Radiation Part 3: Radiation Exchange Between Surfaces

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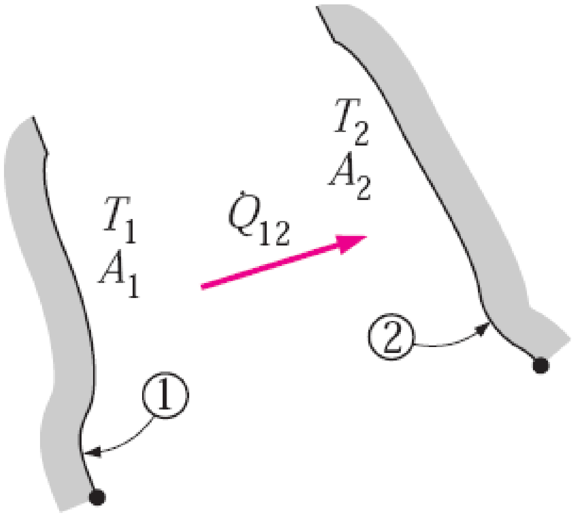
Blaclbody Radiation Exchange

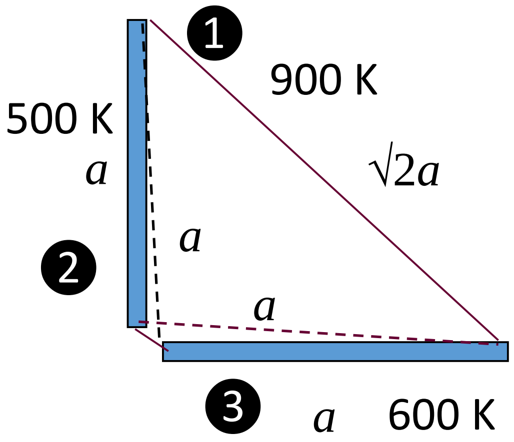
$$\dot{Q}_{1\to 2} = \begin{pmatrix}\text{Radiation leaving}\\ \text{the entire surface 1}\\ \text{that strikes surface 2}\end{pmatrix} - \begin{pmatrix}\text{Radiation leaving}\\ \text{the entire surface 2}\\ \text{that strikes surface 1}\end{pmatrix}$$
$$= A_1 E_{b1} F_{1\to 2} - A_2 E_{b2} F_{2\to 1} \qquad \text{(W)}$$



But by reciprocity: $A_1 F_{1\to 2} = A_2 F_{2\to 1}$

Therefore: $\dot{Q}_{1\to 2} = A_1 F_{1\to 2}\, \sigma (T_1^4 - T_2^4)$

$$\dot{Q}_2 = \dot{Q}_{3\to 2} + \dot{Q}_{1\to 2} = \begin{bmatrix}(A_3 F_{32} E_{b3} - A_2 F_{23} E_{b2})\\ +(A_1 F_{12} E_{b1} - A_2 F_{21} E_{b2})\end{bmatrix}$$

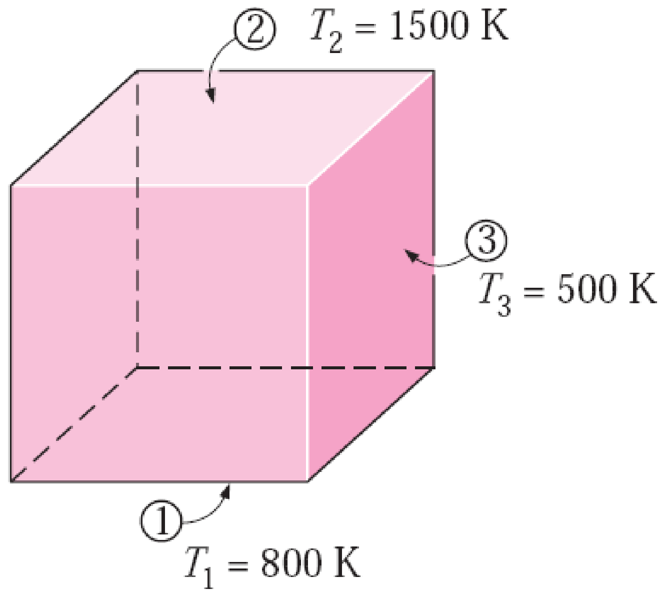


$$F_{12} = F_{13} = \frac{1}{2}$$
$$F_{11} = F_{22} = F_{33} = 0,$$
$$F_{21} = F_{31} = \frac{1}{\sqrt{2}},$$
$$F_{23} = F_{32} = 1 - \frac{1}{\sqrt{2}}$$

Find the net heat received by the vertical plate

Example

Consider the 5-m × 5-m × 5-m cubical furnace shown in Fig. 22–19, whose surfaces closely approximate black surfaces. The base, top, and side surfaces of the furnace are maintained at uniform temperatures of 800 K, 1500 K, and 500 K, respectively. Determine (*a*) the net rate of radiation heat transfer between the base and the side surfaces, (*b*) the net rate of radiation heat transfer between the base and the top surface, and (*c*) the net radiation heat transfer from the base surface.

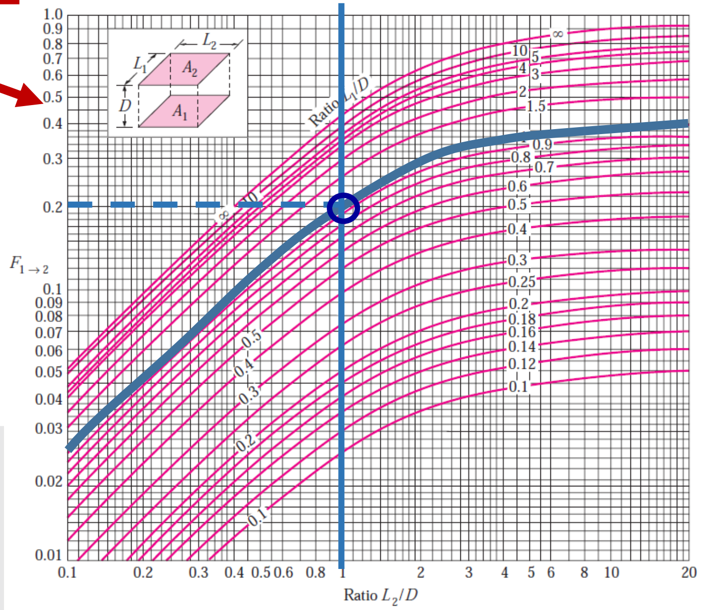


$$\dot{Q}_{1\rightarrow 3} = A_1 F_{1\rightarrow 3}\sigma(T_1^4 - T_3^4)$$

$$F_{1\rightarrow 1} + F_{1\rightarrow 2} + F_{1\rightarrow 3} = 1$$

$$F_{1\rightarrow 3} = 1 - F_{1\rightarrow 1} - F_{1\rightarrow 2} = 1 - 0 - 0.2 = 0.8$$

$F_{1\text{-}2}$ = 0.2 from the graph

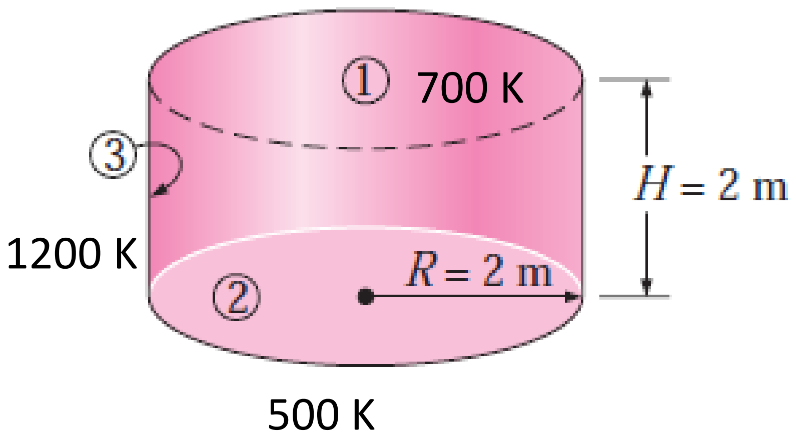


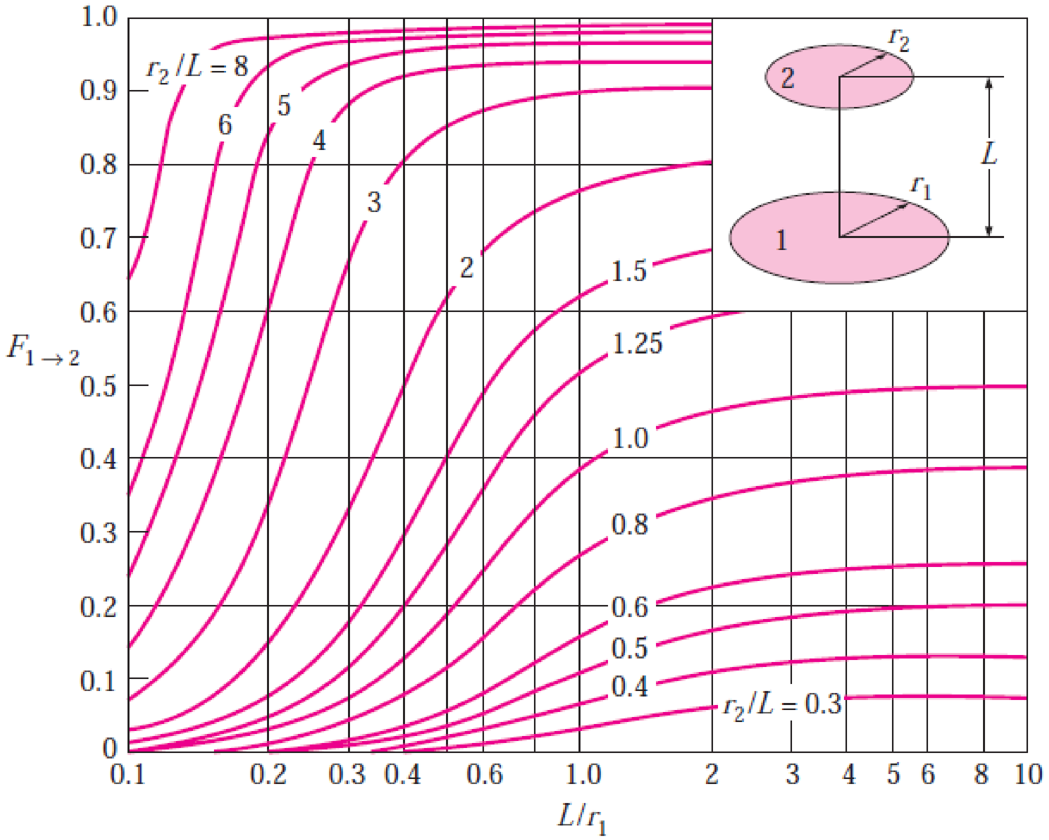
$$\dot{Q}_{1\rightarrow 3} = (25\ \text{m}^2)(0.8)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\ \text{K})^4 - (500\ \text{K})^4]$$
$$= \mathbf{394 \times 10^3\ W = 394\ kW}$$

$$\dot{Q}_{1\rightarrow 2} = A_1 F_{1\rightarrow 2}\sigma(T_1^4 - T_2^4)$$
$$= (25\ \text{m}^2)(0.2)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\ \text{K})^4 - (1500\ \text{K})^4]$$
$$= \mathbf{-1319 \times 10^3\ W = -1319\ kW}$$

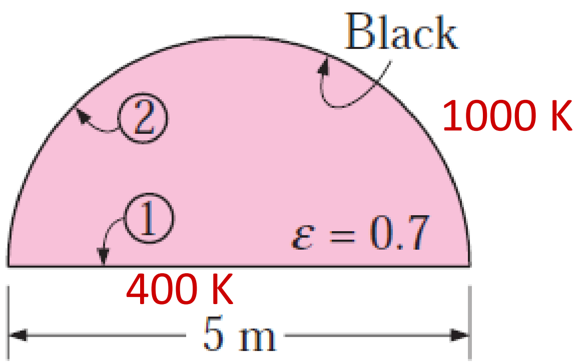
Example

A furnace is of cylindrical shape with R = H = 2 m. The base, top, and side surfaces of the furnace are all black and are maintained at uniform temperatures of 500, 700, and 1200 K, respectively. Determine the net rate of radiation heat transfer to or from the top surface during steady operation.





Example: Consider a hemispherical furnace of diameter $D = 5$ m with a flat base. The dome of the furnace is black, and the base has an emissivity of 0.7. The base and the dome of the furnace are maintained at uniform temperatures of 400 and 1000 K, respectively. Determine the net rate of radiation heat transfer from the dome to the base surface during steady operation.



Net radiation heat received by 1 from 2:

$$\dot{Q}_{net,2\to 1} = \alpha A_2 E_{b2} F_{21} - \varepsilon A_1 E_{b1} F_{12}$$

From Reciprocity rule $A_2 F_{21} = A_1 F_{12}$

$$\dot{Q}_{net,2\to 1} = \alpha A_1 E_{b2} F_{12} - \varepsilon A_1 E_{b1} F_{12}$$

From Kirchoff's Law, $\alpha = \varepsilon$

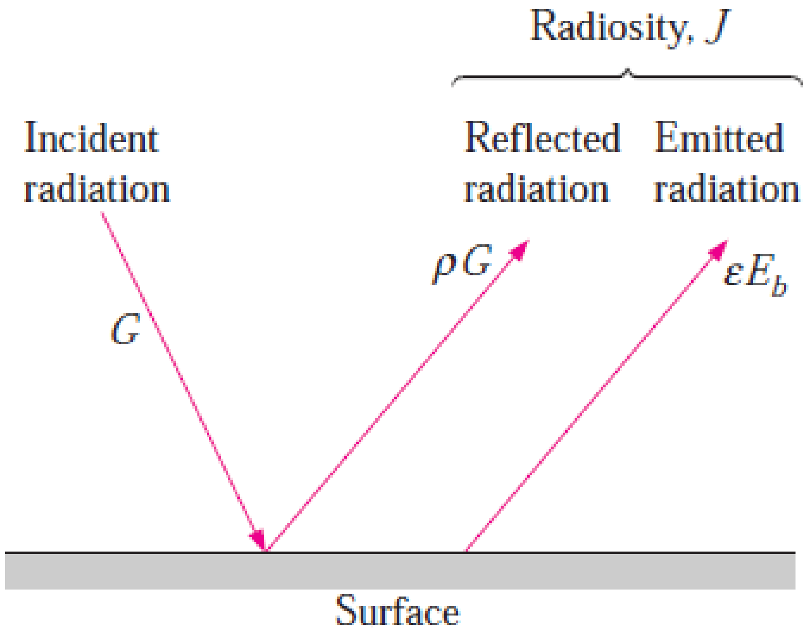
$$\dot{Q}_{net,2\to 1} = \varepsilon A_1 F_{12}\left(E_{b2} - E_{b1}\right) = \varepsilon A_1 \times 1 \times \sigma\left(T_2^{\,4} - T_1^{\,4}\right)$$

Radiation exchange between diffuse grey, opaque surfaces

Grey Surface $\Rightarrow \alpha_i = \varepsilon_i$

Any Surface $\Rightarrow \alpha_i + \rho_i + \tau_i = 1$

Opaque Surface $\Rightarrow \alpha_i + \rho_i = 1$



$$J_i = \begin{pmatrix} \text{Radiation emitted} \\ \text{by surface } i \end{pmatrix} + \begin{pmatrix} \text{Radiation reflected} \\ \text{by surface } i \end{pmatrix}$$

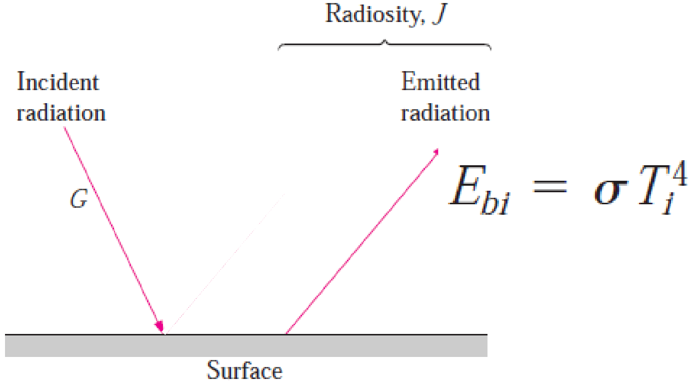
$$= \varepsilon_i E_{bi} + \rho_i G_i$$

$$= \varepsilon_i E_{bi} + (1 - \varepsilon_i)\, G_i \qquad (\text{W/m}^2)$$

$$G_i = \frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}$$

For Blackbody Surfaces, $\rho_i = 0$

$$J_i = E_{bi} = \sigma T_i^4$$



Net radiation transfer from a surface

$$\dot{Q}_i = \begin{pmatrix}\text{Radiation leaving} \\ \text{entire surface } i\end{pmatrix} - \begin{pmatrix}\text{Radiation incident} \\ \text{on entire surface } i\end{pmatrix}$$

$$= A_i(J_i - G_i) \qquad \text{(W)}$$

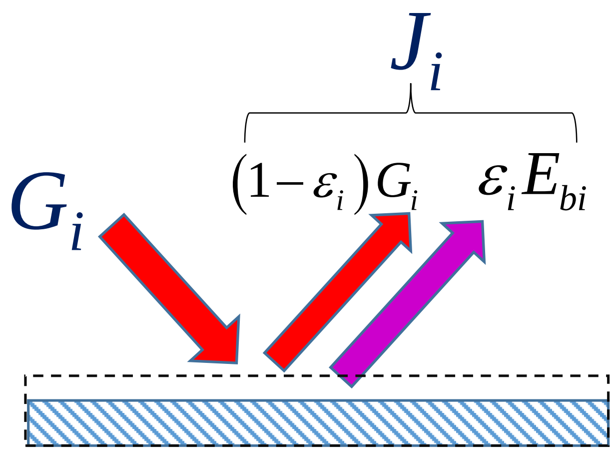
But we know, $G_i = \dfrac{J_i - \varepsilon_i E_{bi}}{1-\varepsilon_i}$

Hence, substituting G_i from Eq. A,

$$\dot{Q}_i = A_i\left(J_i - \frac{J_i - \varepsilon_i E_{bi}}{1-\varepsilon_i}\right) = \frac{A_i\varepsilon_i}{1-\varepsilon_i}(E_{bi} - J_i)$$

$\dot{Q}_i = \dfrac{E_{bi} - J_i}{R_i}$, where $R_i = \dfrac{1-\varepsilon_i}{A_i\varepsilon_i}$ **Surface Resistance to radiation**

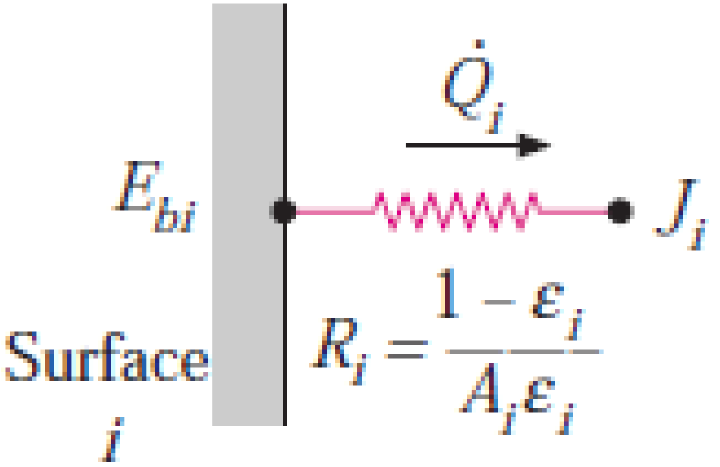
$$E_{bi} = \sigma T_i^4$$



$$J_i = \varepsilon_i E_{bi} + (1-\varepsilon_i)G_i$$

Note: Unit of J and G is W/m²
Unit of $\dot{Q}_i$ is W

Electric Circuit Analogy



$\dot{Q}_i = 0 \Rightarrow$ Re-radiating surface ($E_{bi} = J_i$)

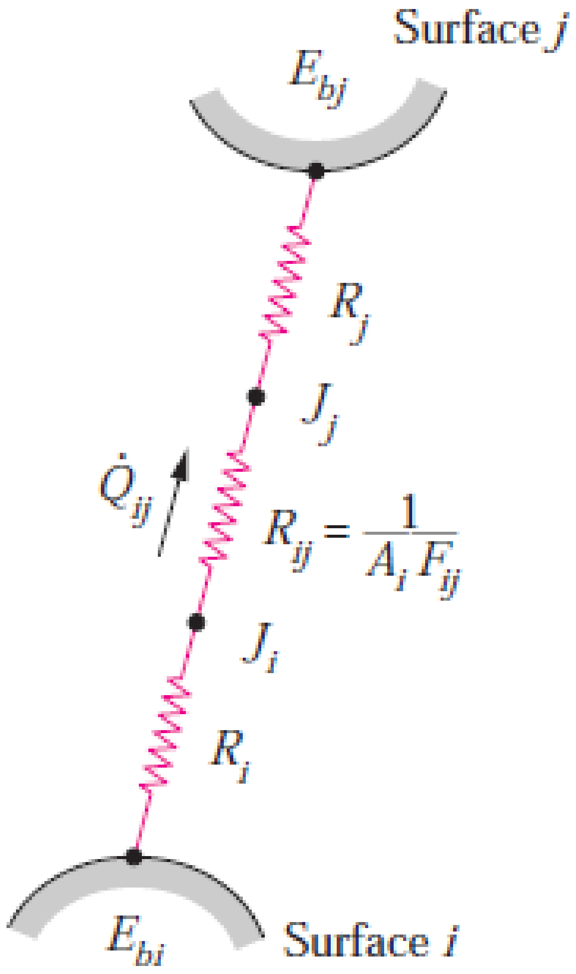
Net radiation exchange between two surfaces

$$\dot{Q}_{i \to j} = \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface } i \\ \text{that strikes surface } j \end{pmatrix} - \begin{pmatrix} \text{Radiation leaving} \\ \text{the entire surface } j \\ \text{that strikes surface } i \end{pmatrix}$$

$$= A_i J_i F_{i \to j} - A_j J_j F_{j \to i} \qquad \text{(W)}$$

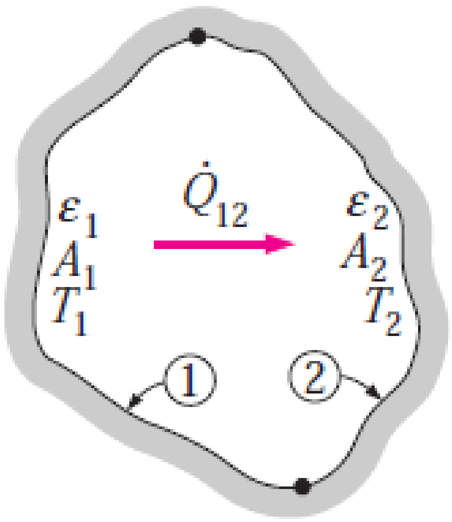
$$A_i F_{i \to j} = A_j F_{j \to i} \Rightarrow \dot{Q}_{i \to j} = A_i F_{i \to j} (J_i - J_j)$$

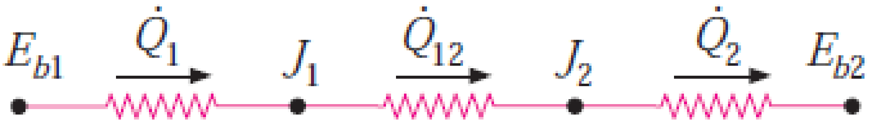
$\dot{Q}_{i \to j} = \dfrac{J_i - J_j}{R_{i \to j}}$, where $R_{i \to j} = \dfrac{1}{A_i F_{i \to j}}$ [Space Resistance]



Net radiation exchange in a 2-surface enclosure

$$\dot{Q}_{i\rightarrow j} = \frac{J_i - J_j}{R_{i\rightarrow j}}$$, where $$R_{i\rightarrow j} = \frac{1}{A_i\, F_{i\rightarrow j}}$$





$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} \qquad R_{12} = \frac{1}{A_1F_{12}} \qquad R_2 = \frac{1-\varepsilon_2}{A_2\varepsilon_2}$$

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

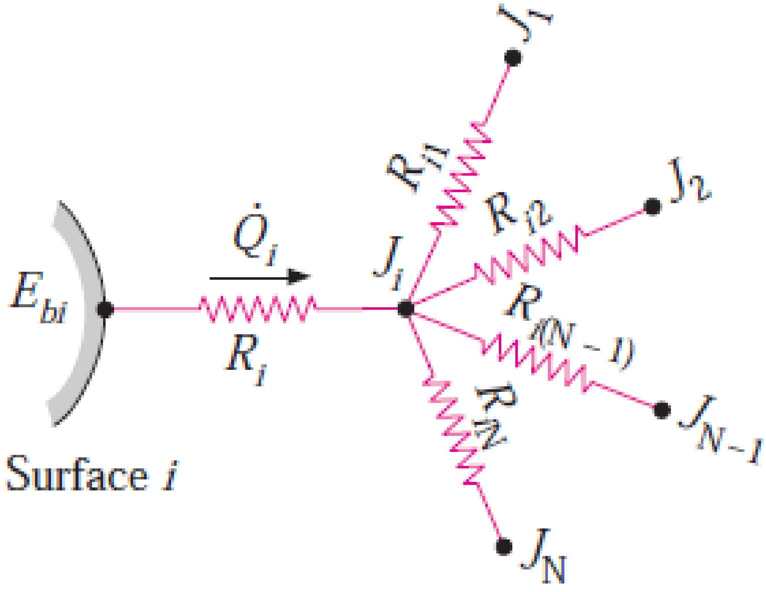
-ve because it is entering surface 2

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2} = \dot{Q}_1 = -\dot{Q}_2$$

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

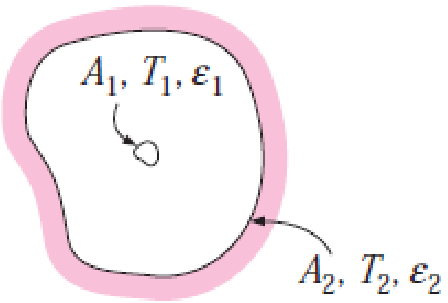
Net radiation exchange in an N-surface enclosure

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\to j} = \sum_{j=1}^{N} A_i F_{i\to j}(J_i - J_j) = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i\to j}}$$



But $\dot{Q}_i = \frac{E_{bi} - J_i}{R_i}$ Therefore, $\frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i\to j}}$

Small object in a large cavity



$$\frac{A_1}{A_2} \approx 0$$

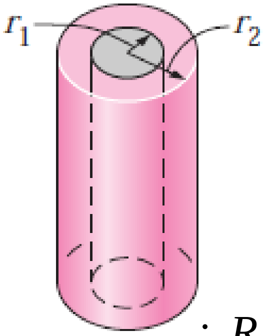
$$F_{12} = 1$$

$$\boxed{\dot{Q}_{12} = A_1 \sigma \varepsilon_1 (T_1^4 - T_2^4)}$$

$$\therefore R_1 + R_{12} + R_2 = \left(\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 \cdot F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2} \right)$$

$$= \frac{1}{A_1}\left(\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + 0 \right) = \frac{1}{A_1}\left(\frac{1}{\varepsilon_1} - 1 + 1 \right) = \frac{1}{A_1\varepsilon_1}$$

Infinitely long concentric cylinders



$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

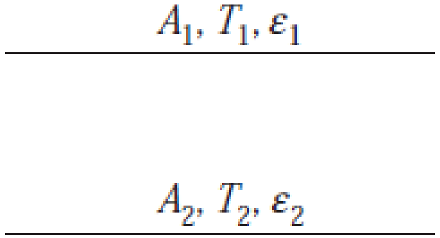
$$F_{12} = 1$$

$$\boxed{\dot{Q}_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)}}$$

$$\therefore R_1 + R_{12} + R_2 = \left(\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 \cdot F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2} \right)$$

$$= \frac{1}{A_1}\left(\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{A_1}{A_2} \right) = \frac{1}{A_1}\left(\frac{1}{\varepsilon_1} - 1 + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{r_1}{r_2} \right)$$

Infinitely large parallel plates



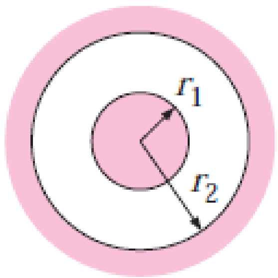
$$A_1 = A_2 = A$$
$$F_{12} = 1$$

$$\boxed{\dot{Q}_{12} = \frac{A \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}}$$

$$\therefore R_1 + R_{12} + R_2 = \left(\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 \cdot F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2} \right)$$

$$= \frac{1}{A}\left(\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2} \right) = \frac{1}{A}\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_1} - 1 \right)$$

Concentric spheres



$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$F_{12} = 1$$

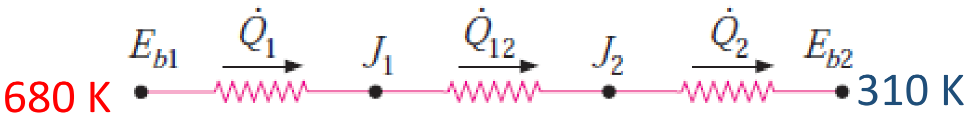
$$\boxed{\dot{Q}_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)^2}}$$

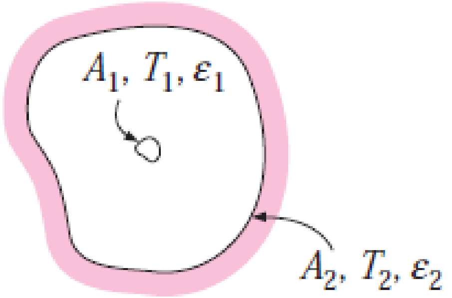
$$\therefore R_1 + R_{12} + R_2 = \left(\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 \cdot F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2} \right)$$

$$= \frac{1}{A_1}\left(\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{A_1}{A_2} \right) = \frac{1}{A_1}\left(\frac{1}{\varepsilon_1} - 1 + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{r_1^2}{r_2^2} \right)$$

Example:

A gray body having a surface area of 0.37 m^2 has ϵ_1 = 0.35 and T_1 = 407 °C. This is completely enclosed by a gray surface having an area of 3.33 m^2, ϵ_2 = 0.75, and T_2 = 37 °C. Find the net rate of heat transfer $q_{1\text{-}2}$ between the two surfaces if $F_{1\text{-}1}$ = 0.



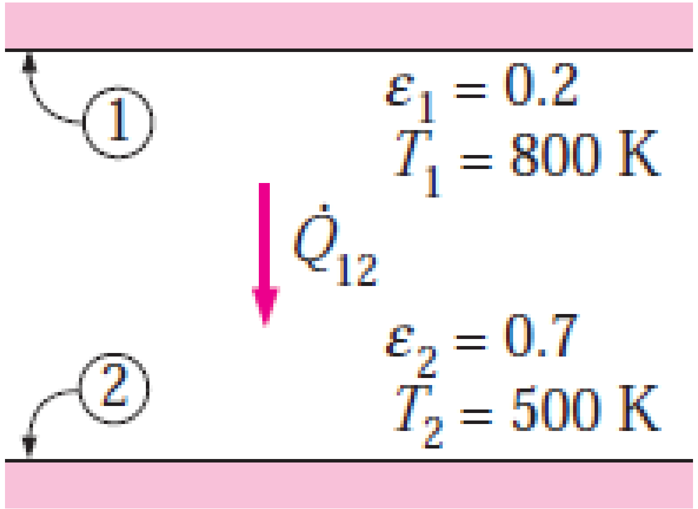


$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} \qquad R_{12} = \frac{1}{A_1F_{12}} \qquad R_2 = \frac{1-\varepsilon_2}{A_2\varepsilon_2}$$

$$\text{Here } R_1 + R_{12} + R_2 = \left(\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 \cdot F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}\right)$$

$$= \frac{1}{A_1}\left(\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2}\cdot\frac{A_1}{A_2}\right) = \frac{1}{A_1}\left(\frac{0.65}{0.35} + 1 + \frac{1-0.75}{0.75}\cdot\frac{0.37}{3.33}\right) = \frac{2.89}{A_1}$$

$$\therefore \dot{Q}_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{2.89/A_1} = \frac{0.37}{2.89}\times 5.65\times10^{-8}\left[680^4 - 310^4\right] = 1483\,W$$



EXAMPLE 22–7 **Radiation Heat Transfer between Parallel Plates**

Two very large parallel plates are maintained at uniform temperatures $T_1 = 800$ K and $T_2 = 500$ K and have emissivities $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.7$, respectively, as shown in Fig. 22–25. Determine the net rate of radiation heat transfer between the two surfaces per unit surface area of the plates.

SOLUTION Two large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates is to be determined.

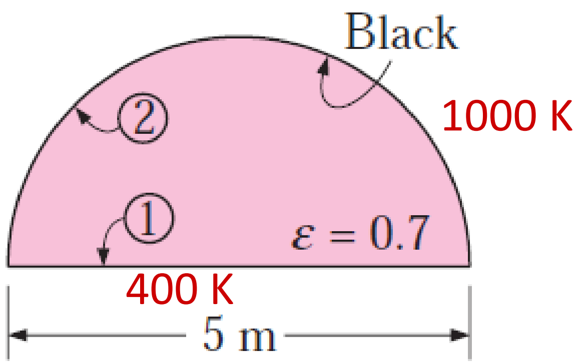
Assumptions Both surfaces are opaque, diffuse, and gray.

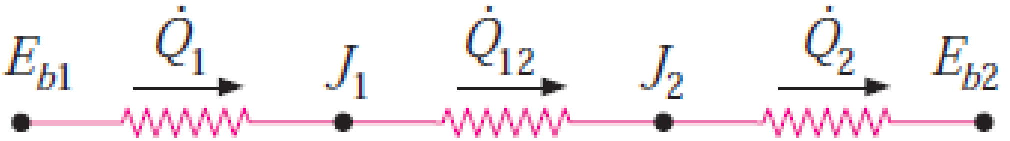
Analysis The net rate of radiation heat transfer between the two plates per unit area is readily determined from Eq. 22–38 to be

$$\dot{q}_{12} = \frac{\dot{Q}_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(800\ \text{K})^4 - (500\ \text{K})^4]}{\dfrac{1}{0.2} + \dfrac{1}{0.7} - 1}$$

$$= \mathbf{3625\ W/m^2}$$

Repeat Example: Consider a hemispherical furnace of diameter $D = 5$ m with a flat base. The dome of the furnace is black, and the base has an emissivity of 0.7. The base and the dome of the furnace are maintained at uniform temperatures of 400 and 1000 K, respectively. Determine the net rate of radiation heat transfer from the dome to the base surface during steady operation.





$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} \qquad R_{12} = \frac{1}{A_1 F_{12}} \qquad R_2 = \frac{1-\varepsilon_2}{A_2\varepsilon_2}$$

Here, $\varepsilon_1 = 0.7$; $\varepsilon_2 = 1$; $F_{12} = 1$; $E_{b1} = \sigma T_{b1}^4$; $E_{b2} = \sigma T_{b2}^4$

$$\therefore R_1 + R_{12} + R_2 = \left(\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1\cdot 1} + \frac{1-1}{A_2\cdot 1}\right) = \frac{1-\varepsilon_1+\varepsilon_1}{A_1\varepsilon_1} = \frac{1}{A_1\varepsilon_1}$$

$$\therefore \dot{Q}_1 = \left(\frac{E_{b1}-E_{b2}}{R_1+R_2+R_3}\right) = \left(\frac{\sigma\left(T_1^4 - T_2^4\right)}{1/A_1\varepsilon_1}\right) = A_1\varepsilon_1\sigma\left(T_1^4 - T_2^4\right)$$

So we get the same result!