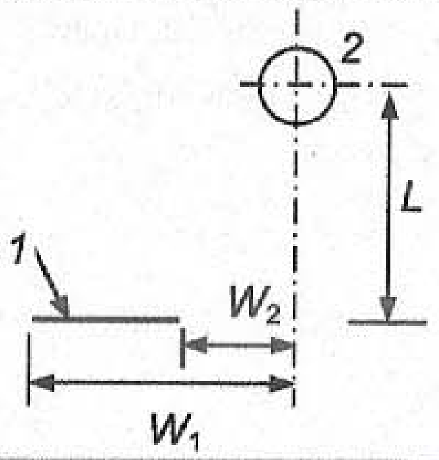
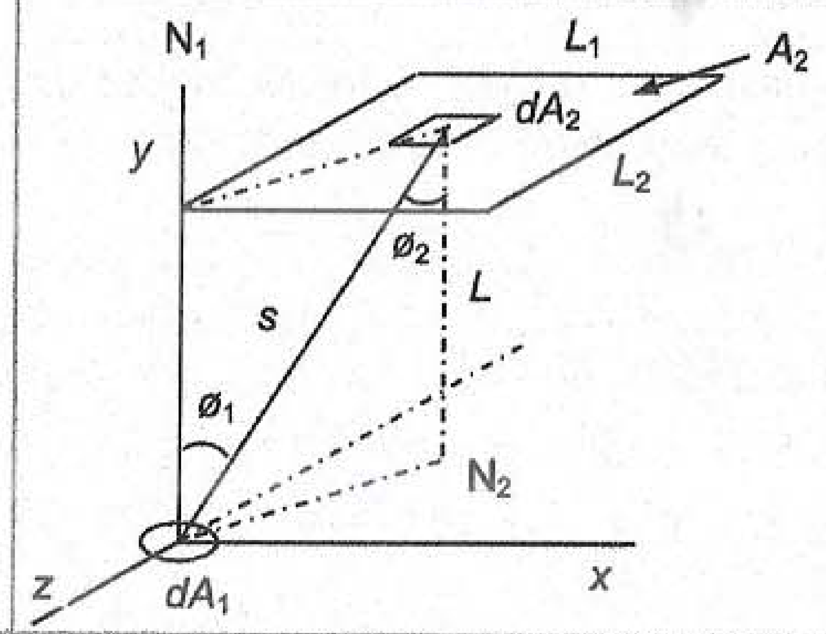
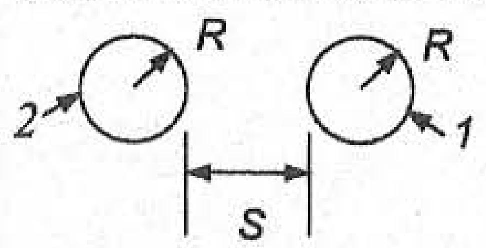
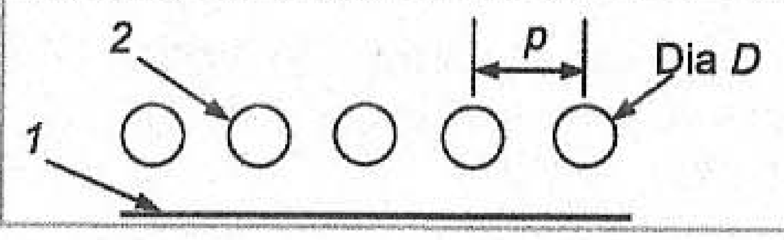
Table 11.1 (continued)

Case	Configuration	Shape, geometric or view factor
9		Infinite cylinder of radius R parallel to an infinite plate of width $W_1 - W_2$: $F_{12} = R/(W_1 - W_2)\ [\tan^{-1}(W_1/L) - \tan^{-1}(W_2/L)]$
10		$F_{12} = (1/2\pi)\left\{\left[L_1/\sqrt{(L_1^2+L^2)}\right] \sin^{-1}\left[L_2/\sqrt{(L_1^2+L_2^2+L^2)}\right] + \left[L_2/\sqrt{(L_2^2+L^2)}\right] \sin^{-1}\left[L_1/\sqrt{(L_1^2+L_2^2+L^2)}\right]\right\}$
11		Two parallel and infinite cylinders $F_{12} = 1/\pi\ [(X^2 - 1)^{1/2} + \sin^{-1}(1/X) - X]$, where $X = 1 + S/2R$
12		Row of infinite cylinders parallel to an infinite plate: $F_{12} = 1 - (1 - x^2)^{1/2} + x \tan^{-1}[(1 - x^2)/x^2]^{1/2}$, where p = pitch and $x = D/p$

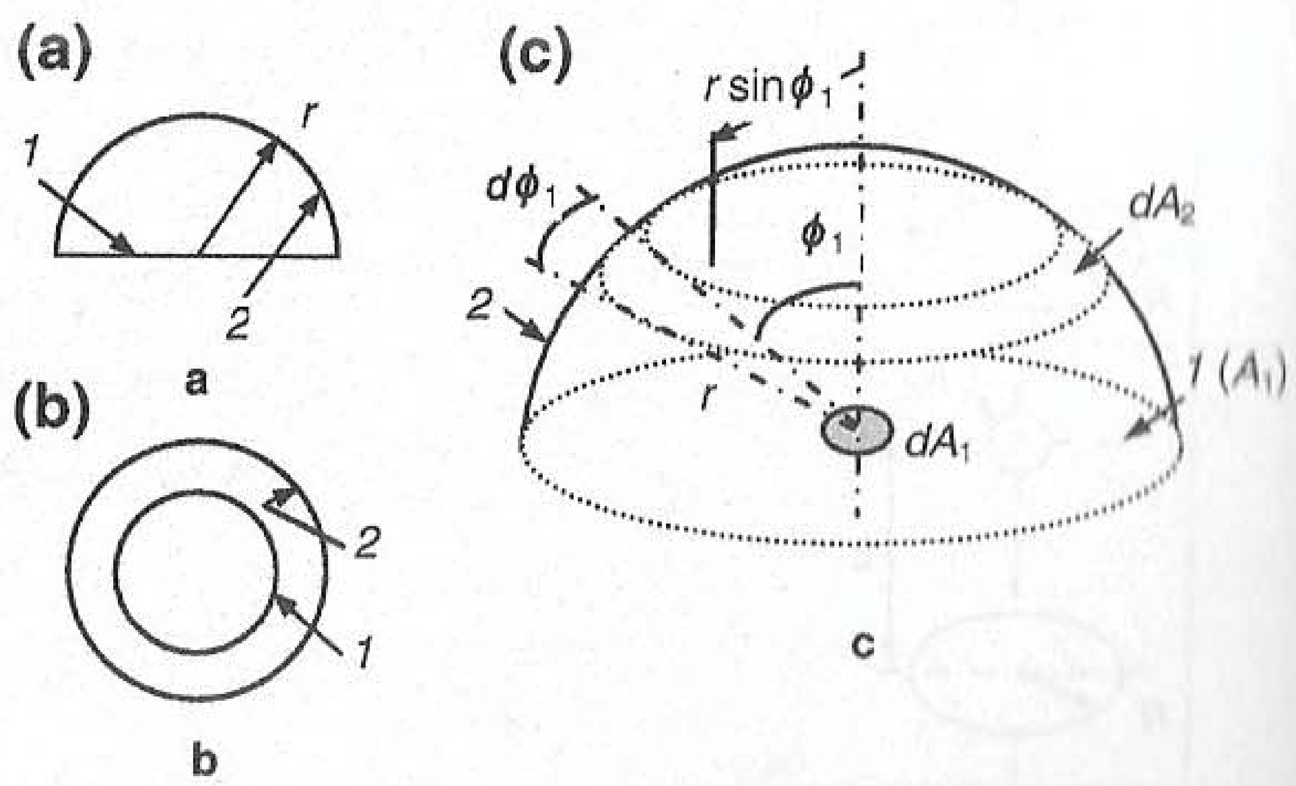


Fig. 11.5 Radiation from a flat or convex surface intercepted by an enclosure

2. **Subdivision of the Emitting Surface**
   In Fig. 11.6, the radiating surface A_1 has been divided into two areas A_3 and A_4. Then,

$$A_1F_{12} = A_3F_{32} + A_4F_{42}$$

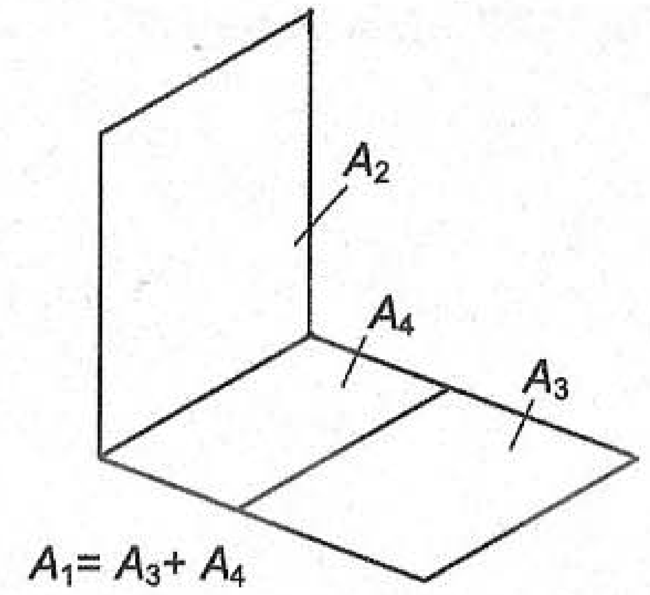


Fig. 11.6 Subdivision of emitting surface

In general, the above equation can be written as

$$A_iF_{ij} = \sum_{n=1}^{n} A_{in}F_{inj} \tag{11.20}$$

where area A_i has been divided into areas $A_{i1}, A_{i2}, \ldots, A_{in}$.

3. **Subdivision of the Receiving Surface**
   In Fig. 11.7, the receiving surface has been divided into areas $A_{2(1)}$ and $A_{2(2)}$. Then,

$$A_1F_{12} = A_1F_{12(1)} + A_1F_{12(2)}$$

or

$$F_{12} = F_{12(1)} + F_{12(2)}$$

In general,

$$F_{12} = \sum_{i=1}^{n} F_{12(i)} \tag{11.21}$$

where area A_2 has been divided into areas $A_{2(1)}, A_{2(2)}, \ldots, A_{2(n)}$. Equation expresses the *additive property* of the shape factor.

4. **Enclosure**
   If a flat or convex surface 1 is completely enclosed by surface areas $A_2, A_3, \ldots, A_n$, refer Fig. 11.8a, then

$$F_{12} + F_{13} + \cdots + F_{1n} = 1 \tag{11.22}$$

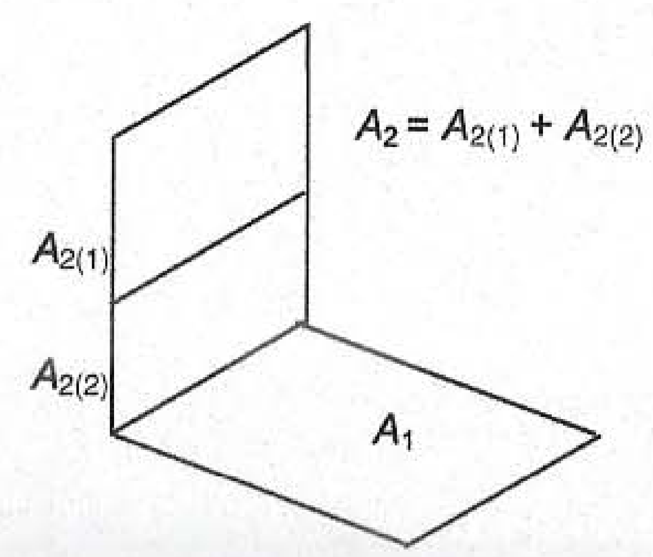


Fig. 11.7 Subdivision of the receiving surface

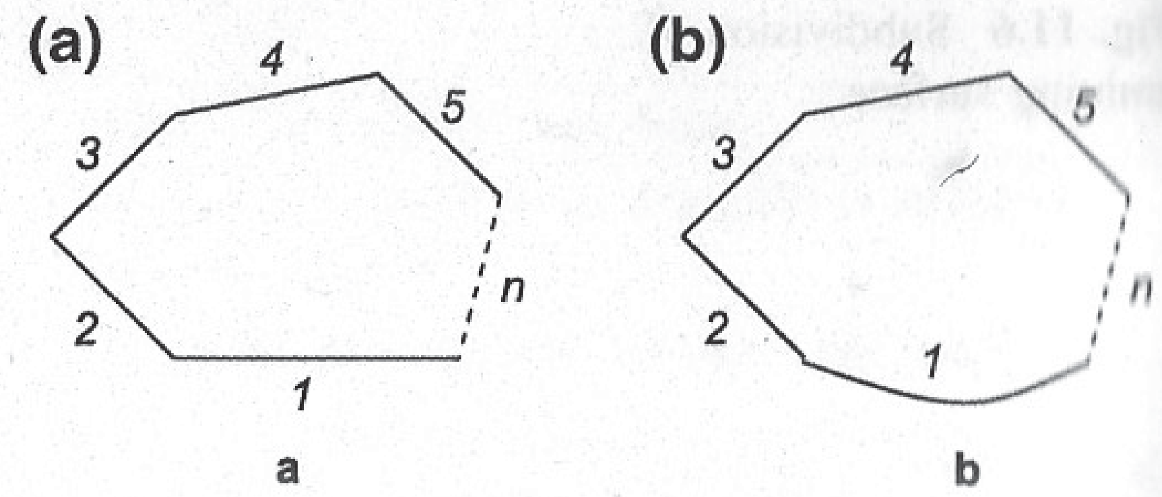


Fig. 11.8 Enclosures

In case of a concave surface, a fraction of the radiant energy emitted by one part of the concave surface will be intercepted by another part of the concave surface, refer Fig. 11.8b. Thus a concave surface has shape factor with respect to itself, which can be termed as F_{11}. It follows that in this case

$$F_{11} + F_{12} + F_{13} + \cdots + F_{1n} = 1 \tag{11.23a}$$

or

$$\sum_{n=1}^{n} F_{1n} = 1 \tag{11.23b}$$

This is known as *summation rule* and is simply based on the principle of conservation of energy. For the convex or a flat surface, F_{11} is zero and Eq. (11.22) results.

5. **Reciprocity Relation**
The reciprocity relation, as presented earlier, is Eq. (11.14)

$$A_1F_{12} = A_2F_{21} \tag{11.14}$$

It indicates that the radiation heat transfer can be determined using the shape factor either ways. If the two surfaces have the same area ($A_1 = A_2$), the shape factor will have the same value when the surfaces 1 and 2 are interchanged.

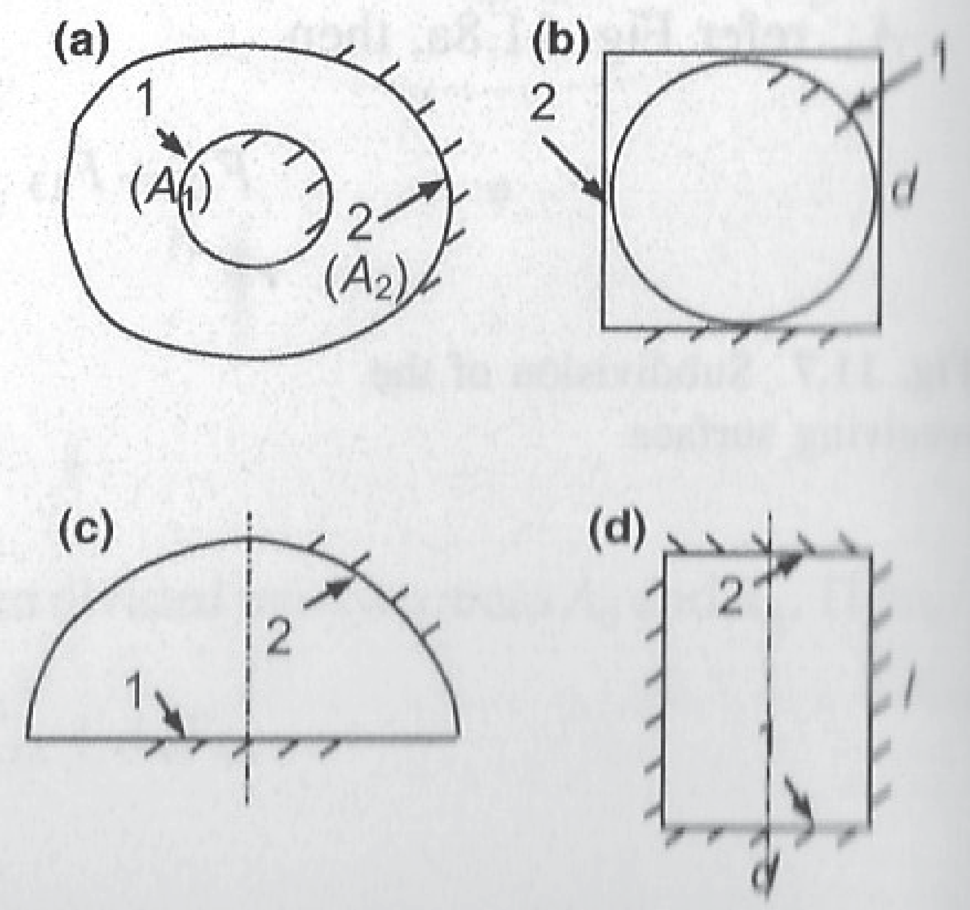


Fig. 11.9 Example 11.1

Example 11.1 Determine the shape factors for

(a) A blackbody inside a black enclosure, Fig. 11.9a.
(b) A black sphere in a cubical box, Fig. 11.9b.
(c) A black hemisphere surface closed by a plane surface, Fig. 11.9c.
(d) A cylindrical cavity closed by a plane surface, Fig. 11.9d.

Solution

The surface 1 in all cases given here is either convex or flat, hence

$$F_{11} = 0.$$

The surface 2 intercepts whole of the radiation emitted by the surface 1, hence

$$F_{12} = 1.$$

From the reciprocity relation,

$$A_1F_{12} = A_2F_{21}$$

or

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{A_1}{A_2}$$

From the summation rule,

$$F_{21} + F_{22} = 1$$

or

$$F_{22} = 1 - F_{21}$$

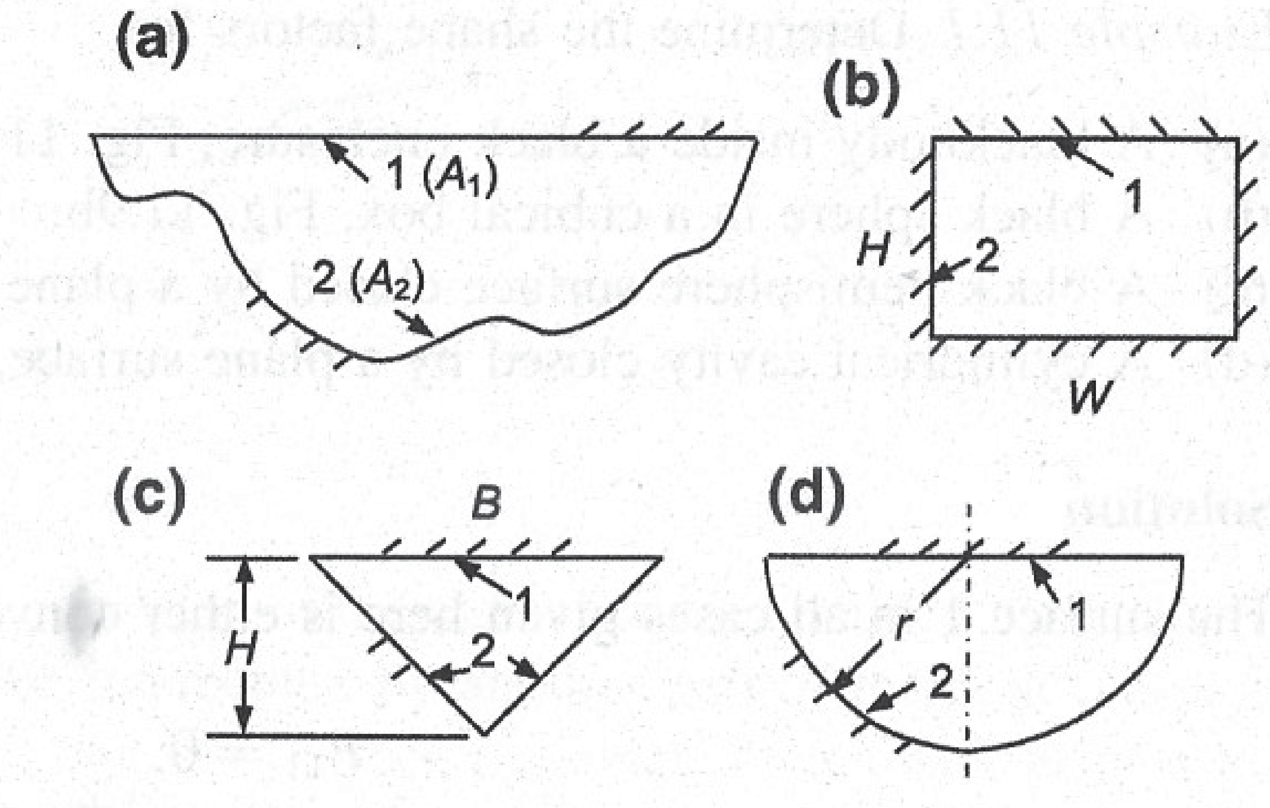
The results for different cases are given in Table 11.2.

Example 11.2 Calculate the shape factor for the ducts, whose cross sections are shown in Fig. 11.10. Length of the duct in all cases is very large and hence the

Table 11.2 Example 11.1

Case	F_{11}	F_{12}	$F_{21} = A_1/A_2$	$F_{22} = 1 - F_{21}$
a	0	1	A_1/A_2	$1 - A_1/A_2$
b	0	1	$\frac{\pi d^2}{6d^2} = \frac{\pi}{6} = 0.5236$	$1 - 0.5236 = 0.4764$
c	0	1	$\frac{(\pi/4)d^2}{(\pi/2)d^2} = 0.5$	$1 - 0.5 = 0.5$
d	0	1	$\frac{(\pi/4)d^2}{\pi dl + (\pi/4)d^2} = \frac{d}{d+4l}$	$1 - \frac{d}{d+4l} = \frac{4l}{d+4l}$

Fig. 11.10 Example 11.2



radiation loss from the ends of the ducts may be neglected. The whole surface enclosing the surface 1 is to be considered surface 2.

Solution

For all the four cases shown in the figure, no part of the radiation leaving the surface 1 falls on the surface itself. Hence,

$$F_{11} = 0.$$

Surface 2 intercepts whole of the radiation emitted by the surface 1, hence

$$F_{12} = 1.$$

From the reciprocity relation,

$$A_1F_{12} = A_2F_{21}$$

or

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{A_1}{A_2}$$

From the summation rule,

$$F_{21} + F_{22} = 1$$

or

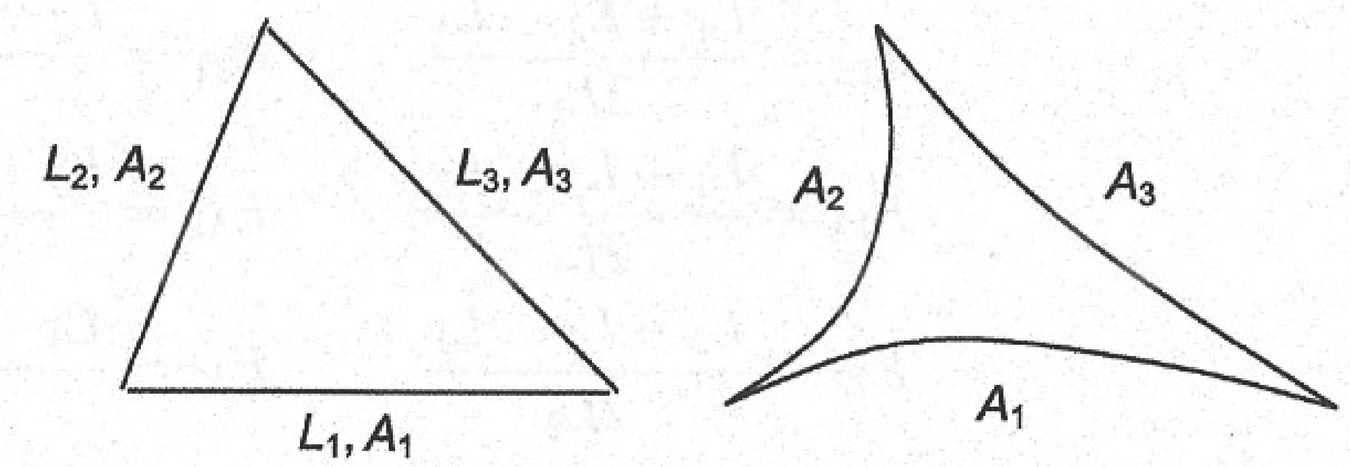
$$F_{22} = 1 - F_{21}$$

The results for different cases are given in Table 11.3.

Table 11.3 Example 11.2

Case	F_{11}	F_{12}	$F_{21} = A_1/A_2$	$F_{22} = 1 - F_{21}$
a	0	1	A_1/A_2	$1 - A_1/A_2$
b	0	1	$\frac{W}{W+2H}$	$1 - \frac{W}{W+2H} = \frac{2H}{W+2H}$
c	0	1	$\frac{B}{2\sqrt{H^2+(B/2)^2}}$	$1 - \frac{B}{2\sqrt{H^2+(B/2)^2}}$
d	0	1	$\frac{2r}{\pi r} = \frac{2}{\pi}$	$1 - \frac{2}{\pi}$

Fig. 11.11 Example 11.3



Example 11.3 Determine shape factors F_{12}, F_{23}, F_{31}, etc., for the triangular cross section enclosure formed by infinitely long plates of different widths as shown in Fig. 11.11.

Solution

Representing the sides of the enclosure by the respective areas A_1, A_2, and A_3, we can write the following:

$$A_1F_{12} + A_1F_{13} = A_1 \qquad \text{(i)}$$

Similarly, we can write

$$A_2F_{21} + A_2F_{23} = A_2$$
$$A_3F_{31} + A_3F_{32} = A_3$$

Using the reciprocity relations, we can write these equations as

$$A_2F_{23} + A_1F_{12} = A_2 \qquad \text{(ii)}$$

$$A_3F_{13} + A_2F_{23} = A_3 \qquad \text{(iii)}$$

Summation of Eqs. (i)–(iii) gives

$$A_1F_{12} + A_1F_{13} + A_2F_{23} = 1/2(A_1 + A_2 + A_3) \qquad \text{(iv)}$$

Subtraction of Eq. (i) from Eq. (iv) gives

$$F_{23} = \frac{A_2 + A_3 - A_1}{2A_2}$$

In terms of the widths of the plates of the enclosure, we can rewrite the above equation as

$$F_{23} = \frac{L_2 + L_3 - L_1}{2L_2}$$

Proceeding in the same manner, we can deduce the following:

$$F_{12} = \frac{L_1 + L_2 - L_3}{2L_1}, \quad F_{21} = \frac{L_2 + L_1 - L_3}{2L_2}$$
$$F_{13} = \frac{L_1 + L_3 - L_2}{2L_1}, \quad F_{31} = \frac{L_3 + L_1 - L_2}{2L_3}$$
$$F_{23} = \frac{L_2 + L_3 - L_1}{2L_2}, \quad F_{32} = \frac{L_3 + L_2 - L_1}{2L_3}$$

The result is listed in Table 11.1. The results can be used for the approximate estimate of triangular enclosure with flat or convex surfaces of area A_1, A_2, and A_3, where length of the duct is very large compared to the widths of the sides.

Example 11.4 Calculate the shape factor for the following:

(a) a very long duct of cross section as shown in Fig. 11.12a
(b) a very long duct with cross section of equilateral triangle, Fig. 11.12b
(c) A conical cavity closed by a plane surface, Fig. 11.12c.

Solution

(a) For surface 1, $F_{11} = 0$. Hence, from the summation rule,

$$F_{12} + F_{13} = 1.$$

Since $A_2 = A_3$, we have

$$F_{12} = F_{13} = 0.5.$$

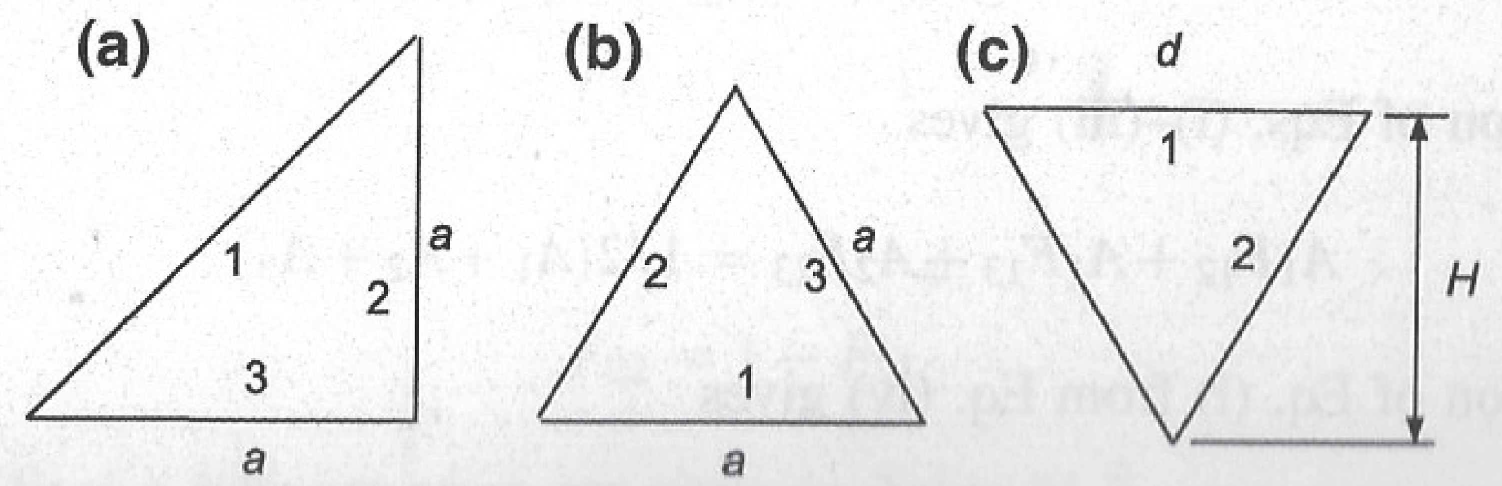


Fig. 11.12 Example 11.4

Similarly, $F_{22} = 0$ and, from the summation rule,

$$F_{23} + F_{21} = 1. \quad \text{(a)}$$

From the reciprocity relation,

$$A_2 F_{21} = A_1 F_{12}.$$

or

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\sqrt{2}a}{a} \times 0.5 = 0.707.$$

From Eq. (a),

$$F_{23} = 1 - F_{21} = 0.293.$$

By symmetry,

$$F_{33} = 0, F_{31} = 0707, \text{ and } F_{32} = 0.293.$$

(b) $F_{11} = 0$ hence, from the summation rule,

$$F_{12} + F_{13} = 1.$$

As $A_2 = A_3$, F_{12} and F_{13} will be equal. This gives

$$F_{12} = F_{13} = 0.5.$$

By symmetry,

$$F_{22} = 0, F_{23} = F_{21} = 0.5$$

and

$$F_{33} = 0, F_{31} = F_{32} = 0.5.$$

(c) $F_{11} = 0$.
Area of the cone surface,

$$A_2 = \frac{\pi d}{2}\sqrt{H^2 + \frac{d^2}{4}}.$$

Area of the plane surface,

$$A_1 = \frac{\pi d^2}{4}.$$

From the reciprocity relation,

$$A_1 F_{12} = A_2 F_{21}.$$

Here $F_{12} = 1$. Hence,

$$F_{21} = \frac{A_1}{A_2} = \frac{d}{\sqrt{4H^2 + d^2}}$$

and

$$F_{22} = 1 - F_{21} = 1 - \frac{d}{\sqrt{4H^2 + d^2}}.$$

The result of part (a) and (b) can also be obtained using the equation presented in Table 11.1 for Case 3.

Example 11.5 Determine the shape factor F_{12} for the configuration shown in Fig. 11.13.

Solution

We have,

$$A_{13} F_{(1,3)-(2,4)} = A_1 F_{1-(2,4)} + A_3 F_{3-(2,4)}$$

where A_{13} = 16 m², A_1 = 12 m², A_3 = 4 m².
This gives

$$4F_{(1,3)-(2,4)} = 3F_{1-(2,4)} + F_{3-(2,4)}.$$

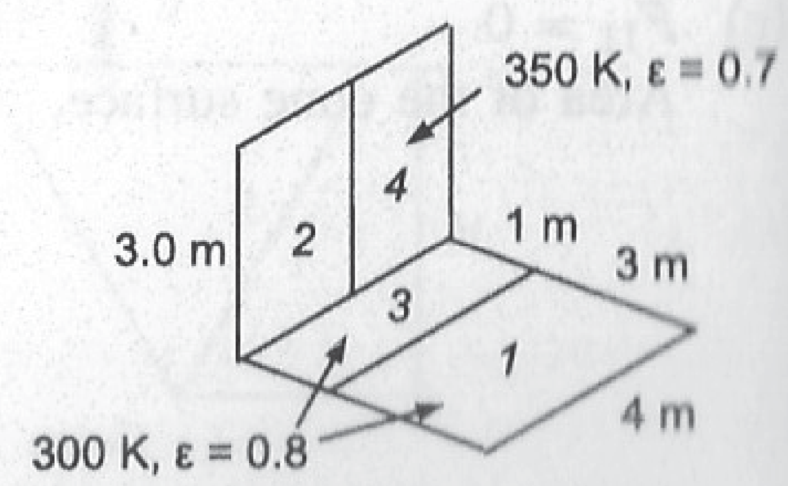


Fig. 11.13 Example 11.5

From symmetry,

$$F_{12} = F_{14}.$$

Hence,

$$F_{1-(2,4)} = 2F_{12}$$

$$4F_{(1,3)-(2,4)} = 6F_{12} + F_{3-(2,4)}. \quad \text{(i)}$$

From Fig. 11.4c for Z/X = 0.75 and Y/X = 1,

$$F_{(1,3)-(2,4)} = 0.18$$

and for Z/X = 0.75 and Y/X = 0.25,

$$F_{3-(2,4)} = 0.35$$

Substitution in Eq. (i) gives

$$F_{12} = (1/6)(4 \times 0.18 - 0.35) = 0.0616.$$

Example 11.6 The ends of concentric cylinders of finite length shown in Fig. 11.14 are covered with flat annular surfaces designated as 3 and 3′. Derive the expression for F_{31}, F_{32}, $F_{33'}$, and F_{13} in terms of F_{11}, F_{12} and areas A_1, A_2, and A_3.

Solution

Surface 1

From the summation rule,

$$F_{11} + F_{12} + F_{13} + F_{13'} = 1.$$

By symmetry,

$$F_{13} = F_{13'}$$

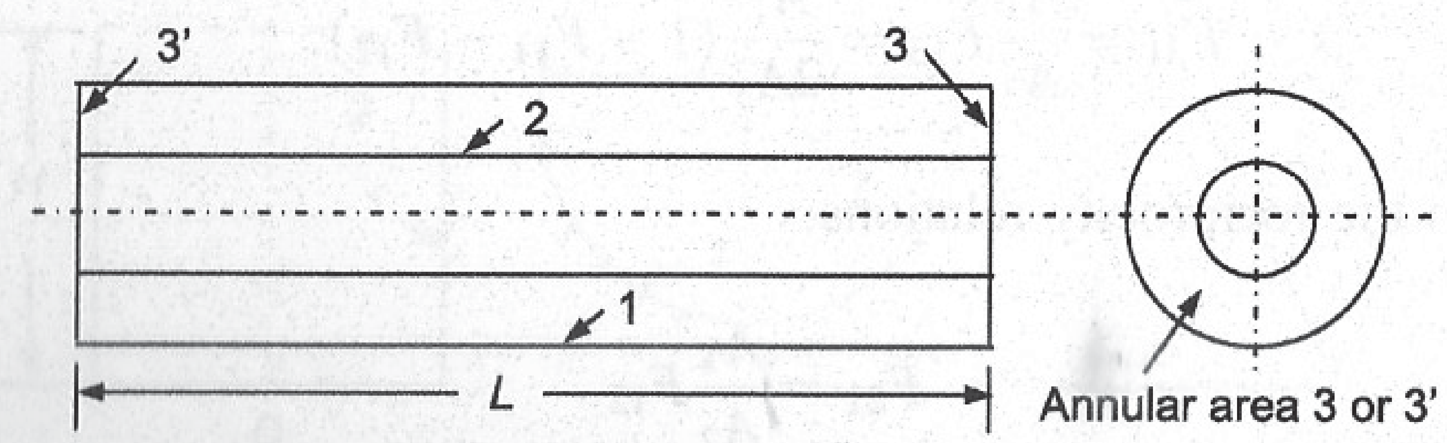


Fig. 11.14 Concentric cylinders of finite length with end covers

Hence,

$$F_{13} = 1/2(1 - F_{11} - F_{12}). \quad \text{(i)}$$

Surface 2

$$F_{22} = 0.$$

Hence, from the summation rule,

$$F_{21} + F_{23} + F_{23'} = 1.$$

By symmetry,

$$F_{23} = F_{23'}.$$

Hence,

$$F_{23} = 1/2(1 - F_{21}). \quad \text{(ii)}$$

Surface 3

$$F_{33} = 0.$$

Hence, from the summation rule,

$$F_{31} + F_{32} + F_{33'} = 1.$$

or

$$F_{33'} = 1 - F_{31} - F_{32}. \quad \text{(iii)}$$

Using reciprocity relation and Eq. (i), we get

$$A_3F_{31} = A_1F_{13}$$

or

$$F_{31} = \frac{A_1}{A_3}F_{13} = \frac{A_1}{2A_3}(1 - F_{11} - F_{12}). \quad \text{(iv)}$$

Again from the reciprocity relations,

$$F_{21} = \frac{A_1}{A_2}F_{12}$$

$$F_{32} = \frac{A_2}{A_3}F_{23}$$

Using these relations and Eq. (ii), we get

$$F_{32} = \frac{A_2}{2A_3}(1 - F_{21}) = \frac{A_2}{2A_3}(1 - \frac{A_1}{A_2}F_{12}) \quad \text{(v)}$$

Substituting the values of F_{31} and F_{32} from Eqs. (iv) and (v), respectively, in Eq. (iii) and rearranging, we obtain

$$F_{33'} = 1 - \frac{A_1 + A_2}{2A_3} + \cdot\frac{A_1}{2A_3}(2F_{12} + F_{11}).$$

Example 11.7 Derive the equation of radiant energy exchange through the openings of the cavities (with black surface) shown in Fig. 11.15. Comment on the result.

Solution

Let T_1 be the temperature of the surface of the cavities and T_2 that of the space above the opening of the cavities. The space acts as a blackbody, hence a black plane surface A_2 can replace the cavity opening.

For the surface 1,

$$F_{11} + F_{12} = 1$$

or

$$F_{11} = 1 - F_{12}$$

Using the reciprocity relation, $A_1F_{12} = A_2F_{21}$, we get

$$F_{11} = 1 - F_{12} = 1 - \frac{A_2}{A_1}F_{21}. \quad \text{(i)}$$

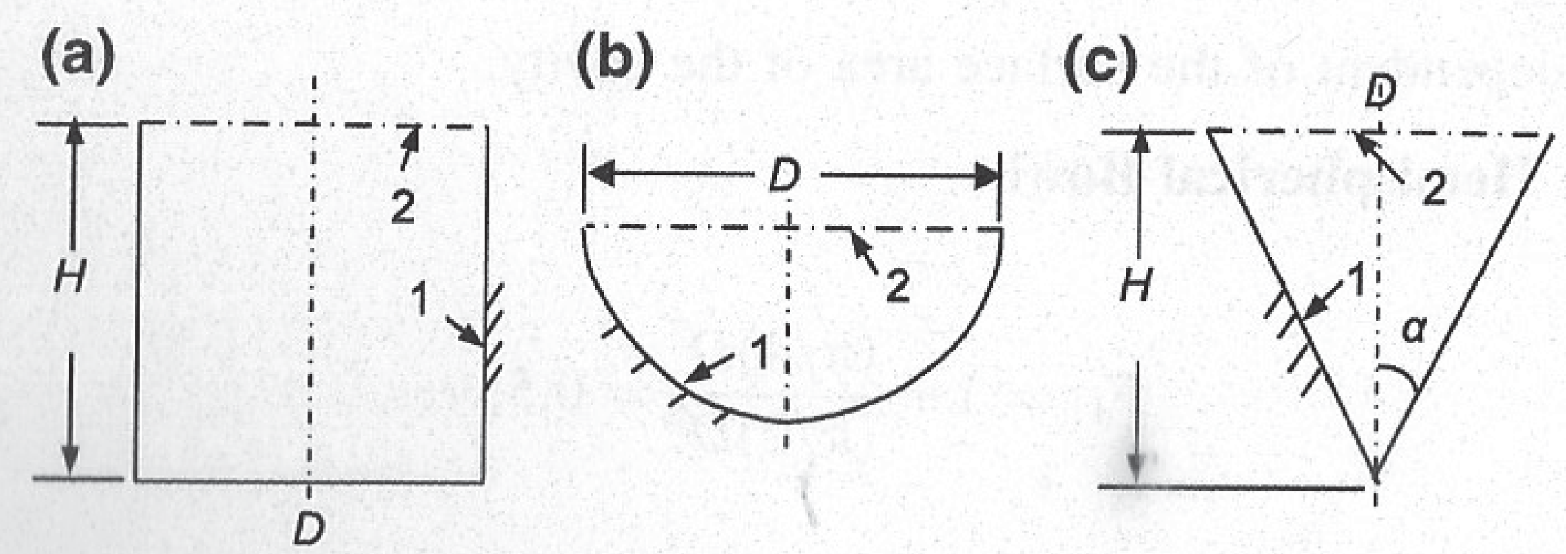


Fig. 11.15 Example 11.7

For the surface 2,

$$F_{21} + F_{22} = 1$$

or

$$F_{21} = 1 - F_{22}$$

Surface 2 is a flat surface, hence $F_{22} = 0$. Hence,

$$F_{21} = 1.$$

Substitution in Eq. (i) gives

$$F_{11} = 1 - \frac{A_2}{A_1}.$$

This is valid for all the cavities shown in Fig. 11.15.

Case (a) A Cylindrical Cavity

$$F_{11} = 1 - \frac{(\pi/4)D^2}{\pi DH + (\pi/4)D^2} = 1 - \frac{D}{D+4H},$$

and

$$F_{12} = 1 - F_{11} = \frac{D}{D+4H}.$$

The net heat exchange (from the cavity to the space) is

$$\begin{aligned} q_{12} &= A_1 F_{12} \sigma (T_1^4 - T_2^4) \\ &= \left[\pi DH + (\pi/4)D^2\right] \times \left(\frac{D}{D+4H}\right) \times \sigma(T_1^4 - T_2^4) \\ &= (\pi/4)D^2 \sigma(T_1^4 - T_2^4) = A_2 \sigma (T_1^4 - T_2^4), \end{aligned}$$

which is independent of the surface area of the cavity.

Case (b) A Hemispherical Bowl

$$F_{11} = 1 - \frac{(\pi/4)D^2}{(\pi/2)D^2} = 0.5,$$

and

$$F_{12} = 1 - F_{11} = 0.5.$$

The net heat exchange (from the cavity to the space) is

$$\begin{aligned} q_{12} &= A_1 F_{12} \sigma (T_1^4 - T_2^4) \\ &= (\pi/2)D^2 \times 0.5 \times \sigma(T_1^4 - T_2^4) \\ &= (\pi/4)D^2 \sigma(T_1^4 - T_2^4) = A_2 \sigma (T_1^4 - T_2^4). \end{aligned}$$

The result is the same as for the case (a).

Case (c) A Conical Cavity

$$F_{11} = 1 - \frac{(\pi/4)D^2}{\frac{\pi D}{2}(H^2 + D^2/4)^{1/2}} = 1 - \frac{D}{2(H^2 + D^2/4)^{1/2}},$$

and

$$F_{12} = 1 - F_{11} = \frac{D}{2(H^2 + D^2/4)^{1/2}}.$$

The net heat exchange (from the cavity to the space) is

$$\begin{aligned} q_{12} &= A_1 F_{12} \sigma (T_1^4 - T_2^4) \\ &= \left(\frac{\pi D}{2}\right)(H^2 + D^2/4)^{1/2} \frac{D}{2(H^2 + D^2/4)^{1/2}} \times \sigma(T_1^4 - T_2^4) \\ &= (\pi/4)D^2 \sigma(T_1^4 - T_2^4) = A_2 \sigma (T_1^4 - T_2^4). \end{aligned}$$

From the analysis of the results of the above cases, we can conclude the following:

(i) In the case of a cavity (i.e., a concave surface), since the parts of the cavity surface can see each other, the radiation escaping the cavity is less than $A_1 \sigma T_1^4$.
(ii) For a cavity with black surface, the escaping radiation is $A_2 \sigma T_1^4$, where A_2 is area of a plane surface (a surface with minimum area) covering the cavity. Thus in such cases, area of the plane surface covering the cavity can be used instead of the surface area of the cavity. This is to be noted that the conclusion is for a cavity with black surface.

Example 11.8 A small sphere of 50 mm diameter is located at the center of a hollow sphere of 200 mm inside diameter. The surface temperatures of the spheres are 600 and 300 K, respectively. Calculate the net exchange of radiation between two spheres. Assume that the surfaces of both spheres behave as blackbody. Also

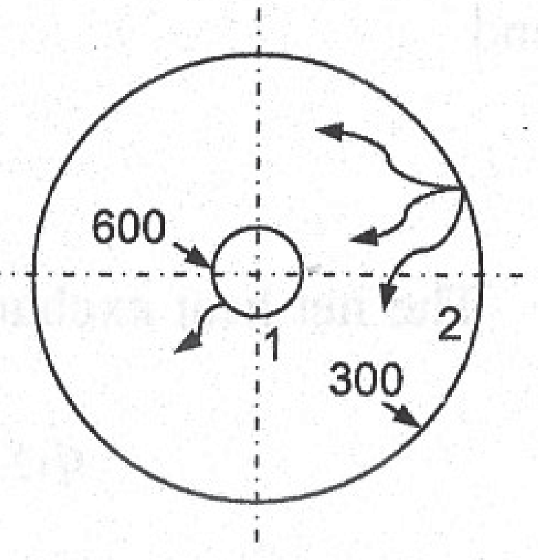


Fig. 11.16 Example 11.8

determine the amount of energy radiated from the surface of the outer sphere incident on the surface of the inner sphere.

Solution

(Refer Fig. 11.16)

The configuration factor F_{12} is unity because whole of the radiation emitted by the inner sphere is falling upon the surface of the outer sphere.

By reciprocity relation

$$A_1F_{12} = A_2F_{21}$$

or

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{A_1}{A_2} = \frac{\pi \times 50^2}{\pi \times 200^2} = 0.0625$$

i.e., only 6.25 % of the radiation emitted by surface of sphere 2 is intercepted by surface of sphere 1. The remaining 93.75 % of the radiation falls upon itself.

The net interchange of the heat between the two spheres is

$$\begin{aligned} q_{13} &= A_1F_{12}\sigma(T_1^4 - T_2^4) \\ &= \pi \times (0.05)^2 \times 1 \times 5.67 \times 10^{-8} \times (600^4 - 300^4) = 54.1\,\text{W}. \end{aligned}$$

Example 11.9 The Sun can be regarded as nearly a spherical radiation source emitting as a blackbody. Its diameter is approximately 1.4×10^9 m and is at a distance of 1.5×10^{11} from the Earth. On a clear day the radiation incident on the Earth's surface was measured to be 1200 W/m^2. If 250 W/m^2 of the solar radiation is estimated to be absorbed by the Earth's atmosphere, estimate the surface temperature of the Sun.

Solution

(Refer Fig. 11.17)

The radiation from the Sun impinging on the Earth is

$$q_{12} = 1200 + 250 = 1450\ \text{W/m}^2.$$

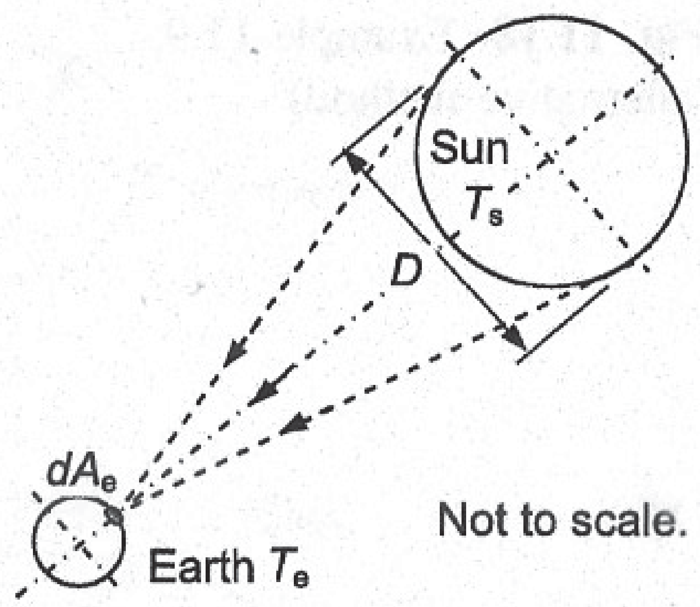


Fig. 11.17 Example 11.9

From Eqs. (11.12) and (11.16), the net radiation heat exchange is given by

$$q_{12} = \sigma(T_s^4 - T_e^4)\frac{\cos\theta_s \cos\theta_e dA_s dA_e}{\pi s^2} \tag{i}$$

where subscript s pertains to the Sun and e to the Earth.

The distance between the Sun and the Earth is very large as compared to the diameter of the Sun, hence

(i) $\cos\theta_s = \cos\theta_e \approx \cos 0° = 1$, refer Fig. 11.17.
(ii) The surface of the Sun emitting radiation can be regarded as a disc of area $dA_s = (\pi/4)\, D_s^2$.
(iii) The radiation measurement refers to the unit area of the Earth's surface, hence $dA_e = 1$ m^2.
(iv) The temperature of the Earth's surface T_e is a very small term compared to T_s and hence can be neglected.

The above conditions transform Eq. (i) to

$$q_{12} = \sigma T_s^4 \frac{(\pi/4)D_s^2}{\pi s^2} = \sigma T_s^4 \frac{D_s^2}{4s^2}$$

or

$$1450 = 5.67 \times 10^{-8} \times T_s^4 \times \frac{(1.4 \times 10^9)^2}{4 \times (1.5 \times 10^{11})^2}$$

This gives

$$T_s = 5851.2\,\text{K}.$$