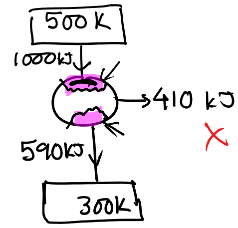
Sample problems

An inventor claims to have developed a power cycle capable of delivering a net work output of 410 kJ for an energy input by heat transfer of 1000 kJ. The system undergoing the cycle receives the heat transfer from hot gases at a temperature of 500 K and discharges energy by heat transfer to the atmosphere at 300 K. Evaluate this claim.



×

$Q_H = 1000$ kJ

$Q_L = 590$ kJ

$\eta_R = 1 - \frac{T_L}{T_H} = 0.4$

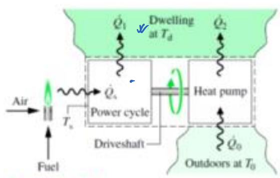
$W_R = 400$ kJ

Clausius inequality

$$\frac{1000}{500} + \frac{-590}{300}$$

$$= 2 - 1.96 > 0 \Rightarrow \boxed{\text{Impossible}}$$

5.22 Figure P5.22 shows a system consisting of a power cycle driving a heat pump. At steady state, the power cycle receives $\dot{Q}_s$ by heat transfer at T_s from the high-temperature source and delivers $\dot{Q}_1$ to a dwelling at T_d. The heat pump receives $\dot{Q}_0$ from the outdoors at T_0, and delivers $\dot{Q}_2$ to the dwelling.



▲ Figure P5.22

(a) Obtain an expression for the maximum theoretical value of the performance parameter $(\dot{Q}_1 + \dot{Q}_2)/\dot{Q}_s$ in terms of the temperature ratios T_s/T_d and T_0/T_d.

(b) Plot the result of part (a) versus T_s/T_d ranging from 2 to 4 for $T_0/T_d = 0.85$, 0.9, and 0.95.

Room heating

Fuel

Performance parameter

$$\beta = \frac{\text{Desired output}}{\text{input}}$$

$$= \frac{\dot{Q}_1 + \dot{Q}_2}{\dot{Q}_s}$$

β is max when cycles are reversible

Clausius inequality:

$$\frac{\dot{Q}_s}{T_s} = \frac{\dot{Q}_1}{T_d} \; ; \; \frac{\dot{Q}_0}{T_0} = \frac{\dot{Q}_2}{T_d}$$

$$\dot{Q}_s - \dot{Q}_1 = \dot{Q}_2 - \dot{Q}_0$$

$$\dot{Q}_1 + \dot{Q}_2 = \dot{Q}_s + \dot{Q}_0$$

$$\boxed{\frac{\dot{Q}_1 + \dot{Q}_2}{\dot{Q}_s} = 1 + \frac{\dot{Q}_0}{\dot{Q}_s}}$$

① HE: $\frac{\dot{Q}_s}{T_s} + \frac{(-\dot{Q}_1)}{T_d} = 0$ — (A)

② HP: $\frac{\dot{Q}_0}{T_0} + \frac{-\dot{Q}_2}{T_d} = 0$ — (B)

HP + HE: $\frac{\dot{Q}_s}{T_s} + \frac{\dot{Q}_0}{T_0} - \frac{\dot{Q}_1 + \dot{Q}_2}{T_d} = 0$

$$\Rightarrow \frac{T_d}{\dot{Q}_s}\left(\frac{\dot{Q}_1 + \dot{Q}_2}{T_d}\right) = \left(\frac{\dot{Q}_s}{T_s} + \frac{\dot{Q}_0}{T_0}\right)\frac{T_d}{\dot{Q}_s}$$

$$\Rightarrow \frac{\dot{Q}_1 + \dot{Q}_2}{\dot{Q}_s} = \left(\frac{T_d}{T_s} + \frac{\dot{Q}_0}{\dot{Q}_s} \times \frac{T_d}{T_0}\right)$$

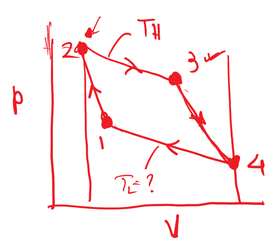
From A $\dot{Q}_s = \frac{\dot{Q}_1}{T_d} T_s$ — ①

From B $\dot{Q}_0 = \frac{\dot{Q}_2}{T_d} T_0$ — ②

$\frac{\dot{Q}_0}{\dot{Q}_s} = \frac{T_0}{T_d} \cdot \frac{\dot{Q}_2}{\dot{Q}_1}$

$\Rightarrow \dot{Q}_1 + \dot{Q}_2$

One kilogram of air as an ideal gas executes a Carnot power cycle having a thermal efficiency of 60%. The heat transfer to the air during the isothermal expansion is 40 kJ. At the end of the isothermal expansion, the pressure is 5.6 bar and the volume is 0.3 m3. Determine **(a)** the maximum and minimum temperatures for the cycle, in K. **(b)** the pressure and volume at the beginning of the isothermal expansion in bar and m3, respectively. **(c)** the work and heat transfer for each of the four processes, in kJ. **(d)** Sketch the cycle on *p*–*v* coordinates.



$Q_{2-3} = Q_H = 40$ kJ

$\eta = 60\% \Rightarrow W = 24$ kJ

$Q_L = 16$ kJ

$T_3 = ?$ $p_3 = 560$ kPa

$m = 1$ kg $V_3 = 0.3\ m^3$

$\eta = 60\% \Rightarrow W = 24\ kJ$

$Q_L = 16\ kJ$

$\Rightarrow T_H = 585.36\ K$

$T_L = ?$

Clausius inequality $\Rightarrow \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0$

if it is Rev.

$\Rightarrow T_L = \frac{Q_L}{Q_H} \times T_H = \frac{16}{40} \times 585.36$

$= 234.14\ K$

$T_3 = ?$ $p_3 = 560\ kPa$

$m = 1\ kg$ $V_3 = 0.3\ m^3$

$p_3 V_3 = m R_a T_3$

$\Rightarrow T_3 = \frac{560 \times 0.3}{1 \times 0.287}$

$= 585.36\ K$

For process 2–3

$Q_{23} = W_{23} = \int_2^3 p\,dV$

$\Rightarrow R T_H \ln \frac{v_3}{v_2} = 40\ kJ$

$\Rightarrow v_2 = v$

	Q	W	ΔU
1–2			
2–3			
3–4			
4–1			

7.86 A combination of a heat engine driving a heat pump (see Fig. P7.86) takes waste energy at 50°C as a source Q_{w1}, to the heat engine rejecting heat at 30°C. The remainder, Q_{w2}, goes into the heat pump that delivers a Q_H at 150°C. If the total waste energy is 5 MW, find the rate of energy delivered at the high temperature.

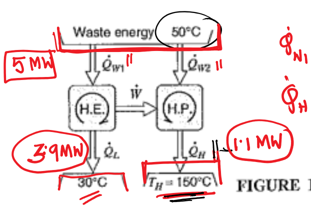
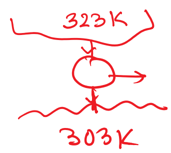


FIGURE P7.86

$\dot{Q}_{W1} + \dot{Q}_{W2} = 5000\ kW$

$\dot{Q}_H = ?$

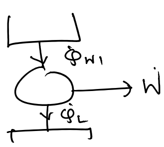
Waste → Energy → Low T



$\eta_R = 1 - \frac{303}{323} = 0.062$

6.2 %

Concentrate the energy from Low T to high T, thus improving its quality.



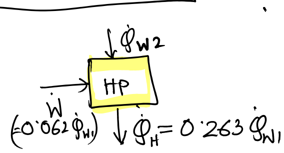
$\dot{W} = \dot{Q}_{W1} \times \eta_{HE} = \dot{Q}_{W1} \times \left(1 - \frac{303}{323}\right) = 0.062\,\dot{Q}_{W1}$

$(COP)_R = \frac{T_H}{T_H - T_L} = \frac{423}{423 - 323} = 4.23 = \frac{\dot{Q}_H}{\dot{W}}$

$\dot{Q}_H = 4.23 \times \dot{W} = 4.23 \times 0.062\,\dot{Q}_{W1}$

$= 0.263\,\dot{Q}_{W1}$

$\boxed{\dot{Q}_H = 0.263\,\dot{Q}_{W1}}$



$\dot{Q}_{W2} + \dot{W} = \dot{Q}_H$

$\dot{Q}_{W2} = \dot{Q}_H - \dot{W}$

$= (0.263 - 0.062)\,\dot{Q}_{W1}$

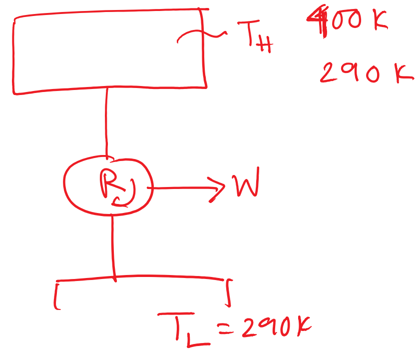
$= 0.2\,\dot{Q}_{W1}$

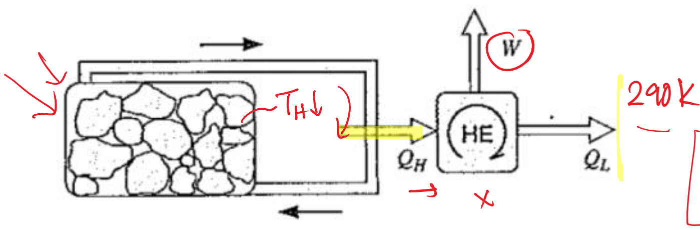
$\dot{Q}_{W1} + \dot{Q}_{W2} = (1 + 0.2)\,\dot{Q}_{W1} = 5\ MW$

$\dot{Q}_H = 0.263 \times \frac{5}{1.2} = 1.1\ MW$

7.61 A thermal storage device is made with a rock (granite) bed of 2 m^3 that is heated to 400 K using solar energy. A heat engine receives a Q_H from the bed and rejects heat to the ambient surroundings at 290 K. The rock bed therefore cools down, and as it reaches 290 K the process stops. Find the energy the rock bed can give out. What is the heat engine efficiency at the beginning of the process, and what is it at the end of the process?

Consider ρ= 2750 kg/ m^3, c = 0.89 kJ/kgK





$M_R = \rho V = 5500$ kg.

$$\boxed{Q_H = M_R \times c \times (400-290)}$$

$= 5500 \times 0.89 \times 110$ kJ

$= 538.45 \times 10^3$ kJ

$\delta Q_H = -M_R c\, dT$

dW

δQ_L

$\eta = \left(1 - \frac{T_L}{T_H}\right)$

$dQ_H = M$

$$dW = dQ_H \left(1 - T_L/T_H\right)$$

$$= -M_R c \; dT_H \left(1 - T_L/T_H\right)$$

$$W = -\int_{T_H=400}^{T_H=290} M_R c \left(dT_H - T_L \frac{dT_H}{T_H}\right) =$$

$$= M_R c \int_{290}^{400} \left(dT_H - T_L \frac{dT_H}{T_H}\right) =$$

7.48 We propose to heat a house in the winter with a heat pump. The house is to be maintained at 20°C at all times. When the ambient temperature outside drops to −10°C, the rate at which heat is lost from the house is estimated to be 25 kW. What is the minimum electrical power required to drive the heat pump?

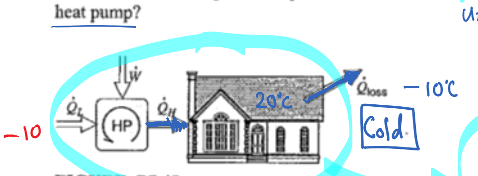


FIGURE P7.48

Corr - I

$\eta_R > \eta_I$
$COP_R > COP_I$

Under steady state

$$\boxed{Q_H = Q_{Loss}}$$

293 K

$Q_H = 25$ kW

$W = ?$

263 K

$COP = \frac{Q_H}{W}$

$W = W_{min}$

$\Rightarrow COP = COP_{max}$

$\Rightarrow$ the cycle is Rev.

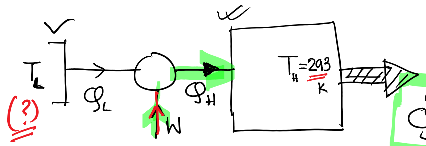
For W = W min the HP has to be

$$(COP)_{R_H} = \frac{Q_H}{Q_H - Q_L} = \frac{T_H}{T_H - T_L} = \frac{293}{30} = 9.76$$

$$\boxed{W = 25/9.76 = 2.56 \text{ kW}}$$

A heat pump has a COP of $\beta' = 0.5\,\beta_{CARNOT}$ and maintains a house at $TH = 20 \circ$ C, while it leaks energy out as $\dot{Q}_{Loss} = 0.6(TH - TL)$[kW]. For a maximum of 1.2 kW power input, find the minimum outside temperature, T_L, for which the heat pump is a sufficient heat source.

$\boxed{COP \equiv \beta}$



T_L $\boxed{\dot{Q}_H = \dot{Q}_L}$

$$\boxed{\dot{Q}_{Loss} = 0.6\,(T_H - T_L)}$$

$$\frac{\dot{Q}_H}{W} = \beta = 0.5\,\beta_{Carnot} = 0.5\left(\underbrace{\frac{T_H}{T_H - T_L}}_{\beta_{Carnot}}\right)$$

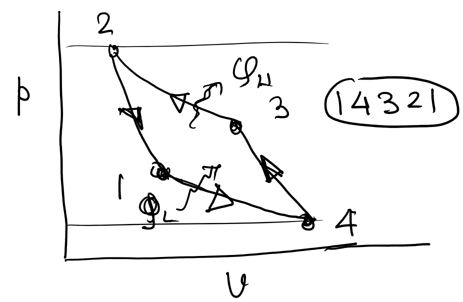
$$\frac{\dot{Q}_{Loss}}{W} = 0.5\left(\frac{293}{293 - T_L}\right)$$

$$\Rightarrow \frac{0.6(293 - T_L)}{W} = 0.5\,\frac{293}{(293 - T_L)}$$

$$\Rightarrow (293 - T_L)^2 = \cancel{\tfrac{5}{6}} \times 293 \times \cancel{1.2}$$

$$\boxed{293^2 - 2 \times 293\,T_L + T_L^2 = 293} \rightarrow \boxed{T_L =}$$

Carnot cycle Heat Pump



7.89 A Carnot heat engine, shown in Fig. P7.89, receives energy from a reservoir at T_{res} through a heat exchanger where the heat transferred is proportional to the temperature difference as $\dot{Q}_H = K(T_{res} - T_H)$. It rejects heat at a given low temperature T_L. To design the heat engine for maximum work output, show that the high temperature, T_H, in the cycle should be selected as $T_H = (T_L T_{res})^{1/2}$.

$T_{res} > T_H$

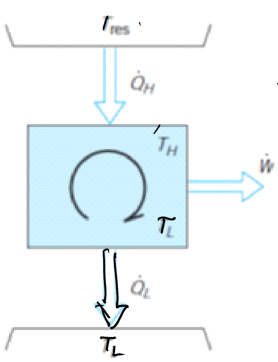


FIGURE P7.89

$$\dot{Q}_H = K(T_{res} - T_H)$$

What is the optimum T_H?

$$\dot{W} = \eta \times \dot{Q}_H$$

If $\eta \uparrow \Rightarrow \dot{W} \uparrow$

If $\dot{Q}_H \uparrow \Rightarrow \dot{W} \uparrow$

To get the max output from the engine, the product $\eta * \dot{Q}_H$ should be max. If $T_H \uparrow$, $\eta \uparrow$ & $\dot{Q}_H \downarrow$

So there must be an optimum point.

Considering the cycle is internally reversible,

$$\eta_{cycle} = \left(1 - \frac{T_L}{T_H}\right)$$

$$\dot{W} = \dot{Q} \times \eta_{cycle} = K(T_{res} - T_H)\left(1 - \frac{T_L}{T_H}\right)$$

$$\dot{W} = k\left[T_{res} - T_L T_{res} \times \frac{1}{T_H} - T_H + T_L\right]$$

$$\dot{W} = k\left[T_{res} - T_L T_{res} * \frac{1}{T_H} - T_H + T_L \right]$$

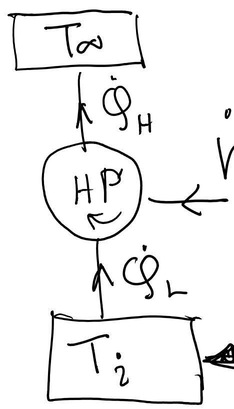
$\dot{W}$ is max when

$$\left.\begin{aligned} \frac{d\dot{W}}{dT_H} &= 0 \\ \frac{d^2\dot{W}}{dT_H^2} &< 0 \end{aligned}\right\} \begin{aligned} &K\left[0 + T_L T_{res} \frac{1}{T_H^2} - 1\right] = 0 \\ &\Rightarrow T_H = \sqrt{T_L T_{res}} \\ &-2K\, T_L T_{res} / T_H^3 \quad < 0 \end{aligned}$$

$\dot{W}$ is max when $T_H = (T_L T_{res})^{1/2}$

Find $\dot{W}_{max} = k\left(\sqrt{T_{res}} - \sqrt{T_L}\right)^2$ [Show]

7.73 A house is cooled by an electric heat pump using the outside as the high-temperature reservoir. For several different summer outdoor temperatures, estimate the percent savings in electricity if the house is kept at 25°C instead of 20°C. Assume that the house is gaining energy from the outside in direct proportion to the temperature difference, as in Eq. 7.17.



$\dot{Q}_{leak} = K(T_\infty - T_i)$

Let T_∞ = outside temp.
T_i = indoor temp.

Heat inleakage $\dot{Q}_{leak} = K(T_\infty - T_i)$

Let the COP of the heat pump be $\beta' = f * \beta_{carnot}$ where $f < 1$ (just to acknowledge that it is a real heat pump)

$$\dot{W} = \dot{Q}_L / \beta' = \frac{\dot{Q}_{leak}}{\beta'}$$ [∵ for the house $\dot{Q}_L = \dot{Q}_{in}$]

* Note: Although here we are using a heat

pump, the expression of COP is

$$\beta' = \dot{Q}_L / \dot{W},$$

because it is used as a refrigerator (AC)

$$\dot{W} = \frac{\dot{Q}_{in}}{f \beta_{carnot}} = \frac{K(T_\infty - T_i)}{f \times \frac{T_i}{T_\infty - T_i}} = \frac{K}{f} \frac{(T_\infty - T_i)^2}{T_i}$$

Here, if we want to find out the change in $\dot{W}$ due to a change

Home work: Assume T_∞ = 300, 305, 310, 315, 320 and 325 K. Calculate the variation of W for the two values of T_i. Plot the same in Excel.