesis is to develop tools that can be used for automatic detection and classification of *cardiac arrhythmias*.

## **1.2 The Focus of The thesis**

The primary focus of this thesis is on the exploration of suitability of employing “Cross Teager-Kaiser Energy Operator” (CTKEO) based techniques for development of intelligent automated medical support systems for diagnosis. In this thesis an attempt has been made to show how these methods can be used for development of suitable bioelectric signal processing algorithms, coupled with support vector machine classifier that can help to diagnose type of CVD that a patient is suffering from.

It is well known that Electrocardiogram (ECG) is a low cost non-invasive test which effectively reveals clinical information related to human heart. ECG is the graphical representation of the electrical activity generated by the heart. Cardiac arrhythmia is an abnormal heart rhythm when electrical impulses in the heart do not work properly. Different variants of arrhythmias can cause different forms of ECG patterns. Therefore, automatic classification of not only the normal heart beats and the irregular heartbeats, but the segregation of different types of irregularities in the beats on the basis of commonalities in the signatures of the beats depending on the type of disease, can serve a powerful medical diagnostic tool.

In general, the task of automated diagnosis of the ailment consists of two steps, namely,

- 1) Extraction of suitable features from the input bioelectric signal, which will bear the signatures of the different types of medical disorders.

- 2) Classifying the bioelectric signals into two or more categories utilizing the extracted feature, such that each class represents a particular disease.

Feature extraction can be carried out from the input signal either directly or after subjecting the signal to a further processing, using various mathematical tools, statistical methods, transforms, etc. This thesis attempts to develop an efficient feature extraction algorithm based employing “Cross Teager-Kaiser Energy Operator” (CTKEO) which measures the similarity between two real time –functions and furthermore, to develop support vector machine based classification algorithm which can classify different arrhythmia beats utilizing the CTKEO features.

### **1.3 Previous works on ECG arrhythmia beat Classification**

In selecting an appropriate feature extraction tool, the primary aim should be to extract the hidden information present in the ECG signals, as much as possible, thereby improving the classification performance. Several feature extraction techniques are used for the analysis of ECG signals, and these are categorized as time-domain, frequency-domain and time–frequency techniques.

In the time domain, the extracted features are the heart-beat interval, duration parameters (QRS, QT, and PR) and amplitude parameters (QRS, ST) [1]. Due to the subtle changes in the amplitude and duration in the ECG, these time-domain parameters do not provide good discrimination [2]. Therefore, frequency-domain methods such as the Fourier transform and the power spectral

density (PSD) are used. However, the frequency-domain methods do not provide information regarding the evolution of the spectral information of the ECG signals with time. A proper time–frequency technique can tackle this problem.

Martis et al. [3] applied a linear method using the discrete wavelet transform (DWT) coefficients with the feature reduction technique of principal component analysis (PCA), to discriminate between normal and arrhythmia classes. A good classification accuracy of 98.78% with an artificial neural network (ANN) was obtained to classify the five main beat classes recommended by the ANSI/AAMI EC57:1998. Most of the linear experiments on ECG have been carried out on noise-free data, providing good classification accuracy. However, these linear methods may not obtain the same maximum accuracy in the presence of noise [4].

Using the fuzzy hybrid ANN classifier and High order Spectral (HOS) analysis features, ECG beats were classified [5] and a classification accuracy of 96.06% was obtained. Martis et al. [7] applied HOS cumulates with PCA; the average classification accuracy obtained was 94.52% using ANN.

Kutlu and Kuntalp [6] proposed an arrhythmia recognition system based on a combination of diverse features including higher order statistics, morphological features, Fourier transform coefficients, and higher order statistics of the wavelet package coefficients. Using a wrapper type feature selection algorithm to determine the optimal features, the optimal features are selected

similarly and then only a vector of data issued to classify the beats into the five main groups defined by the Association for the Advancement of Medical Instrumentation (AAMI). The classification accuracy values of the  $k$ -nearest neighbor were 85.59%, 95.46%, and 99.56%.

#### **1.4 Overview of The Thesis**

The work in this thesis describes the progress towards the development of CTKEO based algorithms for the classification of bioelectric signal like ECG arrhythmia signal. Section 1.2 introduced the importance of the work undertaken and summarized the approaches in this thesis to develop such classification tools. Section 1.3 discussed several significant previous attempts made to classify ECG arrhythmia beats. The rest of the thesis is organized as follows:

- Chapter 2 describes a detailed study of the ECG signal and one of the cardiovascular diseases, namely, cardiac arrhythmia. It is seen that there are several types of arrhythmic ECG beats. In this chapter the different available ECG databases have also been discussed.
- Chapter 3 presents a detailed study of “Teager Kaiser Energy Operator” (TKEO) for automatic classification of bioelectric signals using a suitable classifier. In this chapter, it has also been pointed out how TKEO can also be used for complex signals. Moreover, the possibility of coming across signals yielding negative energy, has also been examined.
- In Chapter 4, the proposed method for classification of arrhythmia beats has been discussed in details. In this method, various features are extracted from the CTKEO of a reference healthy beat and the other beats. A detailed discussion on the support vector machine (SVM) classifier is presented, which can efficiently classify arrhythmia beats into normal beats (N), paced beats (P), premature ventricular contraction beats (V) and other beats

corresponding to left bundle block (LB) and right bundle block (RB). At the end of this chapter, the results of findings using different methods are also summarized.

- Chapter 5 concludes the thesis with a general discussion and ideas for future work.



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# CHAPTER 2

## Discussions on Electrocardiogram (ECG)

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An electrocardiogram (ECG) is a record of the electric activity of the heart and has been widely used for detecting heart diseases due to its simplicity and non-invasive nature. By analyzing the electrical signal of each heartbeat, i.e., the combination of action impulse waveforms produced by different specialized cardiac tissues found in the heart, it is possible to detect some of its abnormalities. The traditional approach in the analysis of ECG recordings (for diagnosis of diseases) has been centered round the use of human expertise directly, based on knowledge acquired in this domain by specialized medical personnel. In the last decades, several works were done directed towards automatic ECG-based heartbeat classification methods. Several computational algorithms have been used, and newer algorithms and refinements are being continuously developed. Very often, not all the researchers involved in the development of this newer computational method have ready access to ECG data in abundance for implementing and testing their tools. Therefore, several institutions/ organizations have developed ECG signal databases, and provide in the following sections, a brief introduction to the ECG signal followed by a short description of cardiac arrhythmia has been given. Subsequently, some of the ECG databases made available by well-known standard institutions and used for evaluation of the

performances of computational tools being developed for automation of the diagnosis, have been discussed. These are the databases recommended in the standard protocol laid down by the “Association for the Advancement of Medical Instrumentation” (AAMI) for assessing the performance of the computational tools being developed for automating the diagnoses of cardiac diseases and described in ANSI/AAMI EC57:1998/(R)2008 (ANSI/AAMI, 2008).

## 2.1 ECG Signal

The heart is a muscle that contracts rhythmically, thereby pumping blood throughout the body. This contraction is initiated at the sinuatrial node that behaves as a natural pace-maker, and travels through the rest of the muscle. This electrical signal propagation follows a pattern [1]. As a result of this activity, electrical currents are generated on the surface of the body, introducing variations in the electrical potential of the skin surface. These signals can be captured or measured with the aid of electrodes and appropriate equipment.

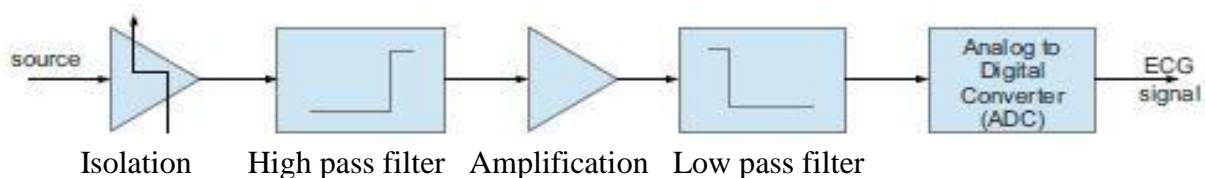


Fig 2.1: Hardware to capture ECG signal

As shown in Fig 2.1 the difference of electrical potential between the points marked by the electrodes on the skin is usually enhanced with the aid of an instrumentation amplifier with optical isolation. Then the signal is fed to a high-pass filter and subsequently conditioned by an anti-aliasing low-pass filter prior to digitization by an analog-to-digital converter (ADC). The graphical registration of this acquisition process is called electrocardiogram (ECG). Although Augustus Desiré Waller demonstrated the first human ECG in 1887 [2], the ability to recognize

the normal cardiac rhythm and/or arrhythmias did not become routine in medical check-ups until 1960. At present, there are many approaches to measure/record the ECG. da Silva et al. [3] provided a taxonomy of the state-of-the-art ECG measurement methods: (i) in-the-person, (ii) on-the-person and (iii) off-the-person.

Within the in-the-person category, there are equipments designed to be used inside human body, such as surgically implanted ones, subdermal applications or even ingestions in the form of pills. These devices are used when less invasive approach are not applicable.

In contrast with the in-the-person category, there is the off-the-person category. Devices of this variety are designed to measure ECG without or with minimal skin contact. According to [3], this class is oriented towards the future when the all-pervasive use of computer systems becomes a reality in medical applications as well. Belonging to this category are those equipment that are based on capacitive devices which measure the electric field changes introduced by the body, permitting ECG measurements at distances of 1 cm or more from the body even with clothing on it [3–5].

Most of the devices used for ECG recording belong to the on-the-person category. Devices in this class normally necessitate the use of electrodes attached to the surface of the skin. Examples of such equipments are bedside monitors and holters. The standard devices used for heart-beat analysis come from this class. In equipments belonging to this group, three or more electrodes are used to obtain the signals, in which one of the electrodes serves as a reference for the others. Usually, the reference electrode is placed near the right leg. As such, there can be different visions of the ECG signal, depending on the pair of electrodes chosen to obtain the signal. These differentiated visions are given the name of leads.

A dominant configuration of electrodes is one consisting of 5 electrodes [6]: (i) one of the electrodes is positioned on the left arm (LA), (ii) one on the right arm (RA), (iii) one on the left leg (LL), (iv) one on the right leg (RL) and (V) one on the chest, to the right of the external (V or V1). Another widely employed setup uses 10 electrodes [6], where 5 extra electrodes (besides V or V1 on the chest and LA, RD, LL and RA on legs and arms) are positioned on the chest (V2 to V6) allowing a formation of 12 leads.



Fig. 2.2 – Morphology of the curve for leads I, II and III.

From these configurations, several different leads can be constructed to visualize the ECG signal. For example, Fig. 2.2 illustrates 3 particular leads formed by: (I) the electrical potential difference between the LA and RA electrodes; (II) the electrical potential difference between the LL and RA electrodes; and (III) the electrical potential difference between the LL and LA electrodes.

Normal rhythm produces four entities such as:

- The P wave representing atrial depolarization.
- The QRS complex representing ventricular depolarization.
- The T wave representing ventricular repolarization, and
- The U wave representing papillary muscle repolarization.

It is to be noted that the U wave is not typically seen and its absence is usually ignored. These waves correspond to the field induced by the electrical phenomena occurring on the surface of the heart. The presence of cardiac arrhythmias in the subjects can deeply change these waves. Lead V and its correlate leads (V1, V2) aid the classification of ventricular related arrhythmias, since there are electrodes positioned on the chest, improving the registry of action potentials on ventricular muscle.

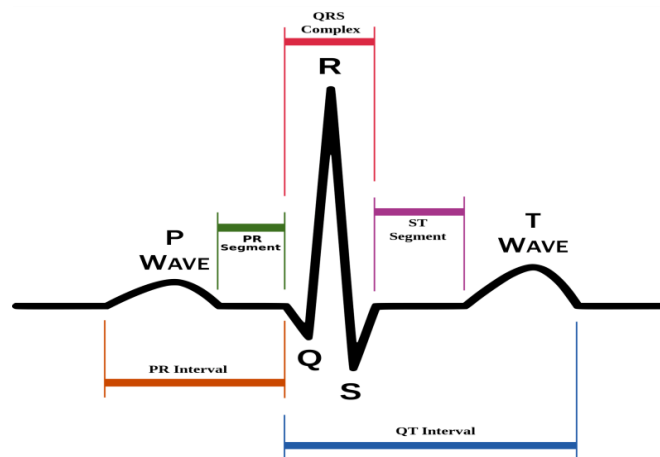


Fig. 4.3: Fiducial points and various usual intervals (waves) of a heartbeat.

Consequently, the leads most used for the automatic heart-beat and arrhythmia classification are leads II and V and the methods that use a combination of these two leads (and other combinations) are the ones that are found to yield the best results [7]. A recent work by Tomasic and Trobec [8] reviews methods working with reduced numbers of leads and approaches for the synthesis of leads, inferring that the traditional 12 lead system can be synthesized from a smaller number of measurements [9]. In contrast, another study published by de Chazal [20] demonstrated that similar effectiveness for ECG arrhythmia classification can be obtained at a lesser computational cost when using only one lead, compared with methods using multiple leads [10].

Although on-the-person equipments are the mainstream devices aiming at heart-disease diagnosis, it has been shown [3] that data acquired by off-the-person devices can be highly correlated to the data acquired with the traditional on-the-person equipments. The authors claim that off-the-person equipments can extend preventive medicine practices by allowing ECG monitoring without interference on daily routine. Hence, they have encouraged researchers to build ECG databases based on off-the-person devices to evaluate and validate heartbeat classification methods for that category.

## **2.2 Arrhythmia**

Cardiac arrhythmia is a cardiac condition when electrical impulses in the heart do not work properly. Cardiac arrhythmia, also known as cardiac dysrhythmia or irregular heartbeat, stands for a group of conditions in which the heartbeat is irregular, too fast, or too slow. A heartbeat that is too fast -above 100 beats per minute in adults - is called tachycardia and a heartbeat that is too slow - below 60 beats per minute - is called bradycardia [18]. There are two types of arrhythmias called atrial and ventricular arrhythmia. About 80% of sudden cardiac deaths result from ventricular arrhythmias. The arrhythmias can be classified into two major categories. The first category is represented by a single irregular heartbeat, and is called morphological arrhythmia. The other category formed by a set of irregular heartbeats, is known as rhythmic arrhythmia.

Arrhythmia may be classified on the basis of:

- Rate (tachycardia, bradycardia)
- Mechanism (automaticity, re-entry, triggered), and

- Duration (isolated premature beats, couplets, runs, that is 3 or more beats, non-sustained= less than 30 seconds or sustained= over 30 seconds).

Although various types of cardiac arrhythmias exist, the “Association for the Advancement of Medical Instrumentation” (AAMI) recommends that only some types should be detected by equipment/methods. There are 15 recommended classes for arrhythmia that are classified into 5 super classes: Normal (N), Supraventricular ectopic beat (SVEB), Ventricular ectopic beat (VEB), Fusion beat (F) and Unknown beat (Q) [17]. Table 1 illustrates the 15 classes and their symbols, as well as the hierarchy of the 5 groups (super classes).

Group	Symbol	Descriptions
N	N	Normal beats
	L	Left bundle branch block beat
	R	Right bundle branch block beat
	e	Atrial escape beat
	j	Nodal (junctional) escape beat
SVEB Supraventricular ectopic beat	A	Atrial premature beat
	a	Aberrated atrial premature beat
	J	Nodal (junctional) premature beat
	S	Supraventricular premature beat
VEB ventricular ectopic beat	V	Premature ventricular contraction
	E	Ventricular escape beat
F Fusion beat	F	Fusion of ventricular and normal beat
Q Unknown beat	P	Paced beat
	f	Fusion of paced and normal beat
	U	Unclassifiable beat

### **2.3 The Databases and The AAMI Standard**

Various databases composed of cardiac heartbeats grouped in patients’ records, are freely available, and have permitted the creation of standardization for the evaluation of automatic arrhythmia classification methods. This standardization, developed by AAMI, is specified in



ANSI/AAMI EC57:1998/(R) 2008 [11] and defines the protocol to perform the evaluations to make sure the experiments are reproducible and comparable.

The use of five databases is recommended by the standardization:

1. **MIT-BIH**: It is unique since it contemplates the five arrhythmia groups proposed by AAMI as described in Table 1. This database contains 48 records of heartbeats at 360 Hz for approximately 30 min of 47 different patients. Each record contains two ECG leads and in the majority of them the principal lead (lead A) is a modification of lead II (electrodes on the chest). The other lead (lead B) is usually lead V1, modified, but in some records, this lead is known to be V2, V5 or V4 [12]. Generally, lead A is used to detect heartbeats, since the QRS complex is more prominent in this lead. Lead B favors the arrhythmic classification of the types SVEB and VEB. More information regarding this database can be found in [13].
2. **EDB**: The EDB database is a collection of 90 records acquired from 79 subjects, sampled at 250 Hz with 12-bit resolution. These records were extracted from 70 men (between 30 and 84 years old) and 8 women (between 55 and 71 years old). As all of these subjects were suffering from a specific cardiac disease (*i.e.*, myocardial ischemia), the database was originally built to allow ST-segment and T-wave analysis. More information regarding this database can be found in [14].
3. **AHA**: The AHA database consists of 155 records, each one composed of two leads, sampled at 250 Hz with 12-bit resolution. Each recording is three hours long and only the final 30 min have been annotated. The database was created to evaluate ventricular

arrhythmia detectors. However, the database does not differentiate normal sinus rhythm from supra ventricular ectopic beats (SVEB).

4. **CU:** The CU database is composed of 35 eight-minute ECG recordings, sampled at 250 Hz with 12-bit resolution. The database was intended to evaluate algorithms aiming at detecting episodes of sustained ventricular tachycardia, ventricular flutter, and ventricular fibrillation. More information regarding this database can be found in [15].
5. **NSD:** NSD database includes 12 half-hour ECG recordings and 3 half-hour noise recordings. The noise inserted in the recordings is typical interferences found in ambulatory care services, such as baseline wander, muscle artefact (EMG) and electrode motion artefact. The ECG recordings available in the NSD database were created based on two clean recordings from MIT-BIH (118 and 119). This database is more detailed described in [16].

The most representative database for arrhythmia is the MIT-BIH, and because of this, it has been used in most of the published research. It was also the first database available for this goal and has been constantly refined over the years [13]. Here in this study we have used MIT-BIH database.

The majority of the heartbeats recorded in these databases have annotations associated with the type of heartbeat or the events. These heartbeat annotations, as much for the class and for the salient points (*e.g.*, point R, maximum amplitude of the heartbeat) are fundamental for the development and evaluation of automatic arrhythmia classification methods.

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## CHAPTER 3

### Teager Kaiser Energy Operator

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In the investigations on non-stationary signals, the estimation of the signal energy in the time-domain as well as in the frequency domain using Time-Frequency distributions is absolutely essential. Such signals are frequently encountered in speech-processing, in radar engineering, and also in geophysical, biological, and transient signal analysis and signal processing. Different types time-frequency distributions have already been examined and implemented by several research workers [1- 3].

Useful signals very often get contaminated with noise that can be extrinsic (picked up from the environment) or intrinsic (arising in sensors or in communication channels) in nature. Consequently, the computation of these time-frequency distributions can be regarded as energy estimation problem in the presence of noise. The present work starts by introducing the concept of signal energy in general, followed by Kaiser's "alternative" definition of energy operator by considering a second order differential equation, which describes the motion of an object suspended by a spring.

### 3.1 Signal Energy

In electrical systems, the instantaneous power of a system can be described as

$$p(t) = |v(t)|^2 / R \quad \text{or} \quad p(t) = |i(t)|^2 \times R \quad (3.1)$$

Where  $R$ ,  $v(t)$ ,  $i(t)$  are the resistance, the voltage and the current in the system, respectively.

Normalizing the above equation by choosing  $R = 1\Omega$ , it is seen that the power is the square of the input signal, regardless of whether one choose to measure the voltage or the current. Therefore expression of the instantaneous power is defined as

$$p(t) = |s(t)|^2 \quad (3.2)$$

Where  $s(t)$  is either voltage or current.

The energy of the signal over a duration extending from  $-T_0$  to  $+T_0$  is

$$E_{T_0} = \int_{t=-T_0}^{T_0} |s(t)|^2 dt \quad (3.3)$$

$\therefore$  The total energy of the signal is defined as

$$E = \lim_{T_0 \rightarrow \infty} E_{T_0} = \lim_{T_0 \rightarrow \infty} \int_{-T_0}^{T_0} |s(t)|^2 dt = \int_{-\infty}^{+\infty} |s(t)|^2 dt \quad (3.4)$$

It is quite difficult to define the term “Energy” of a signal in the context of signal processing. In physics, energy is a measure of the work done or work that can be done in a system. However, people are often unaware or do not have access to the system in which the signal arises. Therefore, the signal processing definition of energy differs from that used in physics. The definition used in signal processing considers only signal amplitude and may yield the same value of energy for two signals with same amplitude but different frequencies, although the work done in generating the two signals may differ [4]. As an example, the energy content of two

sinusoids with same amplitude but with frequencies of 100 Hz and 1000 Hz turns out to be same over a time-span of say 0.01 s.

To alleviate this shortcoming, Kaiser [5] introduced a measure of energy, based on an unpublished work by Teager [6] that takes into account the amplitude as well as the frequency of the signal. Thus, according to this Teager-Kaiser definition of energy, signals with different frequencies but identical amplitudes contain different energy. Such a definition, often described as a nonlinear-energy operator, is different from the classical energy measure also because it is an instantaneous measure of energy. In other words, it is a function of time and can therefore track the changes in the signal, and consequently the system-energy.

### **3.2 Teager Kaiser Energy Operator**

As already pointed out Teager-Kaiser Energy Operator was first proposed by Teager [6] and investigated by Kaiser [5]. It has several applications, e.g. in separation of Amplitude Modulation (AM) from Frequency Modulation (FM) [10], in the detection of transient signals [13], for suppression of spike noise in radio receivers [11], and in the detection of bearing fault in industrial applications [12]. Kaiser used the following differential equation as the starting point for introducing the non-linear energy operator.

$$\frac{d^2x(t)}{dt^2} + \frac{k}{m}x(t) = 0 \quad (3.5)$$

This second order differential equation is derived from Newton's law of motion of an object with mass  $m$  suspended by a spring with stiffness  $k$ . This is a simple model of a mechanical-acoustical system, where the object may oscillate thus creating pressure waves in the surrounding medium, although none of the medium's characteristics are included in the model.

The solution of equation 3.5 is a periodic oscillation given by  $x(t) = A \cos(\omega t + \phi)$  where  $x(t)$  is the position of the object at time  $t$ ,  $A$  is the amplitude of the oscillation,  $\omega = \sqrt{\frac{k}{m}}$  is the angular frequency of the oscillation; and  $\phi$  is the initial phase. If  $\phi \neq 0$  then that the object is not initially in equilibrium.

The total energy of the object is the sum of the potential energy of the spring and the kinetic energy of the object, given by

$$E(t) = \frac{1}{2} kx^2(t) + \frac{1}{2} mv^2(t) \quad (3.6)$$

By substituting  $v(t) = \frac{dx(t)}{dt}$ , one gets and  $x(t) = A \cos(\omega t + \phi)$ , we get

$$\begin{aligned} E(t) &= \frac{1}{2} kA^2 \cos^2(\omega t + \phi) + \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \phi) \\ &= \frac{1}{2} m\omega^2 A^2 [\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)] \quad [ \because k = m\omega^2 ] \\ \therefore E &= \frac{1}{2} m\omega^2 A^2 \end{aligned} \quad (3.7)$$

From the above equation it is seen that the energy ( $E$ ) of the object is proportional to both  $A^2$  and  $\omega^2$ . It is also a function of time.

From equation (3.7) one can easily see that the energy to generate a simple sinusoidal signal varies as a function of both amplitude and frequency. This observation is what Kaiser used to derive the Teager Energy Operator.

### **3.2.1 The Continuous Teager Kaiser Energy Operator**

“Teager Kaiser Energy Operator” (TKEO), can be defined in a manner so as to serve as a local energy measure for oscillating (simple harmonic) signals. TKEO computes the energy of a real-



valued signal  $x(t)$  using the follows expression.

$$\psi(x(t)) = \left( \frac{dx(t)}{dt} \right)^2 - x(t) \frac{d^2x(t)}{dt^2} \quad (3.8)$$

This energy operator characterizes the localized property of the signal and depends only on the values assumed by the signal and its first and second time derivatives at the particular instant of time under consideration, and not on any span of time. TKEO is an energy tracking method that has been found to have low computational complexity, quite good time-resolution and ease of efficient implementation.

### 3.2.2 The Discrete Operator

When equation (3.8) is translated to an equivalent discrete-time form by using the backward-difference approximation

$$\dot{x}[n] \approx \frac{x[n] - x[n-1]}{T_s}, \text{ for small value of } T_s,$$

It becomes

$$\psi[x[n]] = \frac{[x[n]]^2 - x[n-1]x[n+1]}{T_s^2} \quad (3.9)$$

where  $T_s$  is the sampling period (i.e. the time-interval between every two adjacent samples),  $x[i]$  and  $x[i+1]$ . In many cases, an assumption of  $T_s = 1$  unit is quite acceptable, since the digital frequency in radian/sample bears the information of  $T_s$ . As a result, a final discrete-time version of TKEO is obtained as

$$\psi[x[n]] = [x[n]]^2 - x[n-1]x[n+1] \quad (3.10)$$

This result is very significant as it shows that the “Teager-Kaiser Energy” (TKE) of a time varying sinusoidal signal can be calculated by considering only three consecutive sampled data.

### 3.2.3 Energies of well-known signals

#### AM Signal:

Amplitude modulated (AM) signal is the combination of two signals, where one signal is the “carrier”, which is a single-frequency sinusoidal signal and the other, is the information we want to transmit, the baseband signal. We can model the amplitude modulated signal as

$$a(t) = A[1 + km(t)]$$
$$s_{AM}(t) = a(t)\cos(\omega_c t)$$

Where  $A > 0$  is the signal amplitude,  $\omega_c$  is the carrier frequency (in radians/second),  $-1 \leq m(t) \leq 1$  is the baseband signal, and  $0 < k \leq 1$  is the modulation factor. These AM signals are called AM signals with carrier, as we require  $a(t)$  being positive.

The Teager Energy of an AM signal is

$$\begin{aligned}\psi(s_{AM}(t)) &= \dot{a}^2(t)\cos^2(\omega_c t) + a^2(t)\omega_c^2 - a(t)\cos^2(\omega_c t)\ddot{a}(t) \\ &= a^2(t)\omega_c^2 + \cos^2(\omega_c t)\psi(a(t))\end{aligned}\tag{3.11}$$

From the above equation it can be seen that the Teager Energy of an AM signal is composed of a term similar to the energy of a sinusoidal signal, and an oscillatory part scaled by the Teager Energy of the amplitude signal. Figure 3.1 shows a sample AM signal and its Teager Energy [14]. It is worth noting the similarity between the envelope of the AM signal and the output of the Teager Kaiser Energy Operator.

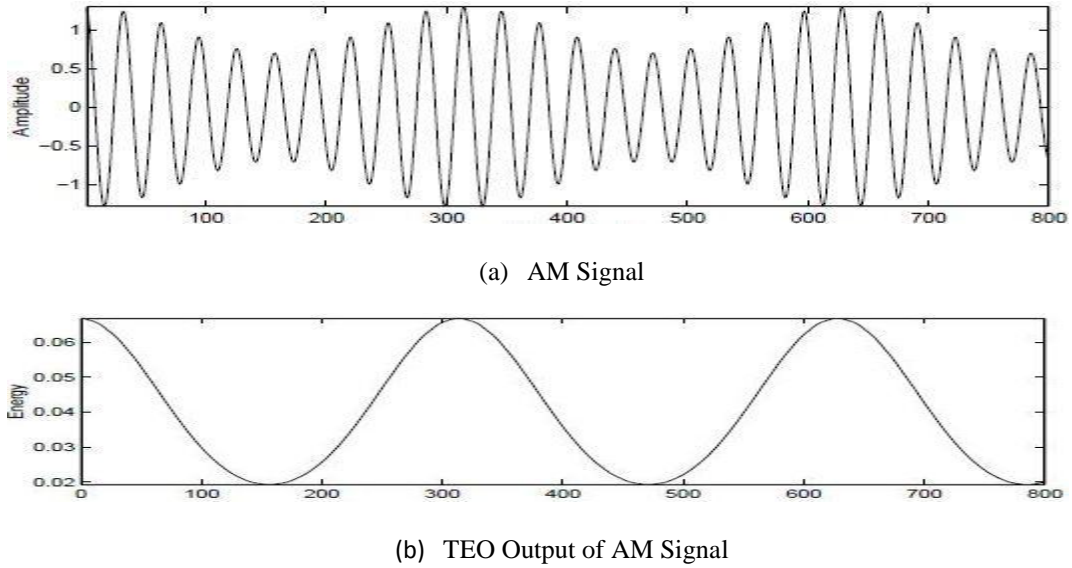


Figure 3.1: Teager Energy of an AM signal

### ✚ FM signals

A frequency modulated signal (FM signal) can be modeled as

$$\phi(t) = \omega_c(t) + \omega_m \int_0^t m(\tau) d\tau + \theta$$

$$s_{FM}(t) = A \cos(\phi(t))$$

The Teager Energy of the above FM signal can be expressed as

$$\psi(s_{FM}(t)) = A^2 (\dot{\phi}^2(t) + \ddot{\phi}(t)) \frac{\sin(2\phi(t))}{2} \quad (3.12)$$

In figure 3.2, a sample FM signal is shown. Here, the baseband signal is a pure sinusoidal signal.

The output of the Teager Energy Operator in this case is a sinusoidal signal with the same frequency as the baseband signal [14].

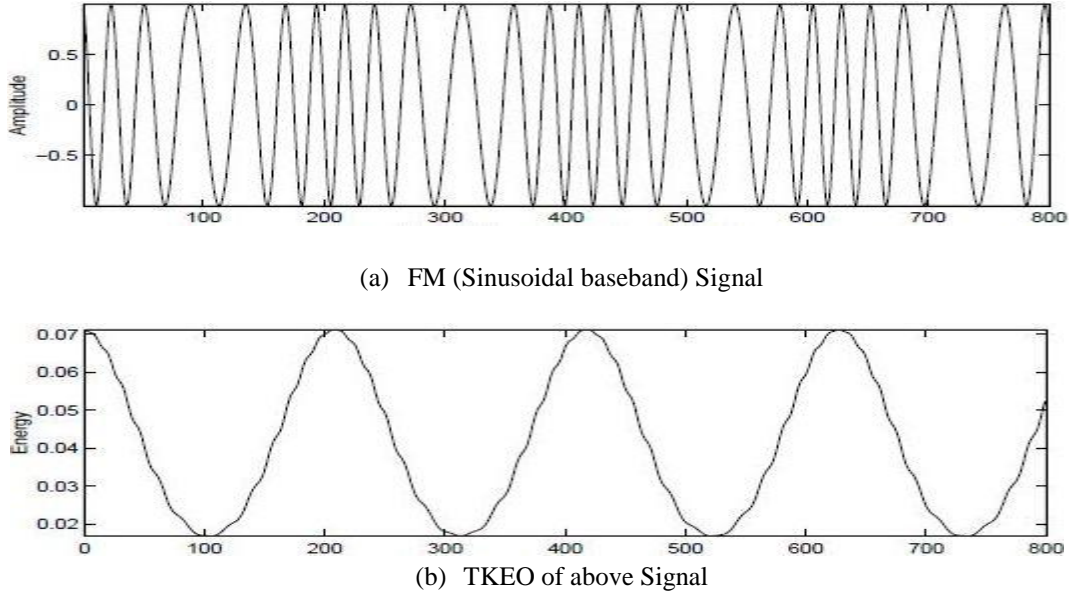


Figure 3.2: Teager Energy of a FM signal where the baseband signal is a sinusoid.

### ✚ AM-FM signals

The most complex and general case is that of the AM-FM signal. The AM-FM signal is the combination of AM and FM signals and it can be modeled in the following manner.

$$a(t) = A[1 + km_{AM}(t)] \quad (\text{For AM Signal})$$

$$\phi(t) = \omega_c(t) + \omega_m \int_0^t m_{FM}(\tau) d\tau + \theta \quad (\text{For FM Signal})$$

$$s_{AM-FM}(t) = a(t) \cos(\phi(t))$$

The Teager Energy of the above AM-FM signal obtained as

$$\psi(s_{AM-FM}(t)) = [a(t)\dot{\phi}(t)]^2 + \frac{1}{2} a^2(t) \ddot{\phi}(t) \sin(2\phi(t)) + \cos^2(\phi(t)) \psi(a(t)) \quad (3.13)$$

Here 2<sup>nd</sup> term of the above equation represents FM signal, while the 3<sup>rd</sup> term represents AM signal. It is observed from the above expression that the similar term from both the AM and FM signal energies (from eq. 3.11 and 3.12) appear in the expression.

In figure 3.3, an AM-FM signal is shown. In this case, the operator's tracking capabilities are not evident, although one can locate some correlation between the peaks of the Teager Energy and the zero-crossings of the AM-FM signal [14].

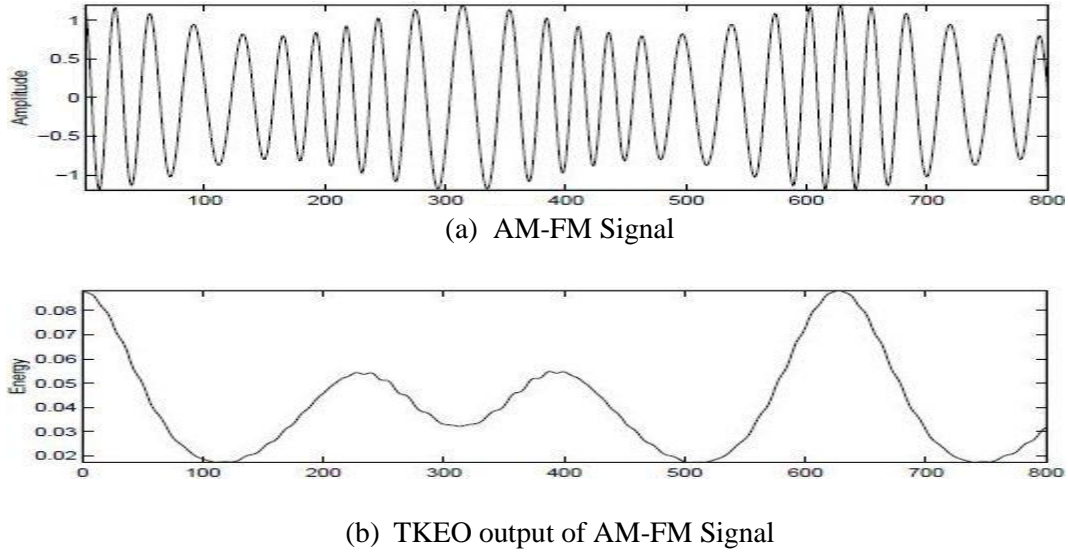


Figure 3.3: Teager Energy of an AM-FM signal

### 3.2.4 Extension to complex signals

The Teager Operator can also be extended to cover complex signals. As the Operator is an energy operator, one can expect it to always give positive values.

P. Maragos *et. al* in their work on image demodulation using multidimensional energy separation [7] have defined the TKEO for complex signals as

$$\Psi_c(x(t)) = \left\| \dot{x}(t) \right\|^2 - \text{Re}[x^*(t) \ddot{x}(t)] \quad (3.14)$$

Hamila *et. al* [8] have defined the operator for complex signals as

$$\Psi_c(x(t)) = \dot{x}(t)^* \dot{x}(t) - \frac{1}{2} [\ddot{x}(t)x^*(t) + x(t)\ddot{x}^*(t)] \quad (3.15)$$

Although written differently, the operators above are equal because

$$\left\| \dot{x}(t)^2 \right\| = \dot{x}(t)^* \dot{x}(t)$$

By definition and omitting  $t$  for clarity, one gets

$$\begin{aligned} \operatorname{Re}[x^*(t) \ddot{x}(t)] &= \operatorname{Re}[(x_r - jx_j)(\ddot{x}_r + j\ddot{x}_j)] \\ &= \operatorname{Re}[x_r \ddot{x}_r + jx_r \ddot{x}_j - jx_j \ddot{x}_r + x_j \ddot{x}_j] \\ &= x_r \ddot{x}_r + x_j \ddot{x}_j \end{aligned} \quad (3.16)$$

and

$$\begin{aligned} \frac{1}{2}[\ddot{x} \dot{x}^* + \dot{x} \ddot{x}^*] &= \frac{1}{2}[(\ddot{x}_r + j\ddot{x}_j)(x_r - jx_j) + (\ddot{x}_r - j\ddot{x}_j)(x_r + jx_j)] \\ &= \frac{1}{2}[x_r \ddot{x}_r + jx_r \ddot{x}_j - jx_j \ddot{x}_r + x_j \ddot{x}_j + x_j \ddot{x}_j + x_r \ddot{x}_r - jx_r \ddot{x}_j + jx_j \ddot{x}_r] \\ &= x_r \ddot{x}_r + x_j \ddot{x}_j \end{aligned}$$

When  $x(t)$  is a real signal, the operator reduces to the conventional definition of the Teager Operator. We can gain more insight if we write the complex operator as the sum of the real and imaginary parts:  $x(t) = x_r(t) + jx_j(t)$

$$\begin{aligned} \Psi_C(x(t)) &= \Psi_C[x_r(t) + jx_j(t)] \\ &= \dot{x}_r(t)^2 + \dot{x}_j(t)^2 - x_r(t) \ddot{x}_r(t) - x_j(t) \ddot{x}_j(t) \\ &= \Psi_R(x_r(t)) + \Psi_R(x_j(t)) \end{aligned} \quad (3.17)$$

Equation (3.17) implies that the Teager Energy of a complex signal is the sum of the energies of the real and the imaginary parts of the signal.

### 3.2.5 Positivity of the Energy Operator

As the Teager Energy Operator is an energy operator, and energy is a non-negative quantity, one must investigate whether the Teager Energy Operator is zero or positive for all signals. This

issue was first addressed by Petros Maragos [9], and was later discussed elaborately by Alan C. Bovik and Petros Maragos [11].

### Real-valued signals

The Continuous Teager Energy Operator has already been defined as

$$\psi(x(t)) = \left( \frac{dx(t)}{dt} \right)^2 - x(t) \frac{d^2x(t)}{dt^2}$$

The only way to have the right hand side nonnegative, is to have

$$\left( \frac{dx(t)}{dt} \right)^2 \geq x(t) \frac{d^2x(t)}{dt^2} \quad (3.18)$$

Thus, for real signals, the Teager Energy Operator is nonnegative if any of the following conditions is satisfied.

1.  $x(t)=0$
2.  $\frac{d^2x(t)}{dt^2}=0$
3.  $x(t) < 0$  and  $\frac{d^2x(t)}{dt^2} > 0$
4.  $x(t) > 0$  and  $\frac{d^2x(t)}{dt^2} < 0$

This is because the right hand side of the inequality 3.18 is negative or zero for all of these cases.

The left hand side is always positive, irrespective of the value of  $\left( \frac{dx(t)}{dt} \right)$ , as it is the square of a quantity. Thus, in these cases,  $\Psi(x(t)) \geq 0$ .

### Complex signals

From the equation 3.15, the complex Teager Energy Operator is defined as

$$\Psi_c(x(t)) = \dot{x}(t)^* \dot{x}(t) - \frac{1}{2} [\ddot{x}(t) x^*(t) + \ddot{x}(t)^* x(t)]$$

The above expression reduces to the traditional form whenever  $x(t)$  is a real signal. Treating  $x(t) = x_r(t) + jx_j(t)$  as the input to the TEO, the output is  $\Psi_c(x(t)) = \Psi_c[x_r(t) + jx_j(t)]$ . It implies that the Teager-energy will be positive as long as the largest absolute term is positive. Effectively, the output of the TEO is positive for a broader range of signals, as one of the terms is now allowed to be negative.

To have an insight of the positivity constraint, Fig. 3.4 sketches the complex plane, with the output of  $\Psi_r(x_r(t))$  as the abscissa and the output of  $\Psi_r(x_j(t))$  as the ordinate. Teager energies in the shaded area are negative.

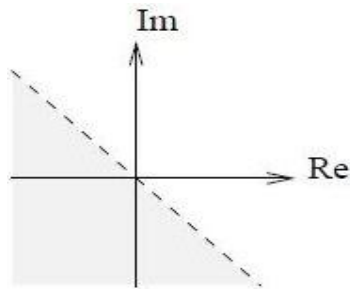


Figure 3.4: Complex plane visualizing the region where the Complex Teager Energy Operator is negative.

### **3.3 Cross Teager Kaiser Energy Operator**

Cross Teager–Kaiser Energy Operator (CTKEO) was introduced by Kaiser [15], primarily to have a measure of the interaction between two real signals  $x(t)$  and  $y(t)$ , and it can be expressed as

$$\Psi(x, y) = \dot{x} \ddot{y} - x \ddot{\dot{y}} \tag{3.19}$$

This function owes its origin to the Teager–Kaiser Energy Operator (TKEO), which is a powerful tool for measuring the temporal changes in sinusoidal energy. The concept of  $\Psi$



operator or CTKEO has been extended to the realm of complex-valued signals [9], yielding a new operator denoted by  $\Psi_C$  and defined as

$$\Psi_C(x, y) = 0.5[\dot{x}^* \dot{y} + \dot{x} \dot{y}^*] - 0.5[\ddot{x} y^* + x^* \ddot{y}] \quad (3.20)$$

Where  $x(t)$  and  $y(t)$  are two complex-valued signals and  $x^*(t)$  and  $y^*(t)$  are their respective complex conjugates. A positive symmetric modification of  $\Psi_C$  represented by  $\Psi_B$  has also been introduced [15], and is defined as

$$\begin{aligned} \Psi_B(x, y) &= 0.5[\Psi_C(x, y) + \Psi_C(y, x)] \\ &= 0.5(\dot{x} \dot{y}^* + \dot{x}^* \dot{y}) - 0.25(\ddot{x} y^* + \ddot{x} y + \ddot{x}^* y + \ddot{x}^* y^*) \end{aligned} \quad (3.21)$$

Where  $x(t) = x_r(t) + jx_j(t)$  and  $y(t) = y_r(t) + jy_j(t)$ .

$$\begin{aligned} \psi_B(x, y) &= \frac{1}{2}[(\dot{x}_r - j\dot{x}_j)(\dot{y}_r + j\dot{y}_j) + (\dot{x}_r + j\dot{x}_j)(\dot{y}_r - j\dot{y}_j)] - \frac{1}{4}[(x_r + jx_j)(\ddot{y}_r - j\ddot{y}_j) + (x_r - jx_j)(\ddot{y}_r + j\ddot{y}_j) \\ &\quad + (y_r + jy_j)(\ddot{x}_r - j\ddot{x}_j) + (y_r - jy_j)(\ddot{x}_r + j\ddot{x}_j)] \\ &= \dot{x}_r \dot{y}_r - \frac{1}{2}[x_r \ddot{y}_r + \ddot{x}_r y_r] + \dot{x}_j \dot{y}_j - \frac{1}{2}[x_j \ddot{y}_j + \ddot{x}_j y_j] \\ &= \Psi_B(x_r, y_r) + \Psi_B(x_j, y_j) \end{aligned} \quad (3.22)$$

So it is observed from the eq. (3.22) that the cross  $\psi_B$  energy operator of  $x(t)$  and  $y(t)$  is equal to the cross-Teager energies of their real and imaginary parts.

$$\Psi_B(x, y) = \Psi_B(x_r, y_r) + \Psi_B(x_j, y_j) \quad (3.23)$$

$$\Psi_B(x_l, y_l) = \dot{x}_l \dot{y}_l - 0.5[\ddot{x}_l y_l + y_l \ddot{x}_l] \text{ Where } l \in \{r, j\} \quad (3.24)$$

The complex form of the signals is obtained using the Hilbert transform. A number of different derivative approximations can be used for the realization of  $\Psi_B$ .

### 3.3.1 Discretizing the Continuous-Time Cross Teager Energy Operator

Backward difference relations can be used to approximate the derivatives and can be combined to obtain an expression closely related to the discrete form  $\Psi_{B_d}$  of the continuous  $\Psi_B$  operator.

$\Psi_{B_d}$  Operates on discrete-time signals, i.e. time-series  $x(n)$  and  $y(n)$ . Although the two-sample backward difference is elaborated here, it is worth mentioning that higher order difference-relations can also be used. For simplicity,  $t$  is replaced by  $nT_s$  where  $T_s$  is the sampling period, and  $x(t)$  with  $x(nT_s)$  or simply  $x[n]$ . The substitutions yield

$$\begin{aligned} \dot{x}(t) &\rightarrow \frac{[x_k[n] - x_k[n-1]]}{T_s} \\ \ddot{x}(t) &\rightarrow \frac{[x_k[n] - 2x_k[n-1] + x_k[n-2]]}{T_s^2} \\ \therefore \Psi_B(x_k(t), y_k(t)) &\rightarrow \frac{x_k[n-1]y_k[n-1]}{T_s^2} - \frac{0.5[x_k[n-1]y_k[n-1] + y_k[n-1]x_k[n-1]]}{T_s^2} \\ \Psi_B(x_k(t), y_k(t)) &\rightarrow \frac{\Psi_{B_d}(x_k[n-1], y_k[n-1])}{T_s^2}, \text{ where } k \in \{j, r\}. \end{aligned}$$

The discrete form of  $\Psi_B(x(t), y(t))$  is given by

$$\Psi_B(x(t), y(t)) \rightarrow \frac{\Psi_{B_d}(x_r[n-1], y_r[n-1]) + \Psi_{B_d}(x_j[n-1], y_j[n-1])}{T_s^2}$$

Where  $\rightarrow$  indicates the mapping from continuous to discrete. Hence, from a knowledge of  $\Psi_B$

its discrete-time version  $\Psi_{B_d}$  shifted by one sample to the left and scaled by  $T_s^{-2}$ , can be obtained.

It can be easily established that using the two-sample forward difference, shifted by one sample

to the left and scaled by  $T_s^{-2}$ , can be obtained from . On ignoring the one-sample shift and also the scaling parameter,  $\Psi_B(x(t), y(t))$  can be converted into  $\Psi_{B_d}(x[n], y[n])$  as follows:

$$\Psi_B(x(t), y(t)) \rightarrow \Psi_{B_d}(x_r[n], y_r[n]) + \Psi_{B_d}(x_j[n], y_j[n]) \quad (3.25)$$

$$\therefore \Psi_{B_d}(x_k[n], y_k[n]) = x_k[n]y_k[n] - 0.5(x_k[n+1]y_k[n-1] + y_k[n+1]x_k[n-1]) \quad (3.26)$$

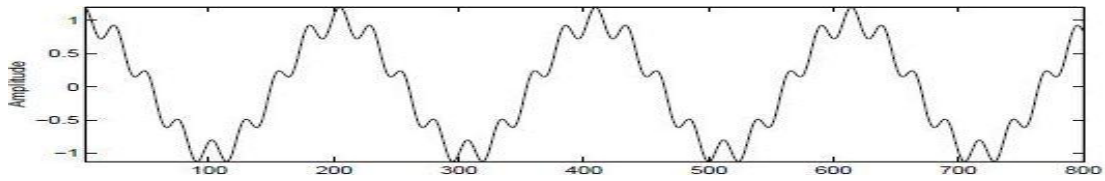
Where  $k \in \{j, r\}$

The three-sample symmetric difference can also be utilized but it results a more complicated expression. Indeed, the asymmetric approximation is less complicated for implementation and faster than the symmetric one because it requires fewer operations. Thus the symmetric difference approximation has not been treated here.

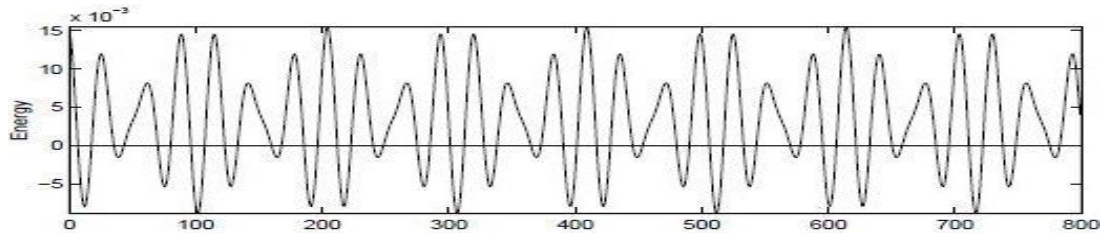
### **3.4 Signals Yielding “Negative Teager Energy”**

Figure 3.5 shows a signal and its’ Teager Energy. It can be seen that at several instants the output of the TKEO is ‘negative’. This is surely a strange behavior for an energy operator. It therefore demands investigations as to how such signals with such TKEO can derived.

The definition of the TKEO makes it amply clear that the operator is aimed at modeling the energy of the source of the signal, and not of the signal. This gives probably a part of the explanation. This has been discussed by Hamila et. al [8].



(a) Two Mixed Sinusoidal Signals



(b) TKEO output of the above signal

Figure 3.5: The Teager Energy Operator output of the signal is negative for some parts of the signal.

The signal in Fig.3.5 has been obtained by combining two sinusoids, where one has significantly higher frequency, but has much smaller amplitude than the other signal. It appears as a model of two separate sources, one high frequency and one low frequency source, where the high frequency source is placed farther away than the low frequency counterpart. However, in this case, if one assumes incorrectly that the signal is generated by a single source, then that might explain why the Teager Energy is negative.

The same signal could indeed have been generated by a single system, but that would also have been a departure from the simple model that has been considered in the definition of the Teager Energy Operator, as the resulting signal consists of two oscillating signals.

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## Results And Discussions

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The present chapter discusses how cross teager kaiser based feature extraction methodology can be associated with support vector machine classifier for efficient classification of arrhythmic heart beat of different class. Analyzing each heartbeat of the ECG records can be troublesome for human being because the ECG record obtained from holter monitor can be affected by noise and also there is a possibility of human error during the analysis of ECG record. To develop SVM based bioelectric signal classification algorithm, ECG signal from MIT/BIH arrhythmia database is considered [25]. Fig 4.1 describes the steps for a full automatic system for arrhythmia classification from signals acquired by an ECG device, and these steps are (1) ECG signal preprocessing; (2) heartbeat segmentation; (3) feature extraction; and (4) learning/classification.

The rest of the chapter is organized as follows: section 4.1 describes the preprocessing steps of ECG signal involve denoising and segmentation. Cross teager kaiser based feature extraction process is detailed in section 4.2. SVM based classification technique is detailed in section 4.3. section 4.4 represents performance results of the classification and summary is presented in section 4.5.

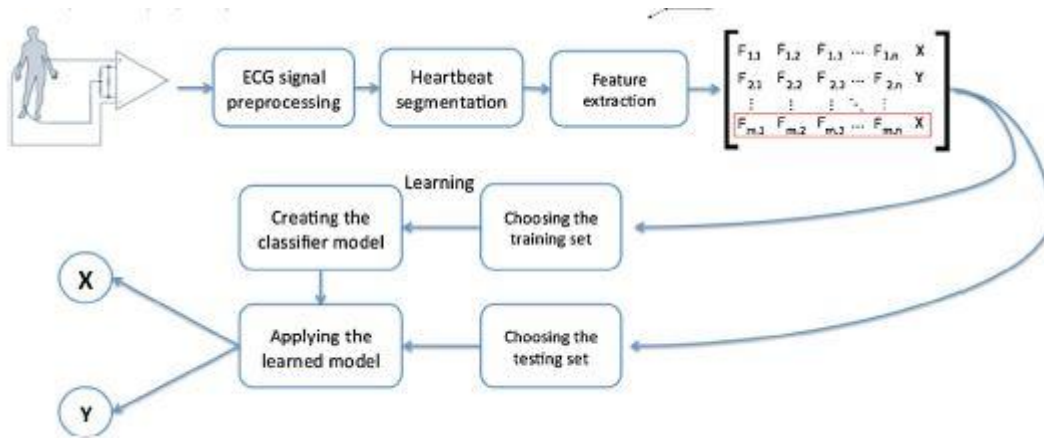


Fig. 4.1: Stages of Classification of Arrhythmia in ECG beats [21]

#### **4.1 Preprocessing:**

The main objective of preprocessing is Denoising of ECG signal and to identify exact cardiac cycle. The ECG signal obtained from holter monitor often contaminated by different types of noise namely Contact noise, baseline drift, muscle artifacts, power line interference, electrode motion artifact, electromyography artifact etc. On the other hand in preprocessing method heartbeat segmentation from the ECG signal is a different approach from automatic classification of arrhythmias. The performance of the ECG signal classification depends on the exact detection of each cardiac cycle [2]. Therefore the choice of suitable preprocessing method affects the final objective of the research.

Among all proposals for reducing noise in ECG signals, the simplest and most widely used is the implementation of recursive digital filters of the finite impulse response (FIR) [4], which was made computationally possible with the advance in microcontrollers and microprocessors. In order to attenuates known frequency bands namely noise coming from the electrical network (50 Hz or 60 Hz), since these methods perform perfectly. But the problem is in most cases the



frequency of noise is unknown. Therefore, to determine the noise frequency one should have to apply different types of filter to the signal for different frequency bands. But doing so one can deform the morphology of the ECG signal. As a result of that the signal becomes unusable for diagnosing cardiac diseases. Other methods have also presented interesting results on noise attenuation. Sameni *et al.* [14] have proposed the use of nonlinear Bayesian filters for ECG signal noise reduction, presenting promising results. A new algorithm is proposed based on the Extended Kalman Filter [13], which do not distort the parameters of the ECG morphology for ECG noise reduction and signal compression, providing a significant contribution because the method showed the greatest effectiveness to date.

In the last decade, many methods based on wavelet transforms have been employed to remove noise, since they maintain the ECG signal properties avoiding loss of its important physiological details and are simple from a computational point of view [5–7].

In this thesis paper the pre-processing module is decomposed into two components as shown in fig 4.1. The first component involves the de-noising of the ECG signal based on the discrete wavelet transform (DWT). For preprocessing of the ECG signal, noise removal involves different techniques for various noise sources [1]. The second component is the segmentation of the signal. Each component is described in brief as follows:

#### **4.1.1 Denoising**

Since DWT is efficient in analyzing non-stationary signals, it is used in this paper. The Daubechies D6 (db6) wavelet basis function is used to denoise the data, with the ECG signals decomposed to nine levels [3]. A frequency range from 0 to 0.351 Hz in the ninth level approximation sub band is mostly the baseline wander. Furthermore, the frequency ranges of 90–180 and 45–90 were not considered because the information after 45 Hz is not important for

arrhythmia detection. The algorithm decomposes the original signal through DWT and the resulting DWT detail coefficients are thresholded by shrinkage (soft) strategy. In this thesis paper the automatic thresholding method is employed for each level of decomposition of the signal. Reconstructing the original sequence from the thresholded wavelet detail coefficients leads to a denoised (smoothed) version of the original sequence.

#### **4.1.2 Segmentation**

Heartbeat segmentation methods have been studied for more than three decades [8 - 11] and the generations of these algorithms and newly developing methods reflect the progress of the processing power of computers.

Here after Denoising, 99 samples were chosen from the left side of the QRS mid-point and 100 samples after QRS mid-point and the QRS mid-point itself as a segment or beat of 200 samples [12]. The five types of beat classes in the ANSI/AAMI EC57:1998 standard database are represented in two preprocessing categories in figure 2, and those are: (a) Signal beats without DWT and (b) smoothing signal beats using DWT + QRS detection of 200 samples. The circle illustrates the presence of non-smoothing signal beats.

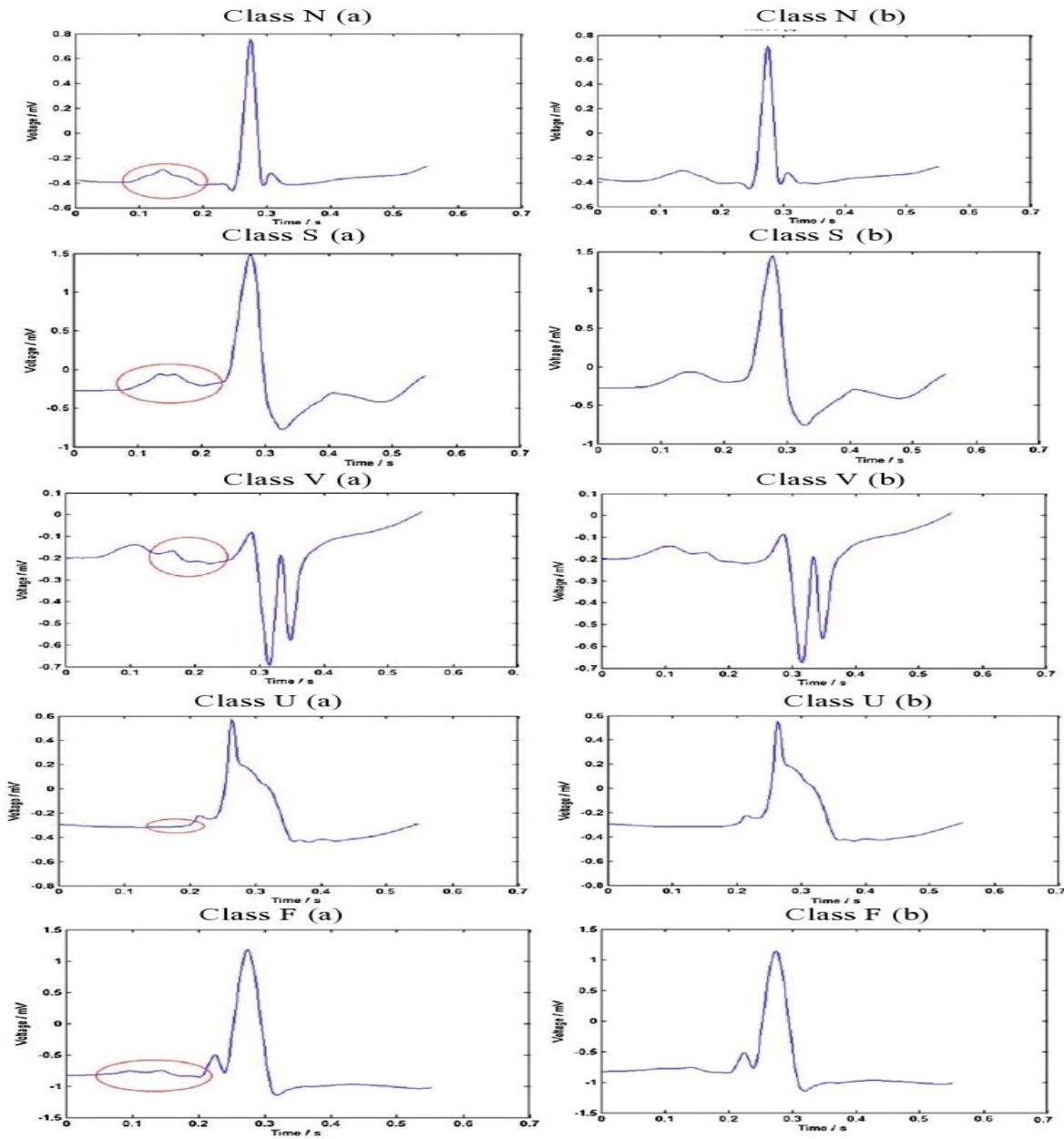


Fig. 4.2: The five types of beat classes in two preprocessing categories: (a) Signal beats without DWT (a) denoised signal beats using DWT and QRS detection of 200 samples

## **4.2 Feature Extraction**

In machine learning tasks such as classification often requires input that mathematically convenient to process. In this motivation feature extraction is the crucial step for automated

classification of bioelectric signal. A feature is piece information of an individual based on which one can differentiate the individual from others. Any information extracted from the heartbeat used to distinguish its type maybe considered as a feature. From the ECG signal's morphology one extract features in the time domain and/or in the frequency domain or from the cardiac rhythm.

Feature extraction and feature selection are two different processes for classification. Feature extraction is defined as the stage of finding the description of an object whereas feature selection can be seen as a search technique for proposing a new feature subset of relevant features in order to improve the performance of classification.

The Cross Teager Kaiser Energy Operator is a mathematical technique which measures the level of similarity of two signals. In this proposed feature extraction technique CTKEO is used to extract feature from the ECG beats. Here the healthy ECG beat  $A(n)$  is correlated with the ECG arrhythmia beat  $B(n)$ . The features can be extracted from resultant sequence are  $E_{AB}$ .

$$E_{AB}(n) = A_k(n)B_k(n) - 0.5[A_k(n+1)B_k(n-1) + A_k(n-1)B_k(n+1)] \quad \text{Where } k \in \{j, r\}$$

Here  $n$  represents the number of samples of each of the signals, then the resultant cross-teager Kaiser sequence will have  $(n-1)$  number of samples. In this study  $n = 200$  sample is used for each heartbeat. For simplicity  $E_n$  has been taken as the Cross-Teager Kaiser sequence.

The features (F1 to F8) [20] that are extracted from the above resultant sequence are as follows:

1. F1= maximum value of the sequence ( $E_{\max}$ )

2.  $F2 = \frac{\sum_{n=w}^w nE_n}{E_{\max}}$

$$3. F3 = \frac{\sum_{n=-w}^w nE_n}{\sum_{n=-w}^w E_n}$$

$$6. F6 = \sum_{n=-w}^w \frac{E_n}{2w+1}$$

$$4. F4 = \frac{\sum_{n=-w}^w |n|E_n}{\sum_{n=-w}^w E_n}$$

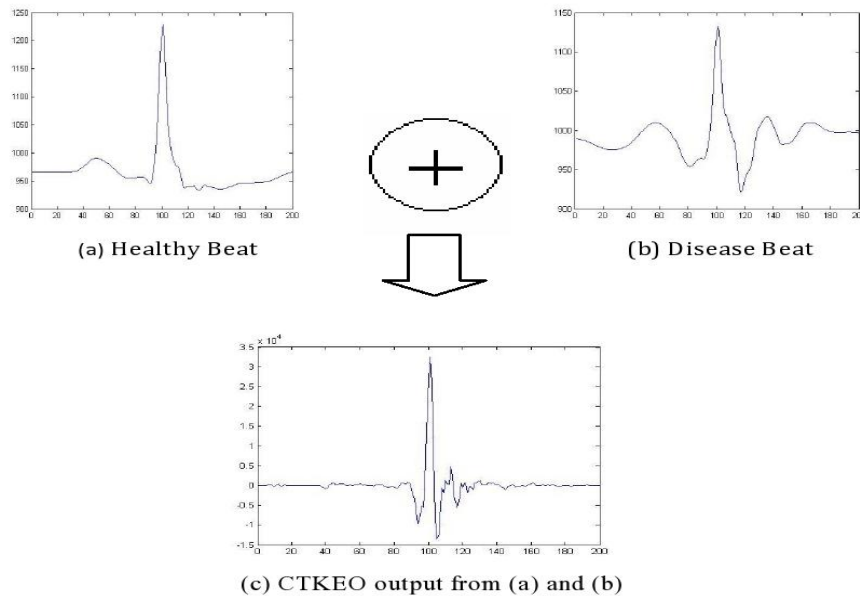
$$7. F7 = \sqrt{\frac{\sum_{n=-w}^w (E_n - F6)^2}{(2w+1)}}$$

$$5. F5 = \sqrt{\frac{\sum_{n=-w}^w n^2 E_n}{\sum_{n=-w}^w E_n}}$$

$$8. F8 = \frac{\sum_{n=-w}^w (E_n - F6)^3}{(w-1) * (F7)^3}$$

Thus, the above eight features are extracted from the cross-teager Kaiser Sequence of each arrhythmia class ECG beat for identification of each disease characteristics.

Here Fig 4.3 represents the Plot of CTKEO between a healthy beat and a disease beat. The above mentioned features are extracted the output of CTKEO in fig 4.3 (c).

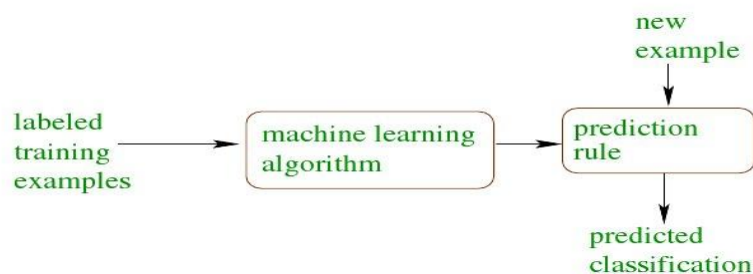


**Fig 4.3: CTKEO Plot of healthy and disease beat**

### **4.3 Classification**

Classification is a general process related to categorization, the process in which ideas and objects are recognized, differentiated, and understood. An algorithm that implements the classification, especially in a concrete implementation, is known as a classifier. The term "classifier" sometimes also refers to the mathematical function, implemented by a classification algorithm that maps input data to a category.

In machine learning and statistics, classification is the problem of identifying to which of a set of categories a new observation belongs, on the basis of a training set of data containing observations whose category membership is known. Here from the fig. 4.4 one can see that with the help of a training set of correctly identified observations machine come up with its own prediction rule based on which a new example or a testing set would be classified.



**Fig 4.4: Machine Learning Classification**

The machine learning tasks typically classified into two categories such as:

1. An instance of supervised learning, i.e. learning where training set of correctly identified observations is available.
2. Unsupervised procedure, also known as clustering, involves grouping data into categories based on some measure of inherent similarity or distance.

Once the set of features have been defined from the arrhythmic heartbeats, models can be built from these data using artificial intelligence algorithms from machine learning for arrhythmia heartbeat classification. Here, in this study the most popular algorithm Support Vector Machine (SVM) is employed.

#### **4.3.1 Support Vector Machines (SVMs)**

In machine learning, support vector machines (SVMs) are supervised learning models with associated learning algorithms, that analyze the data used for classification and regression analysis. Suppose there are a set of training examples, each marked for belonging to one of two categories. An SVM training algorithm builds a model based on the features of training example that assigns new examples into one category or the other, making it a non-probabilistic binary linear classifier. More accurately, one can think of an SVM model as representing the examples as points in space, mapped so that each of the examples of the separate classes are divided by a gap that is as wide as possible. New examples are then mapped into that same space and predicted to belong to a category based on which side of the gap they fall on.

The original SVM algorithm was invented by Vladimir N. Vapnik and Alexey Ya. Chervonenkis in 1963[24]. In 1992, Bernhard E. Boser, Isabelle M. Guyon and Vladimir N. Vapnik suggested a way to create nonlinear classifiers by applying the kernel trick to maximum-margin hyperplanes[4].

More accurately, a support vector machine constructs a hyperplane or set of hyperplanes in a high or infinite-dimensional space, which can be used for classification, regression analysis. Intuitively, a good separation is achieved by the hyperplane that has the largest distance to the nearest training-data point of any class (so called functional margin), since in general the larger the margin the lower the generalization error of the classifier. The machine achieves this

desirable property on the basis of the principle of Structural Risk Minimization (SRM) principle. The SRM principle seeks to minimize the upper bound of the generalization error consisting of the sum of the training error and a confidence interval [22].

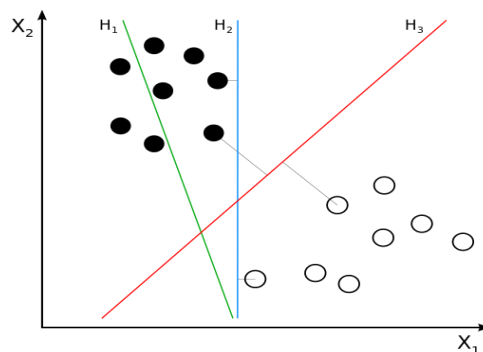
In addition to performing linear classification, SVMs can efficiently perform a non-linear classification using the kernel trick by mapping their inputs into high-dimensional feature spaces.

### **Linear SVM**

Suppose there are a training dataset of  $n$  points of the form

$$(\vec{x}_1, y_1), \dots, (\vec{x}_n, y_n)$$

Where  $\vec{x}_i$  is an N-dimensional is input feature vector and  $y_i$  are either +1 or -1 indicates the class to which the points  $\vec{x}_i$  belongs. For linearly separable case one can select two parallel hyperplane that separates the two classes of data, so that the distance between them is as large as possible. The region bounded by these two parallel hyperplanes is called “margin” and the hyperplane that lies in middle of them is called maximum-margin hyperplane. The support vectors denote those data points, which are closest to the maximum-margin hyperplane, and they are most difficult to classify [16].



**Fig 4.5: SVM separating hyperplanes**



From fig 4.5 one can see that the hyperplane  $H_1$  does not separate the classes. On the other hand  $H_2$  separates the classes but distance from data points is small whereas in case of  $H_3$ , this hyperplane separates the classes with maximum margin.

The equation of hyperplane for a set of data point  $\vec{x}$  can be written as

$$w.x + b = 0$$

$w$  is the adjustable weight vector and  $b$  is the bias. For linearly separable case the two parallel hyperplanes can be formulated as follows

$$w.x + b = 1$$

And

$$w.x + b = -1$$

Now if one tries to prevent data points from falling on the margin, then the hyperplanes can be reformulated as

$$w.x + b \geq 1 \text{ if } y_i = 1 \tag{4.1}$$

And

$$w.x + b \leq -1 \text{ if } y_i = -1 \tag{4.2}$$

Now combining eq. (4.1) and eq. (4.2) one can write that

$$y_i(w.x + b) - 1 \geq 0 \tag{4.3}$$

The distance from origin to the optimal hyper plane is  $\frac{\|b\|}{\|w\|}$  and  $\|w\|$  is the Euclidean norm of  $w$

[15]. As shown in fig. 4.6 that for some data points  $(\vec{x}_i, y_i)$ ,  $w.x + b = +1$  is satisfied and for some other data points  $(\vec{x}_i, y_i)$ ,  $w.x + b = -1$  is satisfied. The main objective here is to maximize the

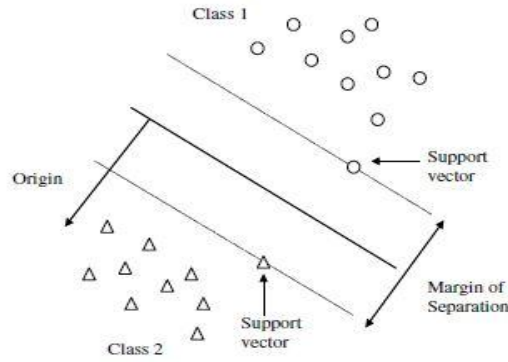


Fig. 4.6: The SVM for the linearly separable case

margin of separation and to do so it is necessary to determine the value of  $\|b\|$  and  $\|w\|$ . As it is known that maximum margin hyperplane is halfway between the two parallel hyperplane, then the distance between two hyper planes is  $\frac{2}{\|w\|}$ . So to maximize the distance between two planes

one should minimize  $\|w\|$ . Hence the constrained optimization problem can be stated as: For the given training samples  $\{(x_i, y_i)\}_{i=1}^n$

$$\text{minimize} \quad \varphi(w) = \frac{1}{2} \|w\|^2 \quad (4.4a)$$

$$\text{subject to} \quad d_i(w.x + b) \geq 1, \quad i = 1, 2, \dots, n \quad (4.4b)$$

But in practice in most cases of classification based SVM the researchers are experienced the pattern of linearly non-separable data i.e. the data points of different classes does overlap. Therefore, in order to deal with such situations, it is necessary to introduce a new set of slack variables  $\{\xi_i\}_{i=1}^n$  that measures the amount of violation of constraints [17]. Then the optimization problem can be reformulated as

$$\text{minimize} \quad \varphi(w, \Xi) = \frac{1}{2} \|w\|^2 + C \left( \sum_i \xi_i \right)^k \quad (4.5a)$$

$$\text{subject to} \quad d_i(w.x + b) \geq 1 - \xi_i, \quad i = 1, 2, \dots, n \quad (4.5b)$$

$$\xi_i \geq 0, i=1, 2, \dots, n$$

Where  $C$  is called the regularization parameter. The second part of the above objective function eq. (4.5b) seeks to penalize the data points located in the incorrect side of the decision boundary. If the data points overlap considerably in feature space, then the penalty term becomes very large, and the hyperplane may not generalize well.

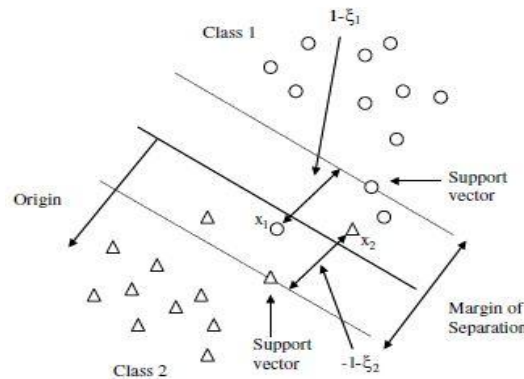


Fig. 4.7: The SVM for the linearly non-separable case

### **Non-Linear SVM**

The basic approaches discussed above can be extended to nonlinear decision surfaces. In case of nonlinear SVM, the same data points are transformed from the input space to the high dimensional feature space with some nonlinear mapping [16]. In this high dimensional feature space the spreading of data points provides a linear hyperplane [18]. Now suppose the training data is mapped to some other (possibly infinite dimensional) Euclidean space  $H$  through a nonlinear function  $\phi$  such that  $\phi: R^n \rightarrow H$ . Therefore in this high dimensional feature space the input vector  $x$  can be characterized as  $\phi(x)$  [18]. Then of course the training algorithm would only depend on the data through dot products in  $H$  i.e. on functions of the form  $\phi(x) \cdot \phi(x_i)$ . Here

the algorithm used to create maximum margin hyperplane is similar to linear SVM except that every dot product is replaced by kernel trick represented as  $k(x, x_i) = \phi(x) \cdot \phi(x_i)$ . The transformation of data from input space to high dimensional feature space is done by kernel trick. The kernel trick is only used in training algorithm. Then the decision function is given by

$$f(x) = \text{sign}\left(\sum_{i=1}^n \alpha_i d_i k(x, x_i) + b\right) \text{ Where } i = 1, 2, \dots, n$$

Here  $\alpha_i$  = Lagrangian Multiplier. For training of the SVM classifier, only the kernel is required.

Gaussian radial basis function (RBF) is one of such kernel and is given as follows:

$$k(x, y) = e^{-\frac{\|x-y\|^2}{2\sigma^2}}$$

Where  $\sigma^2$  = kernel parameter or width. The values, chosen for the two kernel parameters  $(C, \sigma^2)$  significantly affect the classification accuracy of SVM classifier. Large values of  $(C, \sigma^2)$  may lead to over fitting problem for the training data [18]. So the values must be chosen carefully.

### **Multiclass SVM**

The aim in Multiclass SVM classification is to assign labels to instances by using support vector machines, where the labels are drawn from a finite set of several elements. In this motivation the dominating approach for doing so is to construct multiple binary classification problems from single multiclass problem [19]. Common methods used for building such multiple binary classifiers are as followed:

- (i) Between one of the labels and the rest (one versus-all)
- (ii) Between every pair of classes (one-versus-one).

For classification of new example in case of one versus all method data points would be classified under a certain class if and only if that class's SVM accepted it and all other classes' SVM rejected it. On the other hand on-versus-one methods creates  $N \frac{(N-1)}{2}$  classifiers where  $N$  = no of classes. Here each classifier is trained with data from two classes to distinguish samples of one class from the other. After that the classification of unknown data is done according to the maximum voting rule where each SVM votes for one class.

### **Ten-fold Cross Validation**

Cross-validation, sometimes called rotation estimation, is a model validation technique for assessing how the results of a statistical analysis will generalize to an independent data set. It is mainly used in settings where the goal is prediction, and one wants to estimate how accurately a predictive model will perform in practice. In a prediction problem, a model is trained using dataset of known data (training dataset), and a dataset of unknown data against is tested against the model. In k-fold cross-validation, the original sample is randomly partitioned into  $k$  equal sized subsamples. In this validation technique, only one subsample is used for testing purpose and the rest  $(k-1)$  subsamples are used to train the model. After that the same process is repeated for  $k$  times for testing each of the  $k$  subsample. Therefore to generate a single estimation one should average the  $k$  no of results. The advantage of this method over repeated random sub-sampling (see below) is that all observations are used for both training and validation, and each observation is used for validation exactly once. 10-fold cross-validation is commonly used but in general  $k$  remains an unfixed parameter.

Here fig 4.8 represents proposed classification technique used for this research work.

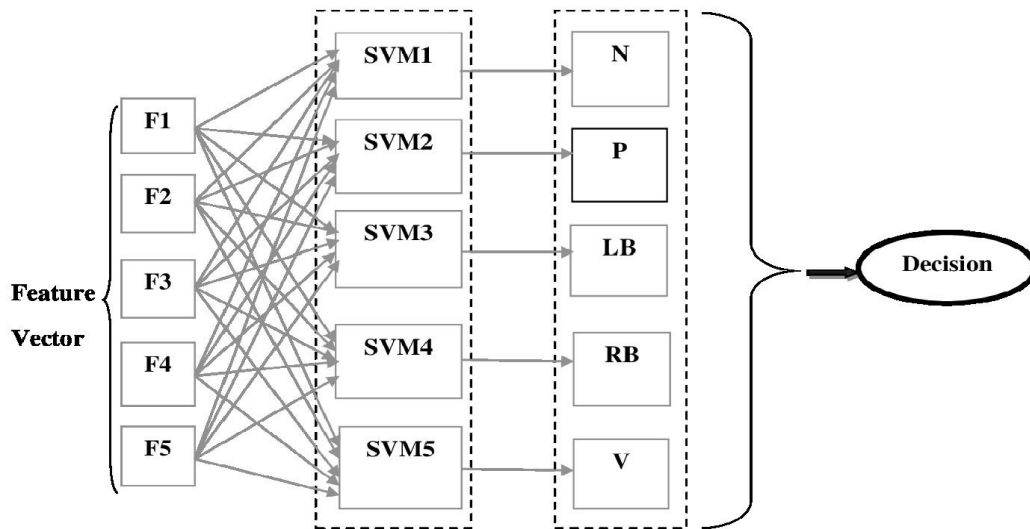


Fig 4.8: Structure of the binary ensemble SVM Classifier

### 4.3.2 Evaluation Method

To evaluate the performance of the classification, one can use the standard statistical indices of sensitivity (Se), specificity, and accuracy (Acc), derived from four parameters of confusion matrix: correctly detected beats (true positives = TP), undetected beats (false negatives = FN), correctly undetected beats (true negatives = TN) and falsely detected beats (false positives = FP) respectively. These statistical indices are defined as follows:

Confusion matrix =

		Actual	
		Yes	No
Predicted	Yes	TP	FP
	No	FN	TN

$$Sensitivity = \frac{TP}{TP + FN} \times 100$$

$$Specificity = \frac{TN}{TN + FP} \times 100$$

$$Accuracy = \frac{TP + TN}{TP + TN + FN + FP} \times 100$$

#### 4.4 Results

The experiments on the automated arrhythmia detection system are conducted using the MIT-BIH arrhythmia database. Here in this study 8 features are used. Then the feature vectors are used to classify arrhythmia beats using SVM-RBF kernel method. The nonlinear SVM was based on the popular Gaussian kernel (referred to as the SVM-RBF). Here the classification process is done by three different approaches.

#### Case - I

(Using SVM-RBF Classifier with tenfold cross-validation technique)

#### Dataset descriptions

In this proposed classification method the dataset is collected from MIT/BIH database. This dataset is distributed into 5 classes where class 1 represent normal ECG beats and class 2-5 represents four different class of arrhythmic beats. In this method 10 different records (from MIT/BIH database) are used. Table 1 shows detail description of the classes, no of beats used for classification.

Table 1: Dataset description of case I			
Class no	Record Example used from MIT-BIH	No. of beats used	Description
1	101,105	20	Normal Beat (N)
2	102,104,107	60	Paced Beat (P)
3	109,111	60	Left Bundle Branch Block Beat (LB)
4	118,124	60	Right Bundle Branch Block Beat (RB)
5	200	60	Ventricular Bigeminy (V)

## Results

Table 2 shows that how accurately one classifies arrhythmic beats from normal beats. To do so here each normal beat is treated as a reference beat and then correlated with every arrhythmic beats. So for each reference beat we get  $80 \times 8$  feature vector. Table 1 shows the averaged classification accuracy obtained with respect to each reference beat.

	Paced Beat (P)	Left Bundle Branch Block Beat (LB)	Right Bundle Branch Block Beat (RB)	Ventricular Bigeminy (V)
Accuracy	98.05%	97.71%	98.11%	95%

## Case - II

(Using SVM-RBF Classifier with tenfold cross-validation technique)

## Dataset descriptions

Also in this proposed classification method the dataset is collected from MIT/BIH database. But this method is different from previous method because here only one normal (reference beat) is selected arbitrarily where 240 beats are selected for each class of arrhythmia. The detailed dataset is described in table 2.

Class no	Record Example used from MIT-BIH	No. of beats used	Description
1	101	1	Normal Beat (N)
2	102,104,107	240	Paced Beat (P)
3	109,111,214	240	Left Bundle Branch Block Beat (LB)
4	118,124,212	240	Right Bundle Branch Block Beat (RB)
5	200	240	Ventricular Bigeminy (V)
<b>Feature Vector</b>		<b><math>960 \times 8</math></b>	

## Results

Here in this method the feature vector is acquired by correlating one reference healthy beat with four other classes of arrhythmic beats. As for this proposed classification technique 8



distinguished features are selected, the size of the feature vector is  $960 \times 8$ . Table 4 shows the performance of classification using SVM-RBF Classifier with tenfold cross-validation technique.

	Paced beat (P)	Left Bundle Branch Block Beat(LB)	Right Bundle Branch Block Beat (RB)	Ventricular Bigeminy (V)																
Accuracy	99.27%	98.75%	96.25%	96.88%																
Sensitivity	97.50%	96.25%	95.83%	92%																
Specificity	99.73%	99.60%	96.40%	98.62%																
Confusion matrix	<table border="1"> <tr> <td>234</td> <td>2</td> </tr> <tr> <td>6</td> <td>718</td> </tr> </table>	234	2	6	718	<table border="1"> <tr> <td>231</td> <td>3</td> </tr> <tr> <td>9</td> <td>717</td> </tr> </table>	231	3	9	717	<table border="1"> <tr> <td>230</td> <td>26</td> </tr> <tr> <td>10</td> <td>694</td> </tr> </table>	230	26	10	694	<table border="1"> <tr> <td>220</td> <td>10</td> </tr> <tr> <td>20</td> <td>710</td> </tr> </table>	220	10	20	710
234	2																			
6	718																			
231	3																			
9	717																			
230	26																			
10	694																			
220	10																			
20	710																			

### **Case – III**

(Using Multiclass SVM-RBF Classifier with Testing Dataset and training dataset)

#### **➤ Dataset description**

In this method multiclass SVM classification technique is use to classify normal ECG beats from four others arrhythmic beats. Here two different datasets is formed by selecting beats from different records. The training set is different from previously describes dataset because here 100 normal ECG beats are selected. And in testing data 100 different beats are picked up for each class from different records used in training dataset.

#### **✚ Training Data:**

Class no	Record Example used from MIT-BIH	No. of beats used	Description
1	101	100	Normal Beat (N)
2	102,104,107	240	Paced Beat (P)
3	109,111,214	240	Left Bundle Branch Block Beat (LB)
4	118,124,212	240	Right Bundle Branch Block Beat (RB)
5	200	240	Ventricular Bigeminy (V)
<b>Feature Vector</b>		<b>1060×8</b>	

## Training Dataset

Class no	Record Example used from MIT-BIH	No. of beats used	Description
1	123	100	Normal Beat (N)
2	217	100	Paced Beat (P)
3	207	100	Left Bundle Branch Block Beat (LB)
4	231	100	Right Bundle Branch Block Beat (RB)
5	106	100	Ventricular Bigeminy (V)
<b>Feature Vector/class</b>		<b>100×8</b>	

## Results

To train the multiclass SVM classifier the feature vector is obtained from training dataset. Here one reference beat (normal beat) is selected arbitrarily and correlated with other beats namely normal beats and arrhythmic beats. As a result of that the size of training feature vector is  $1060 \times 8$ . Applying similar process for testing we get  $100 \times 8$  feature vector for each class.

Table 7 shows the result by performing multiclass classification. Here one can see that the diagonal elements represent correctly identified rhythms.

Confusion matrix =

Classes	Predicted Classes				
	N	P	LB	RB	V
N	83	0	9	0	8
P	7	87	0	6	0
LB	5	0	84	5	6
RB	2	3	0	88	7
V	10	0	7	4	79

## **4.5 Summary**

In this study, it is observed that the proposed combination of 8 features are able to extract the hidden information from the non-stationary ECG signal and discriminant features very well using the experimental MIT-BIH data. The technique provides a good extraction of the most discriminant features and clearly distinguishes the different arrhythmia classes. The features of

the ECG beat are extracted using cross teager kaiser method. To carry out these computations, MATLAB 2015b was used. All the methods including Denoising, segmentation, feature extraction and classification algorithms are developed in MATLAB with the custom software.

Based on the performance evaluation, the sensitivity, specificity and accuracy of the confusion matrix with the SVM-RBF classifiers are summarized. Here three different approaches are used to classify arrhythmia beats and those are as follows:

- ✚ Case 1 describes Classification between healthy and disease beats using SVM-RBF classifier with tenfold cross validation technique.
- ✚ Case 2 describes Classification among 4 different class beats of arrhythmia using SVM-RBF classifier with tenfold cross-validation technique.
- ✚ Case 3 describes classification among 4 different class beats and healthy beats of arrhythmia using multi class SVM-RBF classifier. Here individually test each class of arrhythmia using a training dataset.

Here comparing above three methods one can see from the result that SVM-RBF tenfold cross validation technique gives better accuracy than multiclass SVM-RBF classification technique. The worst class accuracy of the proposed features was obtained for the RB and V beats (approx 97%), while approx 99% was obtained for the P and LB beats.

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## Conclusions and Future work

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### 5.1 General Discussion

In the present work, a Teager Kaiser Energy operator based technique is developed as a feature extraction methodology for automatic classification of bioelectric signals. As elaborated earlier, millions of people around the globe are suffering from cardiovascular disease and researchers are trying to develop a computer aided automatic detection system to detect them. Against this backdrop the scope of this thesis are kept bounded for development of such effective algorithm for classification of arrhythmia from ECG beats. Bioelectric signals were processed using eight different features acquired from cross teager Kaiser Algorithm and two different SVM-RBF classifiers are developed.

The main contributions of this research work can be outlined as follows:

- The development of computer aided diagnostic tool for automatic classification of cardiac arrhythmia. An accurate and reliable Cross Teager Kaiser Algorithm has been developed for feature extraction. In this context, SVM classifier is presented which can efficiently classify arrhythmia beats into normal beats (N), paced beats (P), Ventricular

Bigeminy (V) and other beats which include left bundle block beats (LB), right bundle block beats (RB).

- To classify arrhythmia beats into different classes three different approaches are adopted. Among them it is observed that classification using binary ensemble SVM-RBF classifier with tenfold classification technique gives acceptable performance in disease classification.

## **5.2 Future Scope of works**

The present work can be extended in several directions. Some of the possible avenues can be summarized as follows:

- The algorithm, that is proposed in this thesis is developed for the classification of heart beats. Researchers may build cross Teager Kaiser energy based algorithm for other bioelectric signal e.g. Electromyogram (EMG), Electroencephalogram (EEG) signal processing.
- The algorithm, that is proposed here may encourage researchers in future to make in-depth study of the feasibility of implementing several other kernel based classifiers including different non linear kernel function based SVM.
- The algorithm used in this thesis can be extended in the future to the two-dimensional biomedical image processing problems e.g. for the detection of the location of tumors or cancer cells inside the human body.

Finally the work presented in this thesis should be of great help to the patients as well as the medical practitioner in future. After all a strong medical support system can not only enhance the capabilities and knowledge of the people involved in this work but also produce direct impact on the quality of service provided.