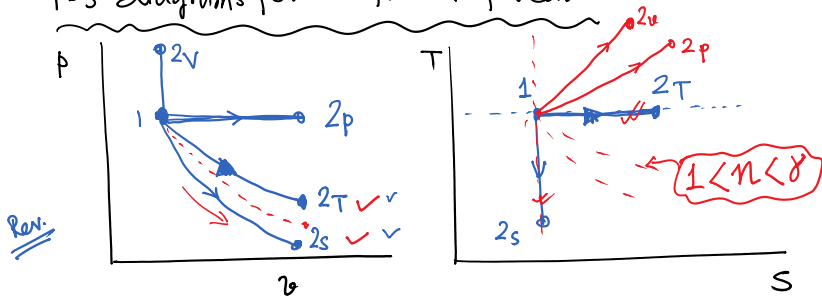


T-s diagrams for different processes

$\delta q = T ds$



isothermal process \Rightarrow δ \uparrow ? \downarrow ?
expansion

Adiabatic expansion from 1 to 2

$\delta q = 0$
 $\delta W = dU$

$-(+ve) = dU \Rightarrow dU$ is -ve
 dT is -ve

Adiabatic + frictionless = Isentropic

Reversible adiabatic = Isentropic

$\delta q - \delta W = dU$

if Reversible $\Rightarrow \delta q = T ds$
 $\delta W = p dv$

$\delta q - \delta W = dU$

$\delta q = T ds$

$\delta W = p dV$

divide by m

$T ds - p dv = du$

or $T ds = du + p dv$

For a constant volume process $du = 0$

$\Rightarrow T ds = du$

$T ds = C_v dT$

or $\left. \frac{dT}{ds} \right|_{v=c} = \frac{T}{C_v}$

$du = C_v dT$

$\Rightarrow T ds = du + p dv + v dp - v dp$

$= du + p dv - v dp$

$= d(u + pv) - v dp$

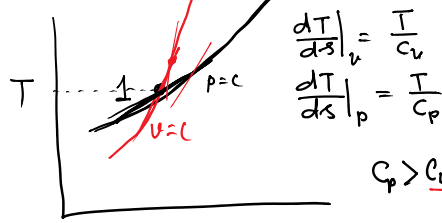
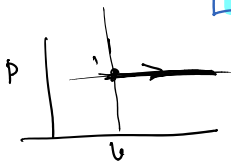
$\Rightarrow T ds = dh - v dp$

$u + pv = h$

Const pr. process $dp = 0 \Rightarrow T ds = dh = C_p dT$

$dh = C_p dT$

$\left. \frac{dT}{ds} \right|_{p=c} = \frac{T}{C_p}$



$\left. \frac{dT}{ds} \right|_{v=c} = \frac{T}{C_v}$
 $\left. \frac{dT}{ds} \right|_{p=c} = \frac{T}{C_p}$

$C_p > C_v$

$\frac{300}{1.004} = 300 \frac{k kJ}{kJ}$
 $\frac{300}{0.7}$

(1.004)

$$\frac{300}{1.004} = 300 \frac{1}{\text{kJ}}$$

$$\frac{dT}{ds}|_p = \frac{T}{c_p} \Rightarrow$$

$$\boxed{\begin{matrix} c_p(T) \\ c_v(T) \end{matrix}}$$

How to compute the change of entropy of pure substances?

$$\boxed{\delta q_r = T ds}$$

for reversible process



$$ds = \frac{\delta q_r}{T}$$

$$\begin{matrix} \Delta p \rightarrow \\ \Delta T \rightarrow \\ \Delta m \rightarrow \end{matrix} \quad \text{100°C}$$

Entropy change of a solid/liquid

$$ds = \delta q_r / T$$

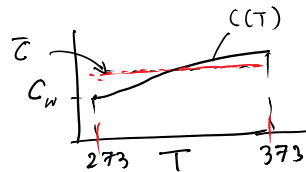
$$\delta q_r = c dT$$

c = sp. heat of the solid/liquid

$$\Rightarrow ds = \frac{c dT}{T}$$

If c = constant over the temperature range

$$\int_1^2 ds = \bar{c} \int_{T_1}^{T_2} \frac{dT}{T}$$



$$\boxed{s_2(T) - s_1(T) = \bar{c} \ln \frac{T_2}{T_1}}$$

average sp. heat

$$\boxed{\text{For } c \neq \text{const} \quad s_2(T) - s_1(T) = \int_{T_1}^{T_2} \frac{c(T)}{T} dT}$$

Prob 50 kg of Fe casting block at 500 K is thrown into a large lake at 285 K. The block attains the temp. of the lake finally. Assuming $\bar{c} = 0.45 \text{ kJ/kgK}$ for steel, determine the Δs_{Fe}

$$\Delta s_{Fe} = m \times \Delta s_F = 50 \text{ kg} \times \bar{c} \ln \frac{T_2}{T_1} = 50 \times 0.45 \times \ln \frac{285}{500}$$

s(T)

$$\boxed{\Delta s_{Fe} = -12.64 \text{ kJ/K}}$$

Entropy change of ideal gases

$$ds = \delta q_r / T$$

$$s(T, p)$$

$$s(T, v)$$

$$\begin{cases} T ds = du + p dv & \text{--- (A)} \\ T ds = dh - v dp & \text{--- (B)} \end{cases}$$

$$\delta q_r = c dT \quad \text{which process}$$

$$\begin{matrix} c = c_v & c = c_p \\ c = 0 & c = \infty \end{matrix}$$

$$\boxed{pv = RT}$$

$$\left\{ \begin{aligned} T ds &= du + p dv \\ T ds &= dh - v dp \end{aligned} \right. \quad \text{--- (b)}$$

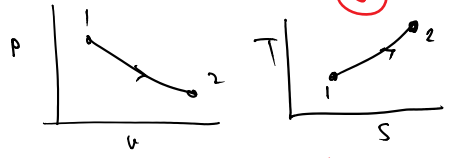
$$p v = R T$$

$$\begin{aligned} ds &= \frac{du}{T} + \frac{p}{T} dv \\ &= \frac{C_v dT}{T} + R \frac{dv}{v} \end{aligned}$$

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{C_v dT}{T} + R \ln\left(\frac{v_2}{v_1}\right) \quad \text{--- (C)}$$

$$\begin{aligned} ds &= \frac{dh}{T} - \frac{v dp}{T} \\ &= \frac{C_p dT}{T} - R \frac{dp}{p} \end{aligned}$$

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{C_p dT}{T} - R \ln\left(\frac{p_2}{p_1}\right) \quad \text{--- (D)}$$



If Temperature const. $T_1 = T_2$
 Isothermal
 $s_2 - s_1 = 0 + R \ln \frac{v_2}{v_1}$
 if $v_2 > v_1 \Rightarrow s_2 > s_1$

if $T = c$
 $s_2 - s_1 = 0 - R \ln(p_2/p_1)$
 if $p_2 > p_1 \Rightarrow s_2 < s_1$

Const vol.

For constant C_p & C_v

$$\text{(C)} \Rightarrow s_2 - s_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \text{--- (E)}$$

$$\text{(D)} \Rightarrow s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \quad \text{--- (F)}$$

Adiabatic & Reversible

$$p v^\gamma = c$$

$\gamma = \text{why?}$

For Isentropic process

$$s_2 = s_1 \Rightarrow \text{from (F)} \quad C_p \ln \frac{T_2}{T_1} = R \ln \frac{p_2}{p_1}$$

$$\frac{r}{r-1} R \ln \frac{T_2}{T_1} = R \ln \frac{p_2}{p_1}$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{r-1}{r}} \quad \text{--- (G)}$$

$$\left. \begin{aligned} C_v &= \frac{R}{\gamma-1} \\ C_p &= \frac{\gamma}{\gamma-1} R \\ C_p - C_v &= R \end{aligned} \right\}$$

From (E)

$$C_v \ln \frac{T_2}{T_1} = -R \ln \frac{v_2}{v_1}$$

$$\frac{R}{r-1} \ln \frac{T_2}{T_1} = R \ln \frac{v_1}{v_2}$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{r-1} \quad \text{--- (H)}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{r-1}{r}} = \left(\frac{v_1}{v_2}\right)^{r-1}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{1}{r}} = \left(\frac{v_1}{v_2}\right)$$

$$\Rightarrow p_1 v_1^r = p_2 v_2^r$$

$$p v^r = c \quad \text{--- (I)}$$

What is the implicit assumption?

C_p & C_v are not varying within the temperature range T_1 to T_2

From D

$$s_2(T, P) - s_1(T, P) = \int_{T_1}^{T_2} \frac{C_p dT}{T} - R \ln \frac{P_2}{P_1}$$

$$u = f(T)$$

$$h = f(T)$$

Find the change of u of 1 kg of O_2 when it is compressed from 1 bar 300K to 10 bar & 400K

$$R_{O_2} = \frac{R_u}{M} = \frac{8.315}{32} = 0.26$$

$$C_p = \frac{\gamma}{\gamma-1} R_{O_2} = \frac{1.4}{0.4} \times 0.26$$

$$s_2(T, P) - s_1(T, P) = \int_{300}^{400} \frac{C_p dT}{T} - 0.26 \times \ln 10$$

$$\Delta S \rightarrow = 0.91 \ln \frac{400}{300} - 0.26 \ln 10$$

$$= \checkmark$$

If C_p is not considered constant

$\int \frac{C_p dT}{T}$ needs to be computed accurately and also the entropy reference needs to be specified.

Calculate the entropy of air at 300K & 10 bar

$$s_{air}(T=300K, P=10bar) = ?$$

What is the ref. entropy?

s_{air} at 1 bar, $T=0$ is treated as zero or reference.

If state 1 \equiv 1 bar, 0K
 " 2 = P bar, T K

$$s_{air}(T, P) - s_{air}(T_0, P_0) = \int_{T_0}^{T_2=T} \frac{C_{p,air} dT}{T} - R_{air} \ln \frac{P}{P_0}$$

$$\text{or } s_{air}(T, P) = \left\{ s_{air,ref} + \int_{T_0}^T \frac{C_{p,air} dT}{T} \right\} - R_{air} \ln \left(\frac{P}{P_0} \right)$$

$$s_{air}(T, P) = s_{air}^0(T) - R_{air} \ln \left(\frac{P}{P_0} \right)$$

$P_0 = 100 \text{ kPa} = 1 \text{ bar}$

Calculate $s_{air}(300K, 10 \text{ bar})$

Go to table for ideal gas property of air
 Find s_{air} at $T=300K$ and use it in

$$s^{\circ}(300) = 1.70203 \text{ kJ/kgK}$$

$$s_1(300 \text{ K}, 10 \text{ bar}) = 1.70203 - 0.287 \ln(10/1)$$

$$= 1.041 \text{ kJ/kgK}$$

Air is heated at const pr. to 600K.
What is the final $s(T, p)$!

$$s_2(600 \text{ K}, 10 \text{ bar}) = s^{\circ}(600) - R_{\text{air}} \ln(10/1)$$

$$= 2.40902 - 0.287 \ln 10$$

What is the change of entropy

$$s_2 - s_1 = 0.707 \text{ kJ/kgK} \rightarrow$$

What if we used eqn. (F) which assumed constant sp. heat $C_p = 1.004 \text{ kJ/kgK}$

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$= 1.004 \ln \frac{600}{300} - R \ln \frac{10}{10}$$

$$= 1.004 \ln 2 - 0$$

$$= 0.696 \text{ kJ/kgK}$$

Entropy change of ideal gas

→ For C_p or $C_v = \text{const}$ $\Delta s = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$
 $= C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$

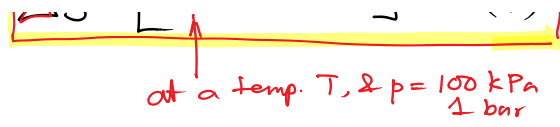
→ For C_p & C_v not constant (Realistic) $\Delta s = \int_{T_1}^{T_2} \frac{C_p dT}{T} - R \ln \frac{p_2}{p_1}$
 $= \int_{T_1}^{T_2} \frac{C_v dT}{T} + R \ln \frac{v_2}{v_1}$

We need $C_p(T)$ & $C_v(T)$ } relations

→ From Ideal gas tables

$$\Delta s = [s^{\circ}(T_2) - s^{\circ}(T_1)] - R \ln \left(\frac{p_2}{p_1} \right)$$

at a temp. T , & $p = 100 \text{ kPa}$

←  →

$$\left. \begin{aligned} \rightarrow s_1(T_1, p_1) &= s^\circ(T_1) - R \ln \frac{p_1}{p_0} \\ \rightarrow s_2(T_2, p_2) &= s^\circ(T_2) - R \ln \frac{p_2}{p_0} \end{aligned} \right\}$$

$$s_2(T_2, p_2) - s_1(T_1, p_1) = \left\{ s^\circ(T_2) - s^\circ(T_1) \right\} - R \left[\ln \frac{p_2}{p_0} - \ln \frac{p_1}{p_0} \right]$$

$$= \left[s^\circ(T_2) - s^\circ(T_1) \right] - R \ln \left(\frac{p_2}{p_0} \times \frac{p_0}{p_1} \right)$$

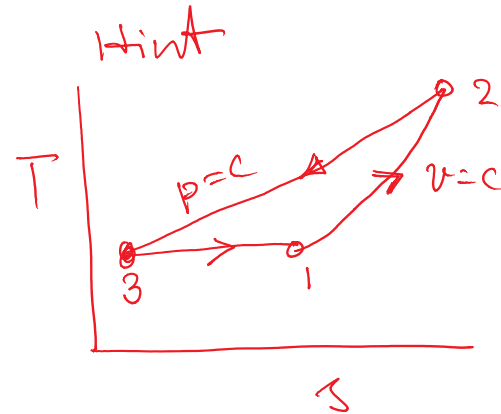
$$\boxed{\Delta S = \left[s^\circ(T_2) - s^\circ(T_1) \right] - R \ln \left(\frac{p_2}{p_1} \right)}$$

6.28 A quantity of air amounting to 2.42×10^{-2} kg undergoes a thermodynamic cycle consisting of three internally reversible processes in series.

Process 1-2: constant-volume heating at $V = 0.02 \text{ m}^3$ from $p_1 = 0.1 \text{ MPa}$ to $p_2 = 0.42 \text{ MPa}$

Process 2-3: constant-pressure cooling

Process 3-1: isothermal heating to the initial state



Employing the ideal gas model with $c_p = 1 \text{ kJ/kg} \cdot \text{K}$, evaluate the change in entropy, in kJ/K , for each process. Sketch the cycle on p - v and T - s coordinates.

Also, using the ideal gas property table of air, find the entropy changes between 1-2, 2-3 & 3-1