Heat conduction with generation

2.23. A prototype nuclear reactor has a fuel element design consisting of a thin aluminum tube (cladding) filled with the nuclear material. The resulting nuclear fuel pin is $2r_s$ in diameter where the subscript s denotes the nuclear material outer surface (or the cladding inner surface). The interfacial temperature T_s is measured via thermocouples. Parameters are:

$$q_f''' = 6.5 \times 10^7\ \text{W/m}^3 \qquad T_s = 550\ \text{K}$$
$$k_f = 2.5\ \text{W/m}\cdot\text{K} \qquad r_s = 0.008\ \text{m}$$

where subscript f denotes the nuclear fuel material. Determine the maximum temperature in the nuclear fuel pin. Compare this with the temperature at $r = 0.004$ m.

From the last problem, the temperature in the nuclear material (rod) is given by

$$T = a - br^2 \qquad (1)$$

where:

$$a = T_s + \frac{r_s^2 q'''}{4k} = 550\ \text{K} + \frac{(0.008\ \text{m})^2(6.5 \times 10^7\ \text{W/m}^3)}{4(2.5\ \text{W/m}\cdot\text{K})}$$
$$= 550 + 416 = 966\ \text{K}$$
$$b = \frac{q'''}{4k} = \frac{6.5 \times 10^7\ \text{W/m}^3}{4\ (2.5\ \text{W/m}\cdot\text{K})} = 6.5 \times 10^6\ \text{K/m}^2$$

Thus, since T is maximum for $r = 0$ by inspection of (A),

$$T_{max} = T_{\text{ሪ}} = T(r = 0) = a = 966\ \text{K}$$

At $r = 0.004$ m,

$$T = a - br^2 = 966\ \text{K} - (6.5 \times 10^6\ \text{K/m}^2)(0.004\ \text{m})^2$$
$$= 862\ \text{K}$$

The reader should verify that eq. (1) yields 550 K at $r = 0.008\ \text{m} = r_s$.

Resistance analogy for conduction and radiation

A flat panel used on a spacecraft is fabricated from a single layer of SiC/SiC composite, 0.010 m thick. The spacecraft has inner air temperature $T_i = 298$ K, and the spacecraft is in orbit with the panel exposed only to deep space, where $T_o = 0$ K (zero Kelvin). This material has $k_c = 5.0$ W/m·K and $\epsilon_c \cong 0.8$. The inner surface of the panel is exposed to airflow resulting in $\bar{h}_i = 70$ W/m²·K. Determine the outer surface temperature T_2. Determine q/A.

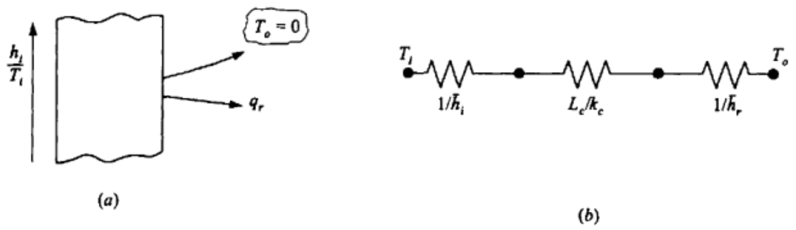


Fig. 2-21

The electrical analog to this is shown in Fig. 2-21. Hence the heat flux is by (2.29) with $\bar{h}_r$ instead of $\bar{h}_o$

$$\frac{q}{A} = \frac{T_i - T_o}{(1/\bar{h}_i) + (L_c/k_c) + (1/\bar{h}_r)}$$

where

$$\bar{h}_r = \epsilon_c \sigma (T_2 + 0)(T_2^2 + 0^2) = \epsilon_c \sigma T_2^3 = (0.8)(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)T_2^3 = 4.536 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\,T_2^3$$

So,

$$\frac{q}{A} = \frac{(298 - 0)\ \text{K}}{[(1/70)(\text{m}^2\cdot\text{K/W}) + (0.01/5)(\text{m}^2\cdot\text{K/W}) + (1/4.536 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\,T_2^3)]} \tag{1}$$

which can also be determined solely by radiation from the outer surface

$$\frac{q}{A} = \epsilon_c \sigma (T_2^4 - T_o^4) = 0.8(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(T_2^4) \tag{2}$$

Combining expressions (1) and (2)

$$\frac{q}{A} = \frac{298\ \text{K}}{[0.014\,28 + 0.002 + (1/4.536 \times 10^{-8}\,T_2^3)]} = 4.536 \times 10^{-8}\,(T_2^4)$$

Solving by trial and error and noting that $T_2 < 298$ K, the results are:

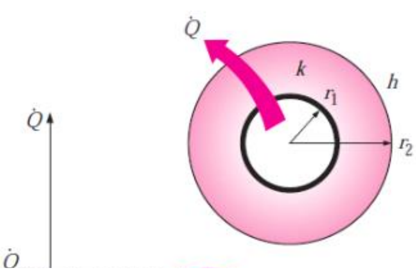
Assumed T_2 (K)	q/A by (1) (W/m²)	q/A by (2) (W/m²)
285	307.6	299.3
287	314.1	307.7
293	333.8	334.3
292	330.5	329.8
292.5	332.1	332.03

A satisfactory solution is $T_2 = 292.5$ K and $q/A = 332$ W/m².

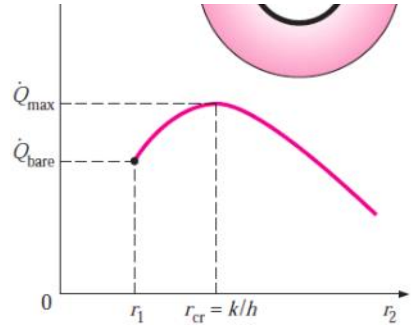
Critical thickness of Insulation

EXAMPLE 17–9 Heat Loss from an Insulated Electric Wire

A 17-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m · °C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30$°C with a heat transfer coefficient of $h = 12$ W/m² · °C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic



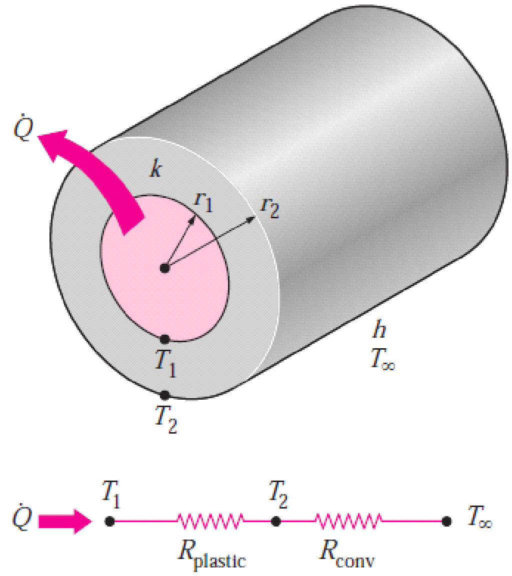
is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30°C$ with a heat transfer coefficient of $h = 12\ W/m^2 \cdot °C$, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.



SOLUTION An electric wire is tightly wrapped with a plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient incorporates the radiation effects, if any.

Properties The thermal conductivity of plastic is given to be $k = 0.15$ W/m · °C.



Analysis Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}_e = VI = (8\ V)(10\ A) = 80\ W$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 17–32. The values of these two resistances are determined to be

$$A_2 = (2\pi r_2)L = 2\pi(0.0035\ m)(5\ m) = 0.110\ m^2$$

$$R_{conv} = \frac{1}{hA_2} = \frac{1}{(12\ W/m^2 \cdot °C)(0.110\ m^2)} = 0.76°C/W$$

$$R_{plastic} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\ W/m \cdot °C)(5\ m)} = 0.18°C/W$$

and therefore

$$R_{total} = R_{plastic} + R_{conv} = 0.76 + 0.18 = 0.94°C/W$$

Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{total} = 30°C + (80\ W)(0.94°C/W) = \mathbf{105°C}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 17–50 to be

$$r_{cr} = \frac{k}{h} = \frac{0.15\ W/m \cdot °C}{12\ W/m^2 \cdot °C} = 0.0125\ m = 12.5\ mm$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will *enhance* heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer $\dot{Q}$ will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.

of the cover reaches 12.5 mm. As a result, the rate of heat transfer Q will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.

Discussion It can be shown by repeating the calculations above for a 4-mm-thick plastic cover that the interface temperature drops to 90.6°C when the thickness of the plastic cover is doubled. It can also be shown in a similar manner that the interface reaches a minimum temperature of 83°C when the outer radius of the plastic cover equals the critical radius.

Composite Walls

EXAMPLE 17–6 **Heat Loss through a Composite Wall**

A 17-m-high and 5-m-wide wall consists of long 16-cm × 22-cm cross section horizontal bricks (k = 0.72 W/m · °C) separated by 17-cm-thick plaster layers (k = 0.22 W/m · °C). There are also 2-cm-thick plaster layers on each side of the brick and a 17-cm-thick rigid foam (k = 0.026 W/m · °C) on the inner side of the wall, as shown in Fig. 17–21. The indoor and the outdoor temperatures are 20°C and −10°C, respectively, and the convection heat transfer coefficients on the inner and the outer sides are h_1 = 10 W/m² · °C and h_2 = 25 W/m² · °C, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

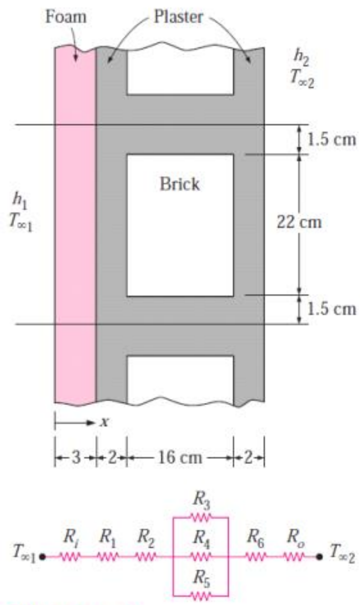


FIGURE 17–21
Schematic for Example 17–6.

SOLUTION The composition of a composite wall is given. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the *x*-direction. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivities are given to be k = 0.72 W/m · °C for bricks, k = 0.22 W/m · °C for plaster layers, and k = 0.026 W/m · °C for the rigid foam.

Analysis There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.

Assuming any cross section of the wall normal to the *x*-direction to be *isothermal*, the thermal resistance network for the representative section of the wall becomes as shown in Fig. 17–21. The individual resistances are evaluated as:

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot °\text{C})(0.25 \times 1 \text{ m}^2)} = 0.4°\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{kA} = \frac{0.03 \text{ m}}{(0.026 \text{ W/m} \cdot °\text{C})(0.25 \times 1 \text{ m}^2)} = 4.6°\text{C/W}$$

$$R_2 = R_6 = R_{\text{plaster, side}} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.22 \text{ W/m} \cdot °\text{C})(0.25 \times 1 \text{ m}^2)}$$
$$= 0.36°\text{C/W}$$

$$R_3 = R_5 = R_{\text{plaster, center}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.22 \text{ W/m} \cdot °\text{C})(0.015 \times 1 \text{ m}^2)}$$
$$= 48.48°\text{C/W}$$

$$R_4 = R_{\text{brick}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.72 \text{ W/m} \cdot °\text{C})(0.22 \times 1 \text{ m}^2)} = 1.01°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(25 \text{ W/m}^2 \cdot °\text{C})(0.25 \times 1 \text{ m}^2)} = 0.16°\text{C/W}$$

The three resistances R_3, R_4, and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03 \text{ W/°C}$$

$$R_4 = R_{\text{brick}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.72 \text{ W/m} \cdot °\text{C})(0.22 \times 1 \text{ m}^2)} = 1.01°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(25 \text{ W/m}^2 \cdot °\text{C})(0.25 \times 1 \text{ m}^2)} = 0.16°\text{C/W}$$

The three resistances R_3, R_4, and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03 \text{ W/°C}$$

which gives

$$R_{\text{mid}} = 0.97°\text{C/W}$$

Now all the resistances are in series, and the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_i + R_1 + R_2 + R_{\text{mid}} + R_6 + R_o \\ &= 0.4 + 4.6 + 0.36 + 0.97 + 0.36 + 0.16 \\ &= 6.85°\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{6.85°\text{C/W}} = 4.38 \text{ W} \qquad (\text{per } 0.25 \text{ m}^2 \text{ surface area})$$

or $4.38/0.25 = 17.5$ W per m^2 area. The total area of the wall is $A = 3 \text{ m} \times 5 \text{ m} = 15 \text{ m}^2$. Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{\text{total}} = (17.5 \text{ W/m}^2)(15 \text{ m}^2) = \mathbf{263 \text{ W}}$$

Of course, this result is approximate, since we assumed the temperature within the wall to vary in one direction only and ignored any temperature change (and thus heat transfer) in the other two directions.

Discussion In the above solution, we assumed the temperature at any cross section of the wall normal to the *x*-direction to be *isothermal*. We could also solve this problem by going to the other extreme and assuming the surfaces parallel to the *x*-direction to be *adiabatic*. The thermal resistance network in this case will be as shown in Fig. 17–22. By following the approach outlined above, the total thermal resistance in this case is determined to be $R_{\text{total}} = 6.97°\text{C/W}$, which is very close to the value 6.85°C/W obtained before. Thus either approach would give roughly the same result in this case. This example demonstrates that either approach can be used in practice to obtain satisfactory results.

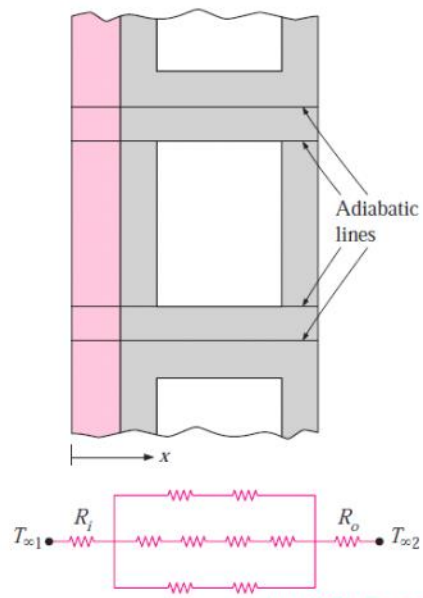


FIGURE 17–22
Alternative thermal resistance network for Example 17–6 for the case of surfaces parallel to the primary direction of heat transfer being adiabatic.

EXAMPLE 17–8 Heat Loss through an Insulated Steam Pipe

Steam at $T_{\infty 1} = 320°\text{C}$ flows in a cast iron pipe ($k = 80 \text{ W/m} \cdot °\text{C}$) whose inner and outer diameters are $D_1 = 5$ cm and $D_2 = 5.5$ cm, respectively. The pipe is covered with 17-cm-thick glass wool insulation with $k = 0.05 \text{ W/m} \cdot °\text{C}$. Heat is lost to the surroundings at $T_{\infty 2} = 5°\text{C}$ by natural convection and radiation, with a combined heat transfer coefficient of $h_2 = 18$ W/m² · °C. Taking the heat transfer coefficient inside the pipe to be $h_1 = 60$ W/m² · °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

SOLUTION A steam pipe covered with glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 80$ W/m · °C for cast iron and $k = 0.05$ W/m · °C for glass wool insulation.

Analysis The thermal resistance network for this problem involves four resistances in series and is given in Fig. 17–29. Taking $L = 1$ m, the areas of the surfaces exposed to convection are determined to be

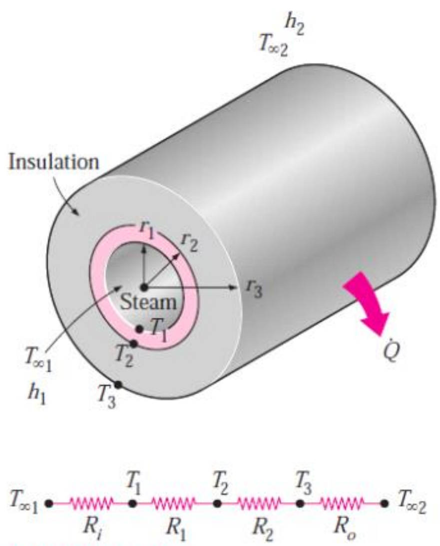


FIGURE 17–29
Schematic for Example 17–8.

$$A_1 = 2\pi r_1 L = 2\pi(0.025\text{ m})(1\text{ m}) = 0.157\text{ m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575\text{ m})(1\text{ m}) = 0.361\text{ m}^2$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(60\text{ W/m}^2\cdot{}^\circ\text{C})(0.157\text{ m}^2)} = 0.106^\circ\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80\text{ W/m}\cdot{}^\circ\text{C})(1\text{ m})} = 0.0002^\circ\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05\text{ W/m}\cdot{}^\circ\text{C})(1\text{ m})} = 2.35^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_3} = \frac{1}{(18\text{ W/m}^2\cdot{}^\circ\text{C})(0.361\text{ m}^2)} = 0.154^\circ\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61^\circ\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)^\circ\text{C}}{2.61^\circ\text{C/W}} = \mathbf{121\text{ W}} \qquad \text{(per m pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length L.

The temperature drops across the pipe and the insulation are determined from Eq. 17–17 to be

$$\Delta T_{\text{pipe}} = \dot{Q}R_{\text{pipe}} = (121\text{ W})(0.0002^\circ\text{C/W}) = \mathbf{0.02^\circ C}$$
$$\Delta T_{\text{insulation}} = \dot{Q}R_{\text{insulation}} = (121\text{ W})(2.35^\circ\text{C/W}) = \mathbf{284^\circ C}$$

That is, the temperatures between the inner and the outer surfaces of the pipe differ by 0.02°C, whereas the temperatures between the inner and the outer surfaces of the insulation differ by 284°C.

Discussion Note that the thermal resistance of the pipe is too small relative to the other resistances and can be neglected without causing any significant error. Also note that the temperature drop across the pipe is practically zero, and thus the pipe can be assumed to be isothermal. The resistance to heat flow in insulated pipes is primarily due to insulation.

Discussion Note that the thermal resistance of the pipe is too small relative to the other resistances and can be neglected without causing any significant error. Also note that the temperature drop across the pipe is practically zero, and thus the pipe can be assumed to be isothermal. The resistance to heat flow in insulated pipes is primarily due to insulation.