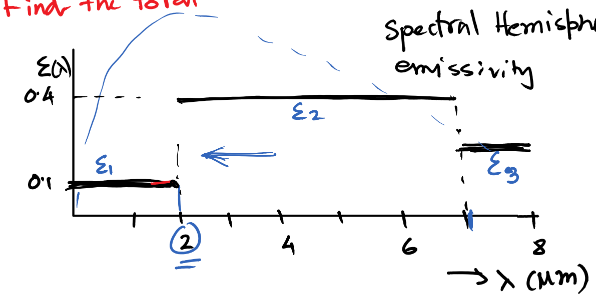
Assignments on Radiation [Spectral & Total Emissivity, Absorptivity & transmissivity]

Tuesday, April 27, 2021 11:43 AM

Find the total



$T = 1000\,K$

$$\varepsilon_\lambda = 0.1 \text{ for } 0<\lambda\leq 2\mu m$$
$$= 0.4 \;\;"\;\; 2<\lambda\leq 7\mu m$$
$$= 0.2 \;\;"\;\; 7<\lambda<\infty$$

1) Find total hemisperical emissivity
2) Find the total emissive power

Use the Blackbody Radiation function table

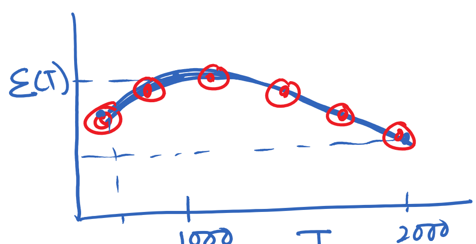
$$\underline{\varepsilon(T)} = \frac{\varepsilon_1 \int_0^{\lambda_1 \to 2\mu} E_{b\lambda}\, d\lambda}{E_b} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2 \to 7\mu} E_{b\lambda}\, d\lambda}{E_b} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda}\, d\lambda}{E_b}$$
$$= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T)$$

$\rightarrow f_{\lambda_2} - f_{\lambda_1} =$

$$\underset{T=1000}{\varepsilon(T)} = \int_0^\infty E_{b\lambda}\cdot\varepsilon_\lambda\, d\lambda = 0.1\times 0.066728 + 0.4\,(0.808109 - .066728) + 0.2\,(1-0.808109)$$
$$= 0.3416 \checkmark$$

$$\underset{T=2000K}{\varepsilon(T)} = 0.1\times 0.480877 + 0.4\,(0.962898 - 0.480877) + 0.2\,(1-0.962898)$$
$$= 0.2483 \checkmark$$

$$\underset{T=500K}{\varepsilon(T)} =$$



Home Work

Find $\varepsilon(T)$ at 500, 1000, 1500, 2000 & 2500 K and explain the variation of $\varepsilon(T)$ with T

Emissive power: $E(T) = E_b(T)\times\varepsilon(T) = \sigma T^4 \times \epsilon(T)$ W/m²

at $T = 1000\,K$, $E(T) = 19368\,W/m^2$

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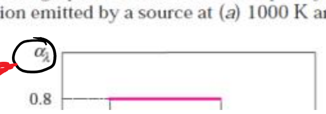
2. 6. The spectral absorptivity of an opaque surface is as shown on the graph. Determine the absorptivity of the surface for radiation emitted by a source at (a) 1000 K and (b) 3000 K.

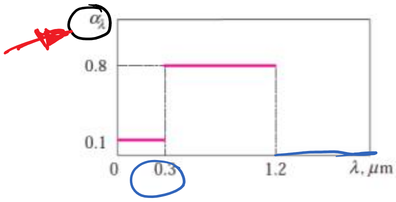


Total Hem. Absorptivity

Blackbody radiation functions f_λ

λT, μm · K	f_λ	λT, μm · K	f_λ
200	0.000000	6200	0.754140
400	0.000000	6400	0.769234
600	0.000000	6600	0.783199
800	0.000016	6800	0.796129
1000	0.000321	7000	0.808109

for radiation emitted by a source at (a) 1000 K and (b) 3000 K.



Total Hem. Absorptivity

How does the ρ_λ vary

400	0.000000	6400	0.769234
600	0.000000	6600	0.783199
800	0.000016	6800	0.796129
1000	0.000321	7000	0.808109
1200	0.002134	7200	0.819217
1400	0.007790	7400	0.829527
1600	0.019718	7600	0.839102
1800	0.039341	7800	0.848005
2000	0.066728	8000	0.856288
2200	0.100888	8500	0.874608
2400	0.140256	9000	0.890029
2600	0.183120	9500	0.903085
2800	0.227897	10,000	0.914199
3000	0.273232	10,500	0.923710
3200	0.318102	11,000	0.931890
3400	0.361735	11,500	0.939959
3600	0.403607	12,000	0.945098
3800	0.443382	13,000	0.955139
4000	0.480877	14,000	0.962898
4200	0.516014	15,000	0.969981
4400	0.548796	16,000	0.973814
4600	0.579280	18,000	0.980860
4800	0.607559	20,000	0.985602
5000	0.633747	25,000	0.992215
5200	0.658970	30,000	0.995340
5400	0.680360	40,000	0.997967
5600	0.701046	50,000	0.998953
5800	0.720158	75,000	0.999713
6000	0.737818	100,000	0.999905

7. The surface in Prob. ~~21-87~~ 6 receives solar radiation at a rate of 820 W/m². Determine the solar absorptivity of the surface and the rate of absorption of solar radiation.

How much is refected?

$$\alpha(T) = \frac{\int_0^\infty E_{b\lambda}(T)\,\alpha_\lambda(T)\,d\lambda}{E_b(T)}$$

Not the radiation from the surface of absorptivity α, but the surface from which the radiation is coming.

$$\alpha(T) = 0.1\,\frac{\int_0^{0.3} E_{b\lambda}(T)\,d\lambda}{E(T)} + \frac{\qquad}{\qquad} + 0$$

$$= 0.1 \times f_\lambda(300) + 0.8\left\{f_\lambda(1200) - f_\lambda(300)\right\}$$

$$= 0 + 0.8\,(0.002134 - 0)$$

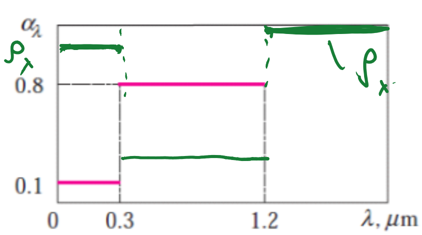
$$= 0.0017$$

Solar Radiation → Blackbody @ ~ 5800 K

Calculate $\alpha(T) = 0.1 \times f_\lambda(1740) + 0.8 \times \left[f_\lambda(6960) - f_\lambda(1740)\right]$

$\dot{Q}_{abs} = G \times \alpha(T)$

Note: How does ρ_λ vary



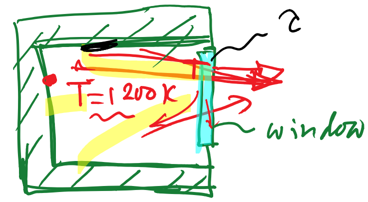
$\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$

ϵ_λ

0 (∵ opaque)

$\rho_\lambda = 1 - \alpha_\lambda$

1. A furnace that has a 25-cm × 25-cm glass window can be considered to be a blackbody at 1200 K. If the transmissivity of the glass is 0.7 for radiation at wavelengths less than 3 μm and zero for radiation at wavelengths greater than 3 μm, determine the fraction and the rate of radiation coming from the furnace and transmitted through the window.



$\tau = 0.7$ for $\lambda \leq 3\,\mu m$

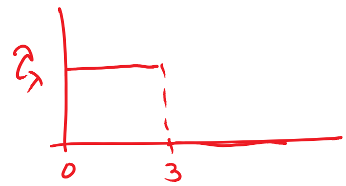
$\quad = 0$ for $\lambda > 3\,\mu m$

$$\tau = \tau_{\lambda_1} f_{\lambda_1} + \tau_{\lambda_2}(1 - f_{\lambda_1})$$

$$= \tau_1 f_\lambda(3600) + 0$$

$$= 0.7 \times 0.403607$$

$$= 0.2825$$



Radiation coming out of the window is

$$\dot{Q}_{out} = G \times \tau \times A = \sigma T^4 \times \tau \times A = 5.67 \times 10^{-8} \times (1200)^4 \times 0.2825 \times (0.25 \times 0.25) \text{ W}$$

T: furnace temp (Not the window temp.)

τ: window

A: window

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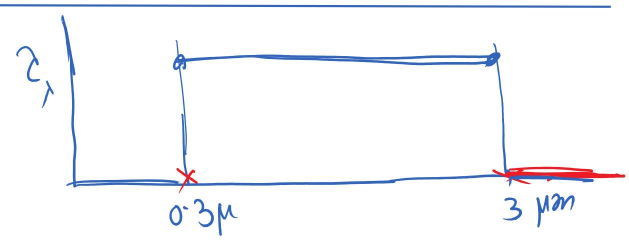
8. The spectral transmissivity of a glass cover used in a solar collector is given as

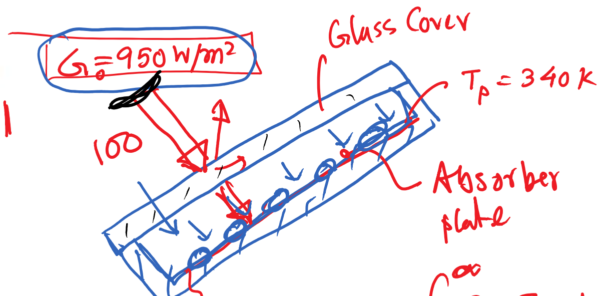
$\tau_1 = 0$ for $\lambda < 0.3\,\mu$m

$\tau_2 = 0.9$ for $0.3 < \lambda < 3\,\mu$m

$\tau_3 = 0$ for $\lambda > 3\,\mu$m

Solar radiation is incident at a rate of 950 W/m^2, and the absorber plate, which can be considered to be black, is maintained at 340 K by the cooling water. Determine (*a*) the solar flux incident on the absorber plate; (*b*) the transmissivity of the glass cover for radiation emitted by the absorber plate; and (*c*) the rate of heat transfer to the cooling water if the glass cover temperature is also 340 K.

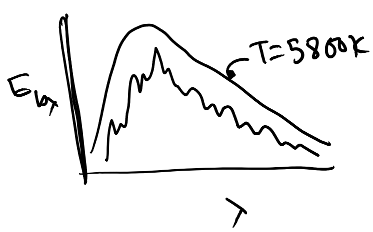




Radiation flux on the absorber plate

$$G_{ab} = \tau_{glass\ cover} \times G_0$$

$$\tau_{glass\ cover} = \frac{\int_0^\infty \tau_\lambda E_{b\lambda}\, d\lambda}{E_\lambda(T)}$$



$$\tau = \tau_\lambda \times [f_\lambda(\lambda T)_2 - f_\lambda(\lambda T)_1]$$

$(\lambda T)_2$: 3μ, T?; $(\lambda T)_1$: 0.3, T?

$$\lambda T_1 = 5800 \times 0.3 = 1740$$

$$\lambda T_2 = 5800 \times 3 = 17400$$

$$\tau \approx 0.9 \times [0.978 - 0.034]$$

$$= 0.8496$$

$= 14400$

$= 0.8496$

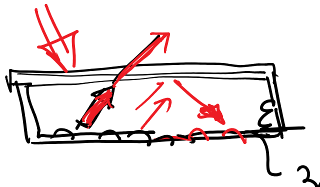
$$\boxed{G_{abs} = 950 \times 0.8496 = 807 \ W/m^2}$$

$$\tau_{T=Abs.} = \tau_\lambda \times \left[f_{\lambda_2}(0.3 \times 340) - f_{\lambda_1}(0.3 \times 340) \right]$$

Tabs

$= 0.9 \times 0.000321$

$\approx \underline{2.8 \times 10^{-4}}$

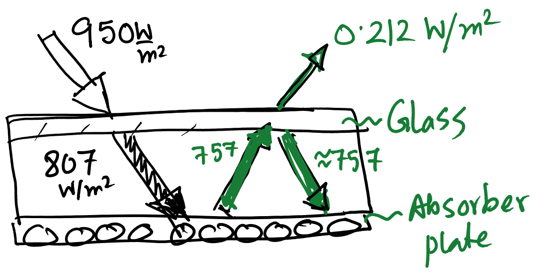


Blackbody Emission Spectra from the absorber plate

Emission from the absorber plate (Blackbody)

$G_p = \sigma T_p^4$

Transmissivity of Glass top to this radiation is 2.8×10^{-4}



$G_p = 5.67 \times 10^{-8} \times 340^4 = 757.7 \ W/m^2$

$\tau G_p \approx 0.212 \ W/m^2$

⇒ Nearly All the radiation emitting from the 340K absorber plate is being reflected back from the glass, since $\tau \to 0$

Hence $\dot{Q}_{absorber} = 807 - \cancel{757} + (\cancel{757} - 0.212)$

$\approx 807 \ W$

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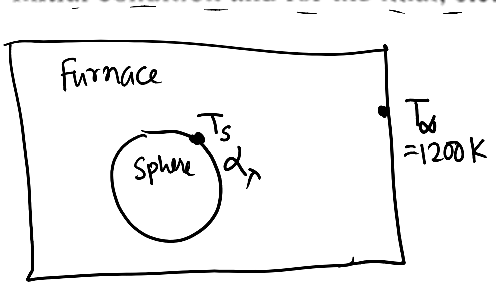
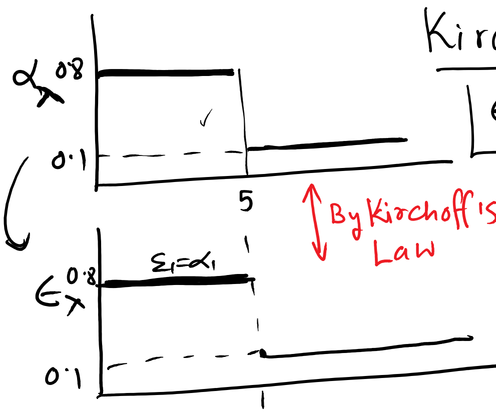
4. A *small*, solid metallic sphere has an opaque, diffuse coating for which $\alpha_\lambda = 0.8$ for $\lambda \le 5 \ \mu m$ and $\alpha_\lambda = 0.1$ for $\lambda > 5 \ \mu m$. The sphere, which is initially at a uniform temperature of 300 K, is inserted into a *large* furnace whose walls are at 1200 K. Determine the total, hemispherical absorptivity and emissivity of the coating for the initial condition and for the final, steady-state condition.





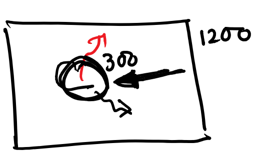
By Kirchoff's Law

Kirchoff's Law

$$\boxed{\epsilon_\lambda = \alpha_\lambda}$$

$\lambda_1 T_\infty = 6000$

Initially



$\alpha_1 f(\lambda T_\infty) + \alpha_2 (1 - f_{\lambda_1}(\lambda_1 T_\infty))$

Finally 

At 300K

$$\underbrace{\alpha(T)}_{T_\infty} = \alpha_1 f_{\lambda_1}(\lambda_1 T_\infty) + \alpha_2\left(1 - f_{\lambda_1}(\lambda_1 T_\infty)\right)$$

$$= 0.8 \times 0.787318 + 0.1\,(1 - 0.787318)$$

$$= 0.6511$$

$$\epsilon(T) = \epsilon_1 \times f_{\lambda_1}(\lambda_1 T_s) + \epsilon_2\left(1 - f_{\lambda_1}(\lambda_1 T_s)\right)$$

T_s

Emission spectrum depends on the sphere surface temp.

$$= 0.8 \times 0.013 + 0.1 \times 0.987$$

$$= 0.1091$$

$$\left.\begin{aligned} \epsilon_1 &= \alpha_1 \\ \epsilon_2 &= \alpha_2 \end{aligned}\right]$$

Note: Always remember the source of the radiation, identify its temperature and then calculate λT based on that Temperature

Since $T_s \neq T_\infty$, $\underbrace{\alpha(T)}_{T_\infty} \neq \underbrace{\epsilon(T)}_{T_s}$

But at 1200 K

$$\alpha(T_\infty) = 0.6511$$

$$\epsilon(T_s) = 0.6511$$

$T_s = T_\infty$

at $T_s = T_\infty$

$$\boxed{\alpha(T) = \epsilon(T)}$$