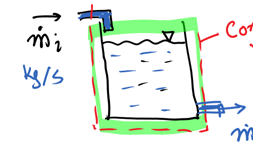
First Law for Open System (Control volume)

Tuesday, September 28, 2021 10:06 AM



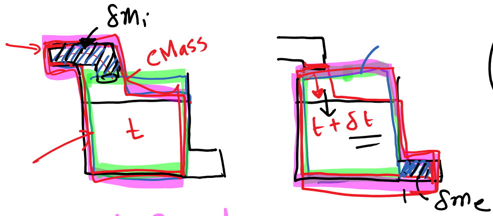
i = inlet
e = exit

$$\boxed{\left(\frac{dM}{dt}\right)_{CV} = \dot m_i - \dot m_e}$$

Closed System
≡ Control mass
Mass is Const.

$$\boxed{\left(\frac{dM}{dt}\right)_{CM} = 0}$$

Conservation of Mass



$$\left(\frac{dM}{dt}\right)_{CM} = \lim_{\delta t\to 0} \frac{M_{CM}(t+\delta t) - M_{CM}(t)}{\delta t}$$

$$= \lim_{\delta t\to 0} \frac{\left(M_{cv}^{(t+\delta t)} + \delta m_e\right) - \left(M_{cv}^{(t)} + \delta m_i\right)}{\delta t}$$

$$= \boxed{\lim_{\delta t\to 0} \frac{M_{cv}(t+\delta t) - M_{cv}(t)}{\delta t}}$$

$$+ \lim_{\delta t\to 0}\left(\frac{\delta m_e}{\delta t} - \frac{\delta m_i}{\delta t}\right)$$

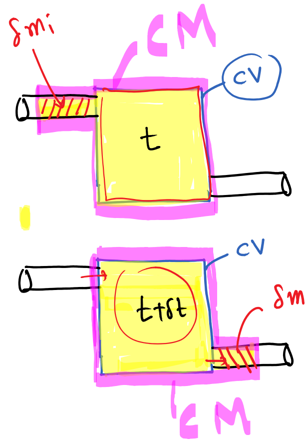
Closed System

$$\frac{dM}{dt} = 0$$

$$\left(\frac{dM}{dt}\right)_{CM} = \left(\frac{dM}{dt}\right)_{CV} + \dot m_e - \dot m_i$$

$$0 = \left(\frac{dM}{dt}\right)_{cv} + \dot m_e - \dot m_i$$

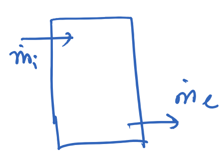
or $$\boxed{\left(\frac{dM}{dt}\right)_{cv} = \left(\dot m_i - \dot m_e\right)}$$

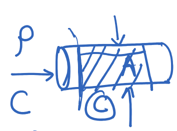


$$M_{CM}(t) = M_{cv}(t) + \delta m_i$$

$$M_{CM}(t+\delta t) = M_{cv}(t+\delta t) + \delta m_e$$

$\dot m$



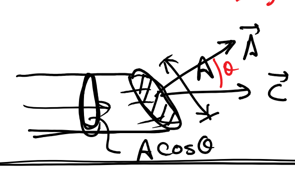


$$\dot m = \rho \times \text{Volume flowrate} \left[\frac{kg}{m^3}\cdot\frac{m^3}{s}\right]$$

$$= \rho \times (CA) \quad \left[\frac{kg}{m^3}\cdot \frac{m}{s} \times m^2\right]$$

$$\boxed{\dot m = \rho A C}$$

Area perpendicular to the velocity



$$\dot m = \rho \vec A \cdot \vec C = \rho A C \cos\theta$$

Conservation of Momentum for a Control Volume → Fluid Mechanics (Reynolds Transport Theorem)

For a closed System the good old Newton's 2nd Law is the conservation of linear momentum for a closed system $\sum \vec{F}_{CM} = m\,\vec{a}_{CM}$

$\boxed{\text{vector eqn.}}$

Conservation of Energy statement (1st Law of Thermo)

$\boxed{CM}$

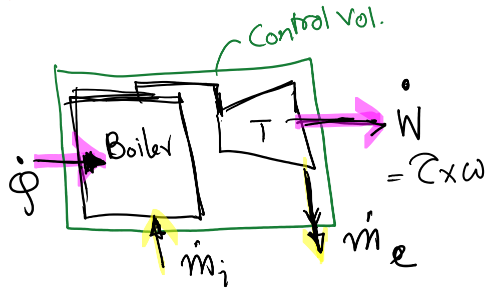
$$\dot{Q}_{CM} - \dot{W}_{CM} = \frac{dE_{CM}}{dt}$$

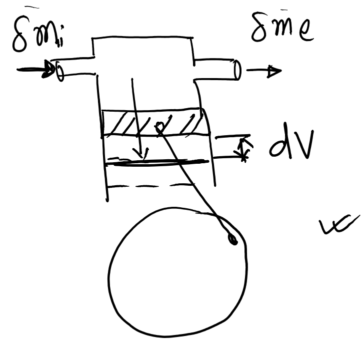
or $Q_{CM} - W_{CM} = \Delta E_{CM}$

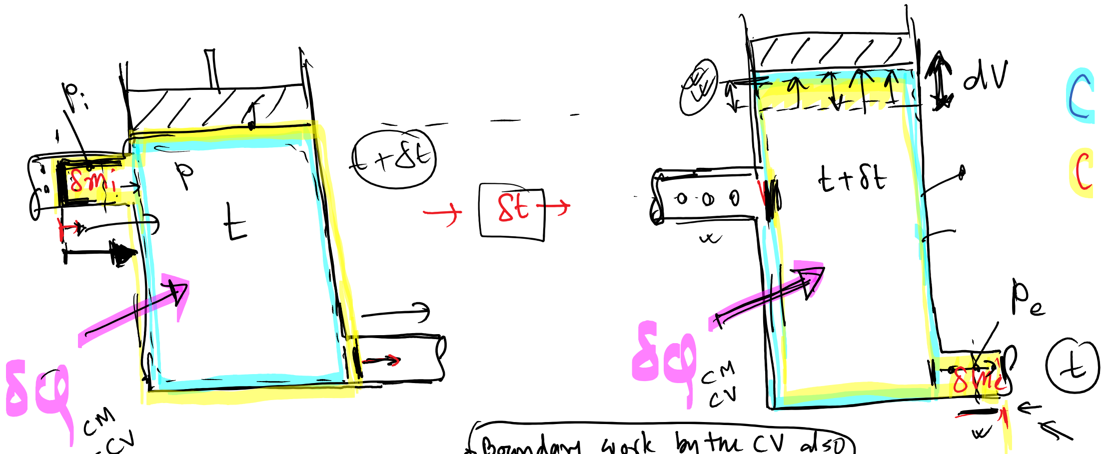
or $\boxed{\delta Q_{CM} - \delta W_{CM} = dE_{CM}}$

For Closed systems only (control mass)

What would be the corresponding statement for Control vol.?







CV Boundary

CM Boundary

(Boundary work by the CV also)

$$\delta W_{CM} = +p\,dV + (-p_i\,dV_i) + p_e\,dV_e$$

+ve dV — −ve dV — +ve dV

$$\underline{dV} = \delta m \times v$$

v → sp. vol.

$kg \cdot \frac{m^3}{kg}$

$$= \delta W_{CV} - p_i\,\delta m_i\,v_i + p_e\,\delta m_e\,v_e$$

$$\boxed{\delta W_{CM} = \delta W_{CV} + \delta m_e\,p_e v_e - \delta m_i\,p_i v_i} \quad (1)$$

$$\delta Q_{CM} = \delta Q_{CV} \quad \text{—— (2)}$$

First Law for Closed System

$$\delta Q_{CM} - \delta W_{CM} = dE_{CM} \quad \text{—— (3)}$$

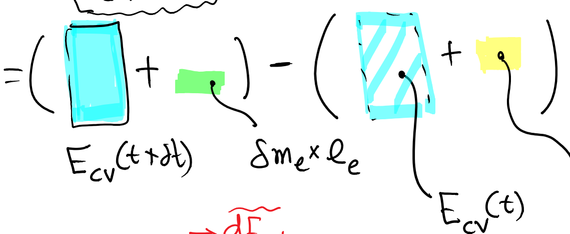
or

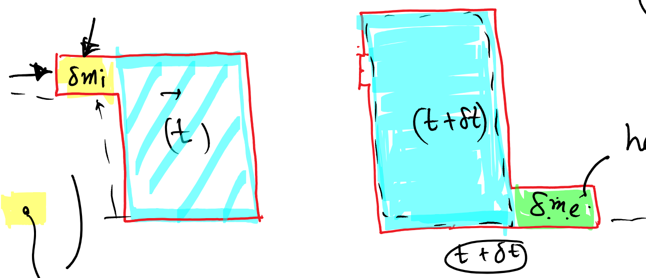
$$\delta Q_{CV} - \delta W_{CV} + \delta m_i p_i v_i - \delta m_e p_e v_e = dE_{CM} \quad \text{—— (4)}$$

From (1) (2) & (3)

$dE_{CM} = ?$

$$dE_{CM} = E_{CM}(t+\delta t) - E_{CM}(t)$$





$$dE_{CM} = \{\underbrace{E_{cv}(t+\delta t) - E_{cv}(t)}_{dE_{cv}}\} + \{\delta m_e e_e - \delta m_i e_i\} \quad \text{—— (5)}$$

Substituting dE_{CM} from (5) in (4)

$$\delta Q_{CV} - \delta W_{CV} + \delta m_i p_i v_i - \delta m_e p_e v_e = dE_{cv} + \delta m_e e_e - \delta m_i e_i$$

or

$$dE_{cv} = \delta Q_{cv} - \delta W_{cv} + \delta m_i (e_i + p_i v_i) - \delta m_e (e_e + p_e v_e)$$

dropping off the cv ~~term~~ suffix

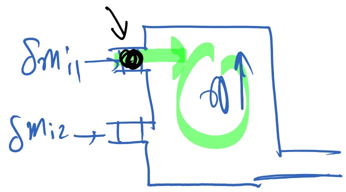
$$dE = \delta Q - \delta W + \delta m_i (e_i + p_i v_i) - \delta m_e (e_e + p_e v_e)$$

or

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \dot{m}_i (e_i + p_i v_i) - \dot{m}_e (e_e + p_e v_e)$$

if there are more than one inlet and or outlet

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum \dot{m}_i (e_i + p_i v_i) - \sum \dot{m}_e (e_e + p_e v_e)$$



$$\left.\frac{dE}{dt}\right|_{cm} = \dot{Q}\big|_{cm} - \dot{W}\big|_{cm} + \qquad -$$

$$\left.\frac{dM}{dt}\right)_{cm} = \sum \dot{m}_i - \sum \dot{m}_e$$

(The rate of change of mass in the CV) = (rate of mass coming in) − (rate of mass going out)

$$\left(\frac{dE}{dt}\right)_{cv} = \dot{Q} - \dot{W} +$$

$\dot{Q}$: energy added as heat

$\dot{W}$: energy drained by work

$$\boxed{e_{i,e} = u + \tfrac{1}{2}C^2 + gz}$$

u: internal energy, $f(T)$

$\tfrac{1}{2}C^2$: KE

gz: PE

$$\boxed{P_e v_e = \text{Flow energy}}$$

$$\boxed{u + pv = h}$$

$$\delta m_i \left(e_i + p_i v_i\right)$$

$$\delta m_i \left(u_i + p_i v_i + C_i^2/2 + g z_i\right)$$

$$\delta m_i \left(h_i + C_i^2/2 + g z_i\right)$$

$$\delta m_e \left(e_e + P_e v_e\right)$$

$$\delta m_e \left(u_e + P_e v_e + \frac{C_e^2}{2} + g_e z_e\right)$$

$$\delta m_e \left(h_e + C_e^2/2 + g_e z_e\right)$$

or

$$\boxed{\left(\frac{dE}{dt}\right)_{cv} = \dot{Q} - \dot{W} + \sum \dot{m}_i \left(\underbrace{h_i}_{\text{Enthalpy}} + C_i^2/2 + g z_i\right) - \sum \dot{m}_e \left(h_e + \frac{C_e^2}{2} + g z_e\right)}$$

= 1st Law for an Open System