## Problem Sheet 4: Second Law of Thermodynamics (From Moran and Shapiro - $5^{\text {th }}$ Ed.)

5.18 The data listed below are claimed for a power cycle operating between reservoirs at $527^{\circ} \mathrm{C}$ and $27^{\circ} \mathrm{C}$. For each case, determine if any principles of thermodynamics would be violated.
(a) $Q_{\mathrm{H}}=700 \mathrm{~kJ}, W_{\text {cycle }}=400 \mathrm{~kJ}, Q_{\mathrm{C}}=300 \mathrm{~kJ}$.
(b) $Q_{\mathrm{H}}=640 \mathrm{~kJ}, W_{\text {cycle }}=400 \mathrm{~kJ}, Q_{\mathrm{C}}=240 \mathrm{~kJ}$.
(c) $Q_{\mathrm{H}}=640 \mathrm{~kJ}, W_{\text {cycle }}=400 \mathrm{~kJ}, Q_{\mathrm{C}}=200 \mathrm{~kJ}$.
5.19 A refrigeration cycle operating between two reservoirs receives energy $Q_{\mathrm{C}}$ from a cold reservoir at $T_{\mathrm{C}}=280 \mathrm{~K}$ and rejects energy $Q_{\mathrm{H}}$ to a hot reservoir at $T_{\mathrm{H}}=320 \mathrm{~K}$. For each of the following cases determine whether the cycle operates reversibly, irreversibly, or is impossible:
(a) $Q_{\mathrm{C}}=1500 \mathrm{~kJ}, W_{\text {cycle }}=150 \mathrm{~kJ}$.
(b) $Q_{\mathrm{C}}=1400 \mathrm{~kJ}, Q_{\mathrm{H}}=1600 \mathrm{~kJ}$.
(c) $Q_{\mathrm{H}}=1600 \mathrm{~kJ}, W_{\text {cycle }}=400 \mathrm{~kJ}$.
(d) $\beta=5$.
5.20 A reversible power cycle receives $Q_{\mathrm{H}}$ from a hot reservoir at temperature $T_{\mathrm{H}}$ and rejects energy by heat transfer to the surroundings at temperature $T_{0}$. The work developed by the power cycle is used to drive a refrigeration cycle that removes $Q_{\mathrm{C}}$ from a cold reservoir at temperature $T_{\mathrm{C}}$ and discharges energy by heat transfer to the same surroundings at $T_{0}$.
(a) Develop an expression for the ratio $Q_{\mathrm{C}} / Q_{\mathrm{H}}$ in terms of the temperature ratios $T_{\mathrm{H}} / T_{0}$ and $T_{\mathrm{C}} / T_{0}$.
(b) Plot $Q_{\mathrm{C}} / Q_{\mathrm{H}}$ versus $T_{\mathrm{H}} / T_{0}$ for $T_{\mathrm{C}} / T_{0}=0.85,0.9$, and 0.95 , and versus $T_{\mathrm{C}} / T_{0}$ for $T_{\mathrm{H}} / T_{0}=2,3$, and 4 .

$$
\text { (a) }=\frac{\begin{array}{c}
\text { Ans } \\
T_{\mathrm{C}}\left[T_{\mathrm{H}}-T_{0}\right]
\end{array}}{T_{\mathrm{H}}\left[T_{0}-T_{\mathrm{C}}\right]}
$$

5.21 A reversible power cycle receives energy $Q_{\mathrm{H}}$ from a reservoir at temperature $T_{\mathrm{H}}$ and rejects $Q_{\mathrm{C}}$ to a reservoir at temperature $T_{\mathrm{C}}$. The work developed by the power cycle is used to drive a reversible heat pump that removes energy $Q_{\mathrm{C}}^{\prime}$ from a reservoir at temperature $T_{\mathrm{C}}^{\prime}$ and rejects energy $Q_{\mathrm{H}}^{\prime}$ to a reservoir at temperature $T_{\mathrm{H}}^{\prime}$.
(a) Develop an expression for the ratio $Q_{\mathrm{H}}^{\prime} / Q_{\mathrm{H}}$ in terms of the temperatures of the four reservoirs.
(b) What must be the relationship of the temperatures $T_{\mathrm{H}}, T_{\mathrm{C}}$, $T_{\mathrm{C}}^{\prime}$, and $T_{\mathrm{H}}^{\prime}$ for $Q_{\mathrm{H}}^{\prime} / Q_{\mathrm{H}}$ to exceed a value of unity?
(c) Letting $T_{\mathrm{H}}^{\prime}=T_{\mathrm{C}}=T_{0}$, plot $Q_{\mathrm{H}}^{\prime} / Q_{\mathrm{H}}$ versus $T_{\mathrm{H}} / T_{0}$ for $T_{\mathrm{C}}^{\prime} / T_{0}=0.85,0.9$, and 0.95 , and versus $T_{\mathrm{C}}^{\prime} / T_{0}$ for $T_{\mathrm{H}} / T_{0}$ $=2,3$, and 4 .
5.22 Figure P5.22 shows a system consisting of a power cycle driving a heat pump. At steady state, the power cycle receives $\dot{Q}_{\text {s }}$ by heat transfer at $T_{\mathrm{s}}$ from the high-temperature source and delivers $\dot{Q}_{1}$ to a dwelling at $T_{\mathrm{d}}$. The heat pump receives $\dot{Q}_{0}$ from the outdoors at $T_{0}$, and delivers $\dot{Q}_{2}$ to the dwelling.


A Figure P5.22
(a) Obtain an expression for the maximum theoretical value of the performance parameter $\left(\dot{Q}_{1}+\dot{Q}_{2}\right) / \dot{Q}_{\mathrm{s}}$ in terms of the temperature ratios $T_{\mathrm{s}} / T_{\mathrm{d}}$ and $T_{0} / T_{\mathrm{d}}$.
(b) Plot the result of part (a) versus $T_{\mathrm{s}} / T_{\mathrm{d}}$ ranging from 2 to 4 for $T_{0} / T_{\mathrm{d}}=0.85,0.9$, and 0.95 .
5.23 A power cycle operates between a reservoir at temperature $T$ and a lower-temperature reservoir at 280 K . At steady state, the cycle develops 40 kW of power while rejecting 1000 $\mathrm{kJ} / \mathrm{min}$ of energy by heat transfer to the cold reservoir. Determine the minimum theoretical value for $T$, in K. Ans: 952
5.24 A certain reversible power cycle has the same thermal efficiency for hot and cold reservoirs at 1000 and 500 K , respectively, as for hot and cold reservoirs at temperature $T$ and 1000 K . Determine $T$, in K.
5.25 A reversible power cycle whose thermal efficiency is $50 \%$ operates between a reservoir at 1800 K and a reservoir at a lower temperature $T$. Determine $T$, in K .
5.26 An inventor claims to have developed a device that executes a power cycle while operating between reservoirs at 800 and 350 K that has a thermal efficiency of (a) $56 \%$, (b) $40 \%$. Evaluate the claim for each case.
5.27 At steady state, a cycle develops a power output of 10 kW for heat addition at a rate of 10 kJ per cycle of operation from a source at 1500 K . Energy is rejected by heat transfer to cooling water at 300 K . Determine the minimum theoretical number of cycles required per minute.
5.30 During January, at a location in Alaska winds at $-30^{\circ} \mathrm{C}$ can be observed. Several meters below ground the temperature remains at $13^{\circ} \mathrm{C}$, however. An inventor claims to have devised a power cycle exploiting this situation that has a thermal efficiency of $10 \%$. Discuss this claim.
5.33 An inventor claims to have developed a refrigeration cycle that requires a net power input of 1.2 kW to remove $25,000 \mathrm{~kJ} / \mathrm{h}$ of energy by heat transfer from a reservoir at $-30^{\circ} \mathrm{C}$ and discharge energy by heat transfer to a reservoir at $20^{\circ} \mathrm{C}$. There are no other energy transfers with the surroundings. Evaluate this claim.
5.35 The refrigerator shown in Fig. P5.35 operates at steady state with a coefficient of performance of 4.5 and a power input of 0.8 kW . Energy is rejected from the refrigerator to the surroundings at $20^{\circ} \mathrm{C}$ by heat transfer from metal coils whose average surface temperature is $28^{\circ} \mathrm{C}$. Determine
(a) the rate energy is rejected, in kW .
(b) the lowest theoretical temperature inside the refrigerator, in K.
(c) the maximum theoretical power, in kW , that could be developed by a power cycle operating between the coils and

## Refrigerator

$\mathrm{COP}=4.5$


Figure P5.35
the surroundings. Would you recommend making use of this opportunity for developing power?
5.36 Determine the minimum theoretical power, in W, required at steady state by a refrigeration system to maintain a cryogenic sample at $-126^{\circ} \mathrm{C}$ in a laboratory at $21^{\circ} \mathrm{C}$, if energy leaks by heat transfer to the sample from its surroundings at a rate of 900 W .

Ans: 900
5.37 For each kW of power input to an ice maker at steady state, determine the maximum rate that ice can be produced, in $\mathrm{kg} / \mathrm{h}$, from liquid water at $0^{\circ} \mathrm{C}$. Assume that $333 \mathrm{~kJ} / \mathrm{kg}$ of energy must be removed by heat transfer to freeze water at $0^{\circ} \mathrm{C}$, and that the surroundings are at $20^{\circ} \mathrm{C}$.
5.47 One kilogram of air as an ideal gas executes a Carnot power cycle having a thermal efficiency of $60 \%$. The heat transfer to the air during the isothermal expansion is 40 kJ . At the end of the isothermal expansion, the pressure is 5.6 bar and the volume is $0.3 \mathrm{~m}^{3}$. Determine
(a) the maximum and minimum temperatures for the cycle, in K .
(b) the pressure and volume at the beginning of the isothermal expansion in bar and $\mathrm{m}^{3}$, respectively.
(c) the work and heat transfer for each of the four processes, in kJ .
(d) Sketch the cycle on $p-v$ coordinates.
5.48 The pressure-volume diagram of a Carnot power cycle executed by an ideal gas with constant specific heat ratio $k$ is shown in Fig. P5.48. Demonstrate that
(a) $V_{4} V_{2}=V_{1} V_{3}$.
(b) $T_{2} / T_{3}=\left(p_{2} / p_{3}\right)^{(k-1) / k}$.
(c) $T_{2} / T_{3}=\left(V_{3} / V_{2}\right)^{k-1}$.


4 Figure P5.48

