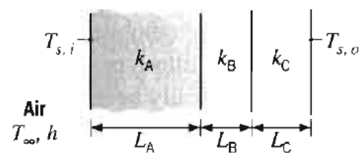
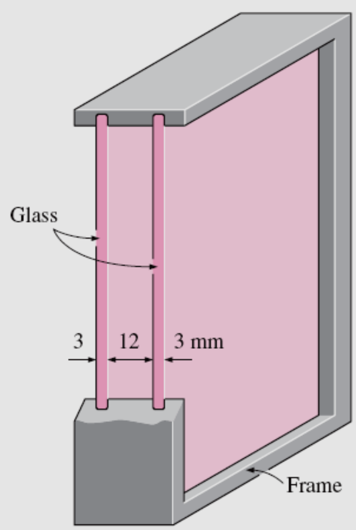
Assignment 3: Steady State Conduction

1 The composite wall of an oven consists of three materials, two of which are of known thermal conductivity, $k_A = 20$ W/m · K and $k_C = 50$ W/m · K, and known thickness, $L_A = 0.30$ m and $L_C = 0.15$ m. The third material, B, which is sandwiched between materials A and C, is of known thickness, $L_B = 0.15$ m, but unknown thermal conductivity k_B.



Under steady-state operating conditions, measurements reveal an outer surface temperature of $T_{s,o} = 20°C$, an inner surface temperature of $T_{s,i} = 600°C$, and an oven air temperature of $T_\infty = 800°C$. The inside convection coefficient h is known to be 25 W/m² · K. What is the value of k_B?

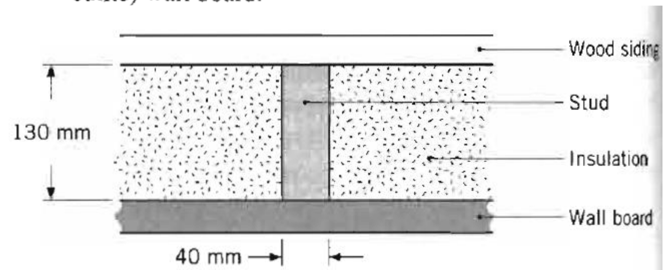
2 Consider a 1.2-m-high and 2-m-wide double-pane window consisting of two 3-mm-thick layers of glass ($k = 0.78$ W/m · °C) separated by a 12-mm-wide stagnant air space ($k = 0.026$ W/m · °C). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 24°C while the temperature of the outdoors is −5°C. Take the convection heat transfer coefficients on the inner and outer



surfaces of the window to be $h_1 = 10$ W/m² · °C and $h_2 = 25$ W/m² · °C, and disregard any heat transfer by radiation.

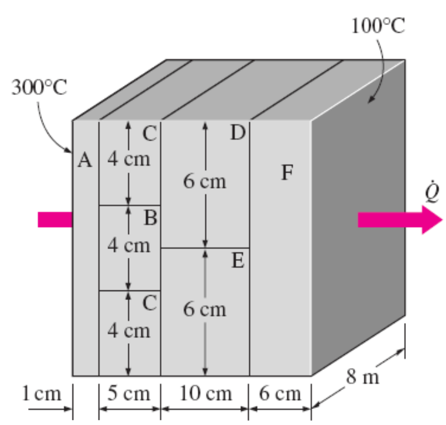
Answers: 114 W, 19.2°C

3 Consider a composite wall that includes an 8-mm-thick hardwood siding, 40-mm by 130-mm hardwood studs on 0.65-m centers with glass fiber insulation (paper faced, 28 kg/m³), and a 12-mm layer of gypsum (vermiculite) wall board.



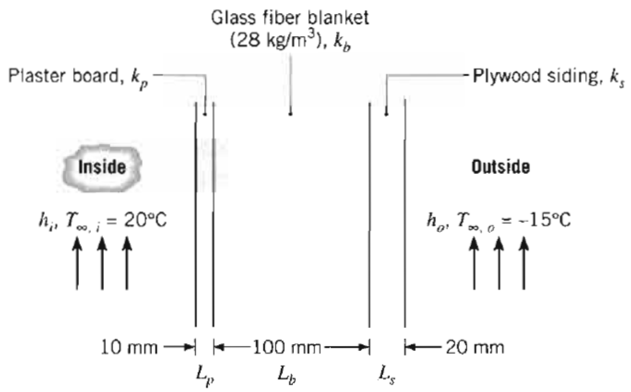
What is the thermal resistance associated with a wall that is 2.5 m high by 6.5 m wide (having 10 studs, each 2.5 m high)?

4 Consider a 5-m-high, 8-m-long, and 0.22-m-thick wall whose representative cross section is as given in the figure. The thermal conductivities of various materials used, in W/m · °C, are $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$. The left and right surfaces of the wall are maintained at uniform temperatures of 300°C and 100°C, respectively. Assuming heat transfer through the wall to be one-dimensional, determine (*a*) the rate of heat transfer through the wall; (*b*) the temperature at the point where the sections *B*, *D*, and *E* meet; and (*c*) the temperature drop across the section *F*. Disregard any contact resistances at the interfaces.



5 Repeat Problem 4 assuming that the thermal contact resistance at the interfaces D-F and E-F is 0.00012 m² · °C/W.

6 A house has a composite wall of wood, fiberglass insulation, and plaster board, as indicated in the sketch. On a cold winter day the convection heat transfer coefficients are $h_o = 60\ \text{W/m}^2 \cdot \text{K}$ and $h_i = 30\ \text{W/m}^2 \cdot \text{K}$. The total wall surface area is $350\ \text{m}^2$.



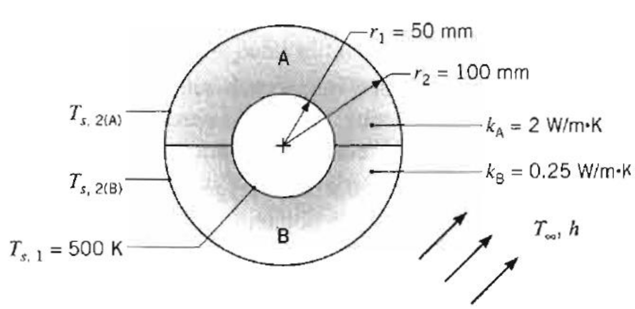
(a) Determine a symbolic expression for the total thermal resistance of the wall, including inside and outside convection effects for the prescribed conditions.

(b) Determine the total heat loss through the wall.

(c) If the wind were blowing violently, raising h_o to $300\ \text{W/m}^2 \cdot \text{K}$, determine the percentage increase in the heat loss.

(d) What is the controlling resistance that determines the amount of heat flow through the wall?

7 Steam flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature of 500 K. The pipe is covered with an insulation blanket comprised of two different materials, A and B.



The interface between the two materials may be assumed to have an infinite contact resistance, and the entire outer surface is exposed to air for which $T_\infty = 300$ K and $h = 25\ \text{W/m}^2 \cdot \text{K}$.

(a) Sketch the thermal circuit of the system. Label (using the above symbols) all pertinent nodes and resistances.

(b) For the prescribed conditions, what is the total heat loss from the pipe? What are the outer surface temperatures $T_{s,2(A)}$ and $T_{s,2(B)}$?

8 An electrical current of 700 A flows through a stainless steel cable having a diameter of 5 mm and an electrical resistance of $6 \times 10^{-4}\ \Omega/\text{m}$ (i.e., per meter of cable length). The cable is in an environment having a temperature of 30°C, and the total coefficient associated with convection and radiation between the cable and the environment is approximately $25\ \text{W/m}^2 \cdot \text{K}$.

(a) If the cable is bare, what is its surface temperature?

(b) If a very thin coating of electrical insulation is applied to the cable, with a contact resistance of $0.02\ \text{m}^2 \cdot \text{K/W}$, what are the insulation and cable surface temperatures?

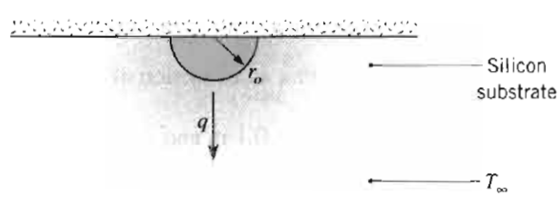
(c) There is some concern about the ability of the insulation to withstand elevated temperatures. What thickness of this insulation ($k = 0.5\ \text{W/m} \cdot \text{K}$) will yield the lowest value of the maximum insulation temperature? What is the value of the maximum temperature when the thickness is used?

9 The outer surface of a hollow sphere of radius r_2 is subjected to a uniform heat flux q''_2. The inner surface at r_1 is held at a constant temperature $T_{s,1}$.

(a) Develop an expression for the temperature distribution $T(r)$ in the sphere wall in terms of q''_2, $T_{s,1}$, r_1, r_2, and the thermal conductivity of the wall material k.

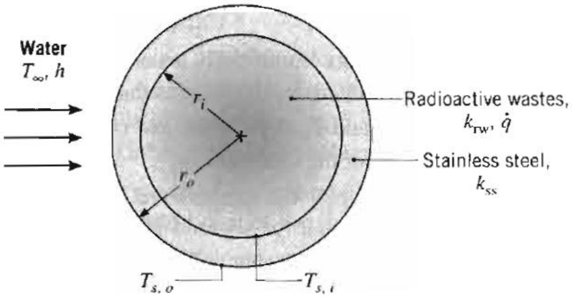
(b) If the inner and outer tube radii are $r_1 = 50$ mm and $r_2 = 100$ mm, what heat flux q''_2 is required to maintain the outer surface at $T_{s,2} = 50$°C, while the inner surface is at $T_{s,1} = 20$°C? The thermal conductivity of the wall material is $k = 10\ \text{W/m} \cdot \text{K}$.

10 A transistor, which may be approximated as a hemispherical heat source of radius $r_o = 0.1$ mm, is embedded in a large silicon substrate ($k = 125\ \text{W/m} \cdot \text{K}$) and dissipates heat at a rate q. All boundaries of the silicon are maintained at an ambient temperature of $T_\infty = 27$°C, except for the top surface, which is well insulated.



Obtain a general expression for the substrate temperature distribution and evaluate the surface temperature of the heat source for $q = 4$ W.

11 Radioactive wastes (k_{rw} = 20 W/m · K) are stored in a spherical, stainless steel (k_{ss} = 15 W/m · K) container of inner and outer radii equal to r_i = 0.5 m and r_o = 0.6 m. Heat is generated volumetrically within the wastes at a uniform rate of $\dot{q} = 10^5$ W/m^3, and the outer surface of the container is exposed to a water flow for which h = 1000 W/m^2 · K and T_∞ = 25°C.

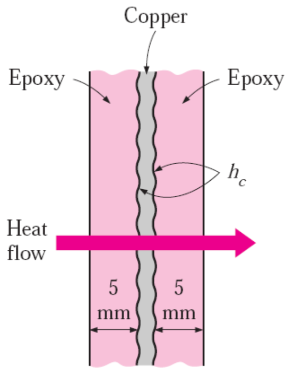


(a) Evaluate the steady-state outer surface temperature, $T_{s,o}$.

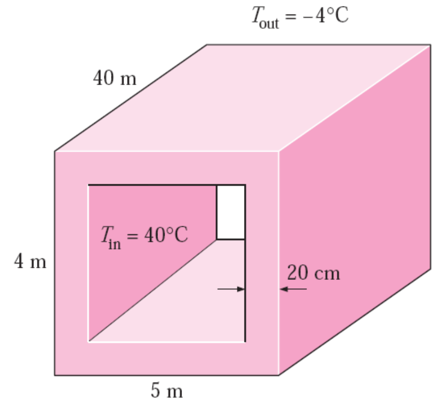
(b) Evaluate the steady-state inner surface temperature, $T_{s,i}$.

(c) Obtain an expression for the temperature distribution, $T(r)$, in the radioactive wastes. Express your result in terms of r_i, $T_{s,i}$, k_{rw}, and $\dot{q}$. Evaluate the temperature at r = 0.

12 A 1-mm-thick copper plate (k = 386 W/m · °C) is sandwiched between two 5-mm-thick epoxy boards (k = 0.26 W/m · °C) that are 15 cm × 20 cm in size. If the thermal contact conductance on both sides of the copper plate is estimated to be 6000 W/m · °C, determine the error involved in the total thermal resistance of the plate if the thermal contact conductances are ignored.

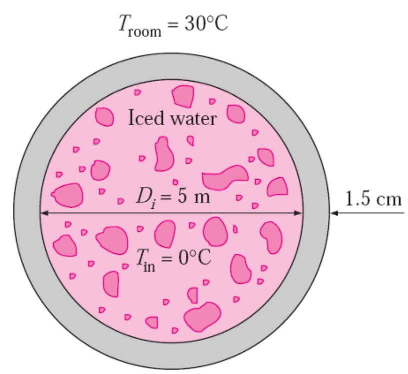


14 A 5-m-wide, 4-m-high, and 40-m-long kiln used to cure concrete pipes is made of 20-cm-thick concrete walls and ceiling (k = 0.9 W/m · °C). The kiln is maintained at 40°C by

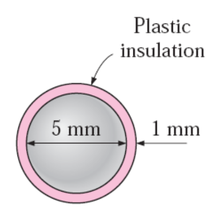


injecting hot steam into it. The two ends of the kiln, 4 m × 5 m in size, are made of a 17-mm-thick sheet metal covered with 2-cm-thick Styrofoam (k = 0.033 W/m · °C). The convection heat transfer coefficients on the inner and the outer surfaces of the kiln are 3000 W/m^2 · °C and 25 W/m^2 · °C, respectively. Disregarding any heat loss through the floor, determine the rate of heat loss from the kiln when the ambient air is at −4°C.

15 A 5-m-internal-diameter spherical tank made of 1.5-cm-thick stainless steel (k = 15 W/m · °C) is used to store iced water at 0°C. The tank is located in a room whose temperature is 30°C. The walls of the room are also at 30°C. The outer surface of the tank is black (emissivity ε = 1), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are 80 W/m^2 · °C and 10 W/m^2 · °C, respectively. Determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period. The heat of fusion of water at atmospheric pressure is h_{if} = 333.7 kJ/kg.



16 A 5-mm-diameter spherical ball at 50°C is covered by a 1-mm-thick plastic insulation (k = 0.13 W/m · °C). The ball is exposed to a medium at 15°C, with a combined convection



13 **A wall is constructed of 10-cm-thick common brick [k = 0.69 W/(m · °C)], 1.5-cm-thick fiber insulating board [k = 0.048 W/(m · °C)], followed by a 5-cm layer of glass wool (k = 0.038) and 1.5-cm-thick insulating board [k = 0.048 W/(m · °C)]. The heat transfer coefficient for both outside and inside is 12 W/(m^2 · °C). Determine the overall heat transfer coefficient U.**

***Answer*: 0.44 W/(m^2 · °C)**

17 A 2-mm-diameter and 10-m-long electric wire is tightly wrapped with a 1-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m · °C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30$°C with a heat transfer coefficient of $h = 24$ W/m² · °C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine if doubling the thickness of the plastic cover will increase or decrease this interface temperature.

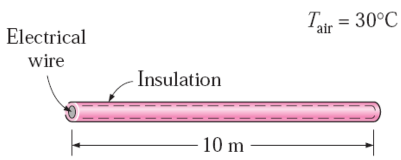
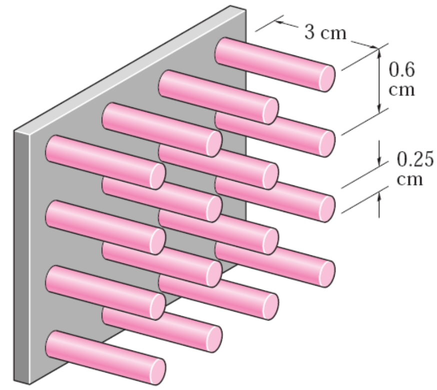


FIGURE P17–88

18 A hot surface at 100°C is to be cooled by attaching 17-cm-long, 0.25-cm-diameter aluminum pin fins ($k = 237$ W/m · °C) to it, with a center-to-center distance of 0.6 cm. The temperature of the surrounding medium is 30°C, and the heat transfer coefficient on the surfaces is 35 W/m² · °C. Determine the rate of heat transfer from the surface for a 1-m × 1-m section of the plate. Also determine the overall effectiveness of the fins.



19 Copper-plate fins of rectangular cross section having thickness $t = 1$ mm, height $L = 10$ mm, and thermal conductivity $k = 380$ W/(m · °C) are attached to a plane wall maintained at a temperature $T_0 = 230$°C. Fins dissipate heat by convection into ambient air at $T = 30$°C with a heat transfer coefficient $h = 40$ W/(m² · °C). Fins are spaced at 8 mm (that is, 125 fins per meter). Assume negligible heat loss from the fin tip.

(*a*) Determine the fin efficiency.

(*b*) Determine the area-weighted fin efficiency.

(*c*) Determine the net rate of heat transfer per square meter of plane wall surface.

(*d*) What would be the heat transfer rate from the plane wall if there were no fins attached?

Answer: (*c*) 26.87 kW/m², (*d*) 8 kW/m²

20. A fin has 5 mm diameter and 100 mm length. The thermal conductivity of fin material is 400 W/mK. One end of the fin is maintained at 130 C and its remaining surface is exposed to ambient air at 30 ºC. If the convective heat transfer coefficient is 40 W/m²K, what is the heat loss (in W) from the fin. Also evaluate its efficiency and effectiveness.