Worked out problems on conduction

Problem 1: The temperature distribution across a copper plate 0.65 m thick experiencing Ohmic heating is given by $T(x) = 70 + 30x - 20x^2$ where T is in K and x is in meters. Calculate the heat flux at $x = 0$, $x = 0.5$ m, $x = 0.75$ m, and $x = 1$m. Thermal conductivity of the material is 386 W/m K. Find the location of maximum temperature. Also calculate the volumetric heat generation, if any.

Solution :

$T = 70 + 30x - 20x^2$

$$\frac{dT}{dx} = 30 - 40\,x$$

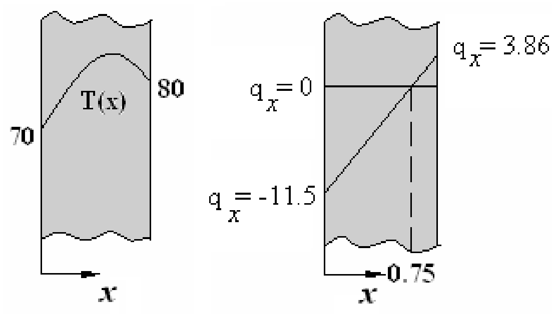
Heat flux $q = -k\frac{dT}{dx} = -386\,(30 - 40x)$ (i)

$q_{x=0} = -386\,(3-0) = -11.580$ kW/m^2

$q_{x=0.5} = -386\,(30-40\,(0.5)) = -3.86$ kW/m^2

$q_{x=0.75} = -386\,(3\,0-40\,(0.75)) = 0$ kW/m^2

$q_{x=1} = -386\,(3\,0-40\,(1)) = 3.86$ kW/m^2



$T=T_{max}$ where $\frac{dT}{dx} = 0$ and $\frac{d^2T}{dx^2}$ is negative. This happens at x=0.75 m

To calculate volumetric heat generation, we use the generalized conduction equation under steady state in 1-D

$$k\frac{d^2T}{dx^2} + \dot{q}_{gen} = 0$$

From Eq. (i), $\frac{d^2T}{dx^2} = -40$, hence, $-386\times40+\dot{q}_{gen} = 0$, $\Rightarrow \dot{q}_{gen} = 14720$ W/m^3

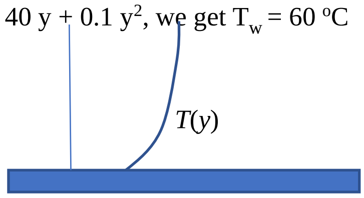
Problem 2: A plate is exposed to an environment containing fluid at 100 ^{o}C. The temperature profile of the fluid is given as T = 60 + 40y + 0.1 y^2. Assume k_{fluid} = 40 W/m K. Determine convective heat transfer coefficient.

Solution :

Given data $T_f = 100$ ^{o}C ; k = 40 W/m K; T = 60 + 40 y + 0.1 y^2

To calculate wall temperature T_w put y = 0 in the expression T = 60 + 40 y + 0.1 y^2, we get $T_w = 60$ ^{o}C

Now convective heat transfer coefficient $h = \dfrac{-k\left.\frac{\partial T}{\partial y}\right|_{wall}}{T_w - T_f}$



Temperature gradient $\left.\frac{\partial T}{\partial y}\right|_{wall}$ = 40 at y = 0

$q'' = h(T_w - T_f)$

$h = \dfrac{-40\times40}{60-100}$ = 40 W/m^2 K

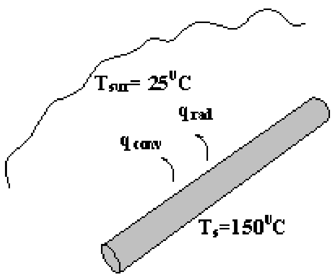
Problem 3: A 25 m long un-insulated steam pipe (10 cm diameter) is routed through a building whose walls and air are at 25^{0}C. Pressurised steam maintains a pipe surface temperature of 150^{0}C, and the coefficient associated with natural convection is h=10 W/m^{2}K. The surface emissivity is 0.8.What is the rate of heat loss from steam line?

Solution :

Given data: L = 25m; D=0.1 m; h=10 W/m^{2}K;

T_w=150 ^{0}C = 423 K;

$T_s = T_f = 25^{0}$C = 298 K; ε =0.8



Total heat loss is due to convection and radiation

$q = q_{conv} + q_{rad}$

$q = h\,(T_w - T_f) + \varepsilon\,\sigma\,(T^4_w - T^4_s)$

Heat transfer area=A=π D L= π x (0.1x 25) = 7.85m^2

Q = q $\times$ A = [10 (423-298)+ 0.8 x 5.67$\times$ 10^{-8}(423^4-298^4)] $\times$ 7.85 = 18405 W=18.4 kW

Problem 4: Find the radiation heat transfer coefficient for a polished aluminium whose temperature exceeds that of surrounding(T_{sur}=25^{0}C) by 10^{0}C. Emissivity value for the surface may be taken as 0.05.

Solution :

Given data : T_s=25+273=298 K ; ε =0.05

$h_r = \sigma\, \varepsilon\, (T_w + T_s)\, (T^2_w + T^2_s)$

T_w=298+10=308 K

h_r =5.67 x 10^{-8} x 0.05(308+ 298) (308^2+ 298^2)

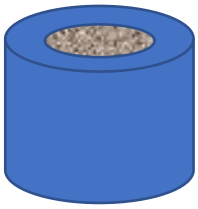
=0.3155 W/m^{2}K

Problem 5: A long cylinder of inner radius 50 mm and outer radius 100 mm has uniform heat generation given by q = 2×10^4 W/m^2. Inside the hollow cylinder ice is kept. What is the rate of melting ice? *k* = 4 W/mK. The outer surface is insulated. $L_{cylinder}$ = 1m. r_i= 50 mm; r_o = 100mm. Latent heat of fusion for ice is 336 kJ/kg. Also find the outer wall temperature.

Solution: Assuming (i) steady-state and (ii) 1-dimensional heat conduction with (iii) constant k,

GDE: $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{q}{k} = 0,$

The general solution is: $T = -\frac{qr^2}{4k} + c_1 \ln r + c_2$ **(1)**



Boundary conditions:

(i) Inner wall: T(r = r_i) = T_i=273 K (since it is in touch with melting ice)

(ii) Outer wall: $\left.\frac{dT}{dr}\right|_{r=r_o} = 0 \Rightarrow \frac{c_1}{r_o} = \frac{qr_o}{2k}$, $c_1 = \frac{qr_o{}^2}{2k} = \frac{2\times10^4\times0.1^2}{2\times4} = 25$

Substituting in Eq. (1), $T(r = r_i) = -\frac{q}{4k}r_i{}^2 + 25\ln r_i + c_2 = 273$

Hence, $c_2 = 273 - 25\,ln(50\times10^{-3}) + \frac{2\times10^4\times(50\times10^{-3})^2}{4\times4} = 351$

Outer wall temperature $T(r = r_o) = -\frac{q}{4k}r_0{}^2 + 25\ln r_o + c_2 = 280$ K

Heat flux at the inner wall $-k\left.\frac{dT}{dr}\right|_{r=r_i} = k\left(\frac{25}{r_i} - \frac{qr_i}{2k}\right) = 4(500\ - 125) =\ 1500\ W/m^2$

Hence the total heat $m_{ice}(L_f)_{ice}$=m_{ice}(336000)

m_{ice} = 1.402×10^{-3} kg/s = 5.04 kg/h

[Alternate method: Since the outer wall is insulated, under steady state, the entire heat generated in the cylinder will be conducted inside to melt the ice.

$\dot{Q} = \pi(r_o^2 - r_i^2)\times L_{cylinder}\ \times q = 471\ W$, Therefore, $m_{ice} = 471/336000\ kg/s = 5.04\ kg/h$

This method will not give you the outer wall temperature and the temperature distribution, but it simply does the steady-state energy balance of the cylinder.]

Problem 6: Two co-axial hollow cylinders are as shown in the diagram below. The temperature on the cylindrical surface at a distance *a* from the axis is T_1 and at a distance *c* from the axis is T_2. Find the temperature at the junction of temperature of the two cylinders. k_1, k_2 are the thermal conductivities of the two cylinder materials.

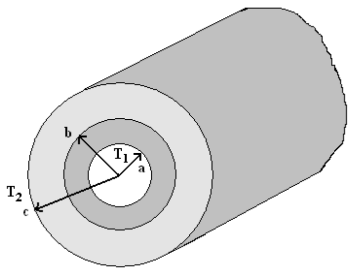
Solution :

The heat flow by conduction in one dimension in a steady state is given by $Q = -kA\frac{dT}{dr}$, which is same through the two insulators.

equating the heat flow through the two cylindrical shells of insulator

$$\frac{T_1 - T}{ln\frac{b}{a}}\ 2\pi k_1 = \frac{T - T_2}{ln\frac{c}{b}}\ 2\pi k_2$$

$$T = \frac{k_1 T_1\, ln\frac{c}{b} + k_2 T_2\, ln\frac{b}{a}}{k_1\, ln\frac{c}{b} + k_2\, ln\frac{b}{a}}$$



Problem 7: A cylinder with heat generation given by q has been exposed to surroundings at 25 ºC. The convective heat transfer coefficient outside is h=15 W/m^{2}K. Outer surface emissivity is 0.8. Find q so that the temperature of outer surface is kept within 200 ºC. Also find temperature distribution inside the cylinder and T (r = 0). Given that k = 25W/mK, r_o = 25 mm

Solution :

From heat equation

GDE: $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{q}{k}=0$ **(1)**

Integrating (1) $r\frac{dT}{dr}=-\frac{q}{2k}r^2=c_1$ **(2)**

And further integrating, $T(r)=-\frac{q}{4k}r^2+c_1\,\boldsymbol{ln}\,r+c_2$ **(3)**

Boundary conditions $\left.\frac{dT}{dr}\right|_{r=0}=0$ **(4)**

And $T(r_o)=T_s$ **(5)**

(From BC 4 and Eq 2) $\Rightarrow c_1=0$

(Hence from BC 5) $\Rightarrow c_2=T_s+\frac{q}{4k}{r_o}^2$

Thus $T(r)=\frac{q{r_0}^2}{4k}\left(1-\frac{r^2}{{r_0}^2}\right)+T_s$ **(3)**

Also, at the outer surface, $-k\left.\frac{dT}{dr}\right|_{r=r_o}=\boldsymbol{h}(T_s-T_\infty)+\boldsymbol{\epsilon}\sigma({T_s}^4-{T_\infty}^4)$

$$\frac{qr_0}{2}=\boldsymbol{h}(T_s-T_\infty)+\in\sigma({T_s}^4-{T_\infty}^4)$$

$\frac{q\times25\times10^{-3}}{2}=2625+1912.76\Rightarrow q=18.15\times10^4W/m^3$

T(r=0) = 201.15$^{\mathbf{o}}$ C.

Note: Here Radiation loss & Convective loss are comparable since T_S is large and h is small.

Heat Transfer Across a slab with variable thermal conductivity

For several materials, it may be generalized that the thermal conductivity varies linearly with temperature. This variation is often approximated as

$$k(T) = k_0(1 + \beta T) \qquad (1)$$

where k_0 and β are constants.

Using equation $\nabla.(k\ \nabla T) = 0$ without heat generation term we arrive at equation for variable thermal conductivity as

$$\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] = 0 \qquad (2)$$

$k(T)$ can be replaced from equation (3.28) to give

$$\frac{d}{dx}\left[k_o(1 + \beta T)\ \frac{dT}{dx}\right] = 0 \qquad (3)$$

Integrating this we get $k_o(1 + \beta T)\ \frac{dT}{dx} = C_1$ (4)

If the prescribed temperature boundary condition is used ($T = T_1$ at $x = 0$ and $T = T_2$ at $x = l$), we can integrate it further to have

$$k_o \int_{T_1}^{T_2}(1 + \beta T)dT = C_1 \int_o^l dx$$

Giving

$$C_1 = \frac{k_0}{l}\left[(T_2 - T_1) + \frac{\beta}{2}\ (T_2^2 - T_1^2)\right]$$

From Fourier's law, the heat transfer from the slab is

$$Q = -kA\ \frac{dT}{dx} = -C_1 A$$

$$Q = \frac{Ak_o}{l}\left[(T_1 - T_2) + \frac{\beta}{2}\ (T_1^2 - T_2^2)\right]$$