Fundamentals of radiation

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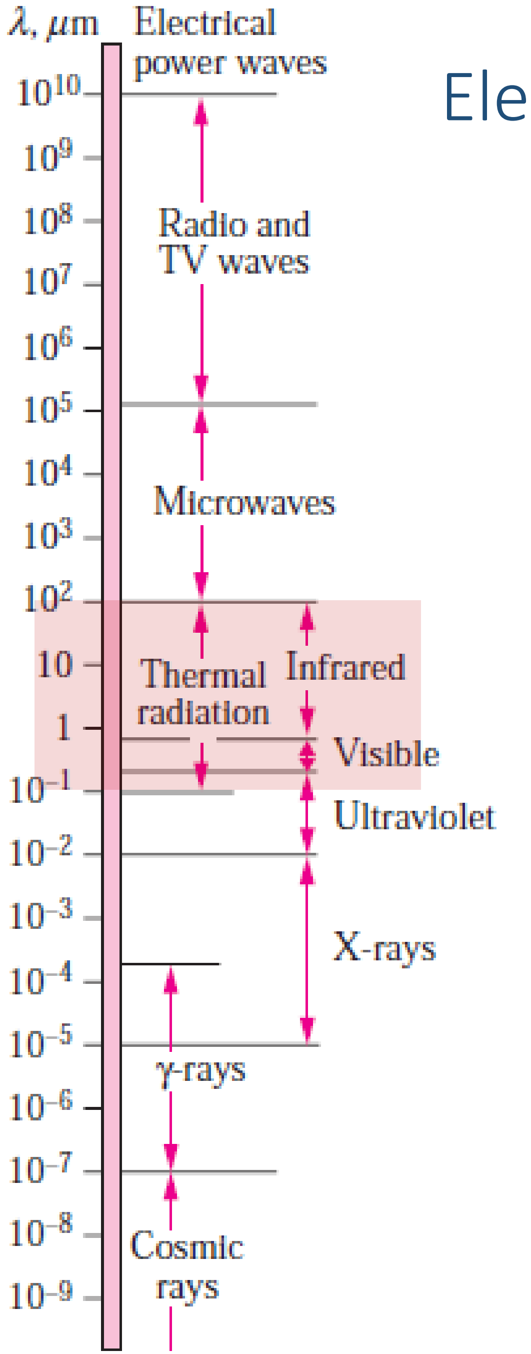
Why consider radiation?

- **Radiation is present in all heat transfer cases, but we neglected them so far if**
  - The temperature differences were low
  - The intervening medium is opaque to thermal radiation – e.g., heat exchange between two plates separated by water (low transmissivity)
  - If the conduction and convection components were very high as compared to radiation (e.g., heat transfer in a high thermal conductivity medium)
- **Radiation plays significant role for the following cases:**
  - Heat transfer in vacuum or near vacuum (e.g., in space)
  - Heat transfer involving large temperature difference (e.g., from a flame)
  - Free convection
  - Applications where radiation property is utilized (e.g., in an IR Camera)



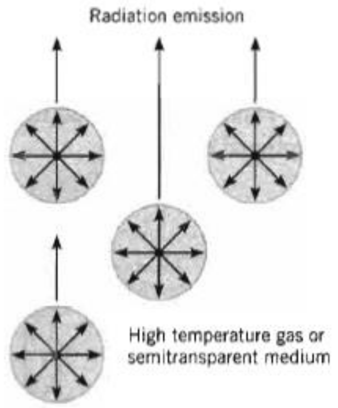
Electromagnetic radiation

Thermal radiation is only a small part of the entire electromagnetic radiation spectrum

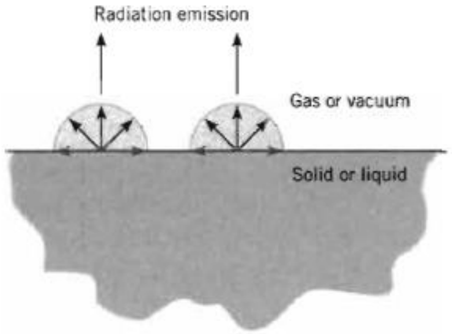
- For 0.1 – 100 μm wavelength
- Solar radiation spans from 0.3 – 3 μm wavelength
- Travels at the speed of light, i.e., 299,792,458 m/s (in vacuum)

The electrons, atoms, and molecules of all solids, liquids, and gases above 0 K are constantly in motion

- Radiation is constantly emitted, as well as being absorbed or transmitted throughout the entire volume of matter. Hence radiation is a **Volumetric phenomenon**.
- For opaque (nontransparent) solids such as metals, wood, and rocks, radiation is considered to be a surface phenomenon
  - Radiation characteristics can be changed by even a thin layer of coating on surfaces

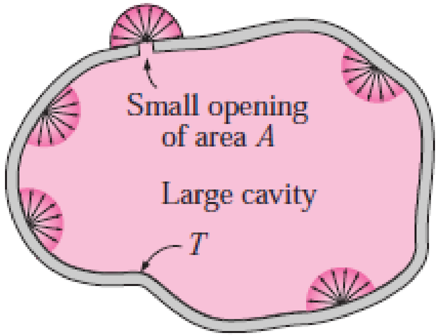


Volumetric radiation



Surface radiation

Planck's law of emissive power from a blackbody



Total Blackbody Emissive Power

$$E_b(T) = \sigma T^4 \qquad (\mathrm{W/m^2})$$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$ (Stefan Boltzmann Constant)

Blackbody surface: (a) Ain ideal emitting surface that emits the largest radiation energy for a given temperature
(b) Ain ideal absorbing surface that absorbs all radiation energy incident on it
(c) Radiation emitted by a blackbody is a function of λ,T. It is independent of direction.

Spectral Blackbody Emissive Power (Power emitted at a λ, per unit wavelength band)

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} \qquad (\mathrm{W/m^2 \cdot \mu m})$$

[Spectral density of electromagnetic radiation, December 1900]

$$C_1 = 2\pi h c_0^2 = 3.742 \times 10^8\ \mathrm{W \cdot \mu m^4/m^2}$$

$$C_2 = h c_0/k = 1.439 \times 10^4\ \mathrm{\mu m \cdot K}$$

h =6.62607004 × 10^{-34} $\mathrm{m^2\ kg / s}$

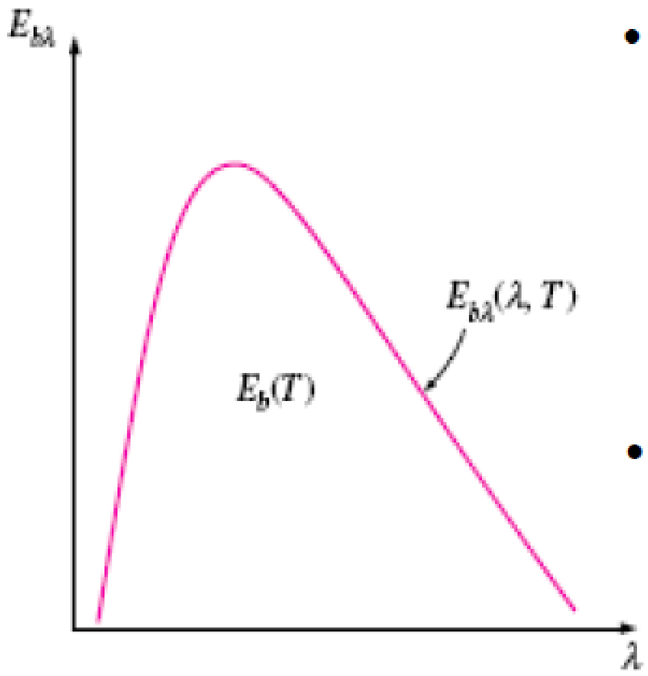
k= 1.38065 × 10^{-23} J/K ($=R_u/N_{AV}$)

(h= Plank's constant, k=Boltzmann constant)

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}$$

Spectral blackbody emission

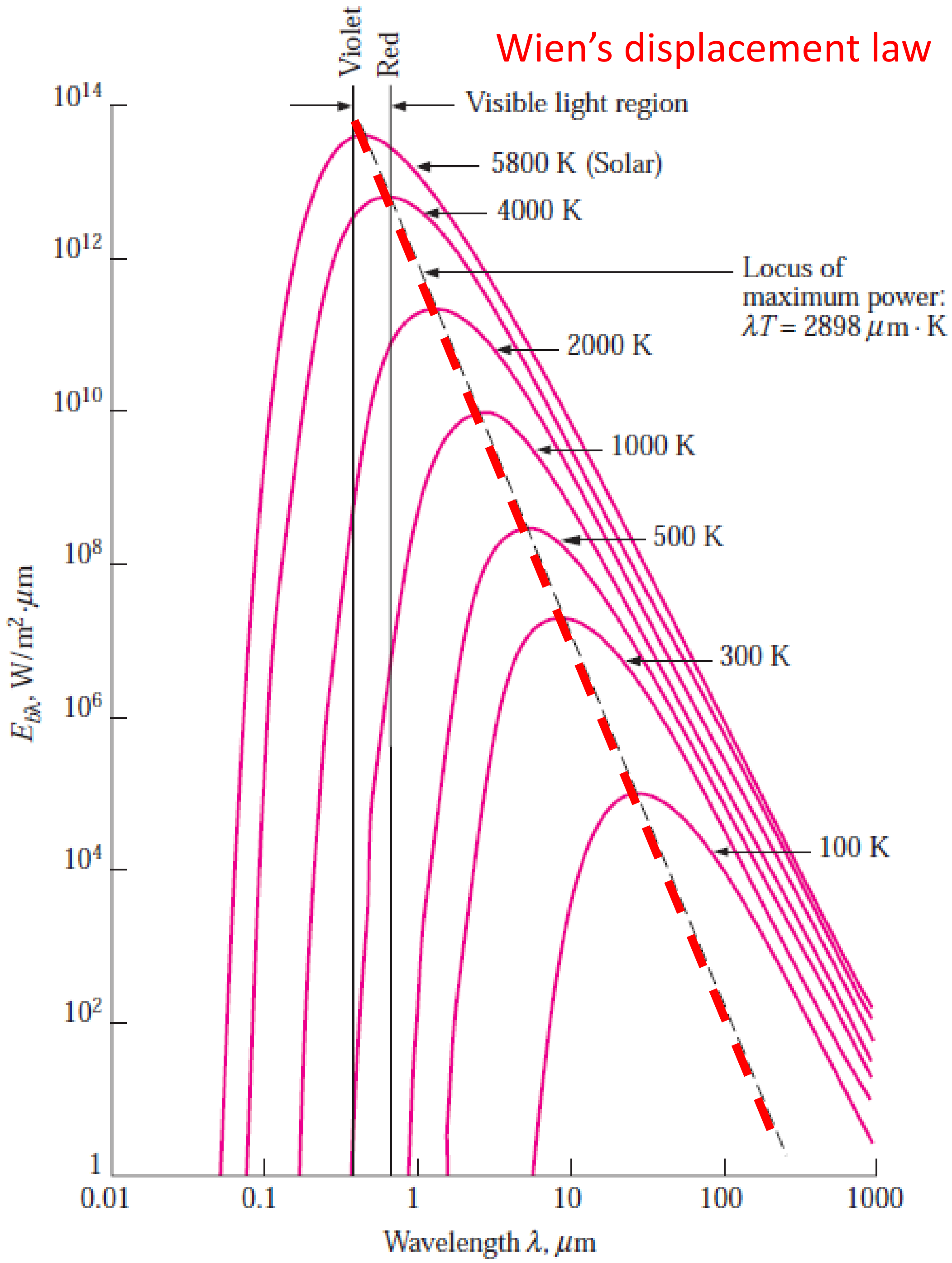
- Energy (W/m^2) emitted varies with wavelength and temperature
- Max. point occurs where $\lambda T = 2897.8\ \mu m \cdot K$ (**Wien's displacement law**) (see next slide for derivation)
- T increase shifts peak to lower λ



- Total energy is integral over all wavelengths

$$E_b = \int_0^\infty E_{b\lambda}\, d\lambda = \sigma T^4$$

- Recall that $\sigma = 2\pi^5 k^4/(15h^3c^2)$



$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}$$

Assignment: Derive Wein's displacement law from Planck's formula

Differentiating Planck's formula,

$$\frac{\partial E_{b\lambda}}{\partial \lambda} = \frac{(e^{C_2/\lambda T} - 1)(-5C_1\lambda^{-6}) - (C_1\lambda^{-5})(e^{C_2/\lambda T})(-C_2.\lambda^2 T)}{(e^{C_2/\lambda T} - 1)^2}$$

For a maximum, the numerator on the right must vanish. This gives, after cancellation of common factors,

$$x - \frac{C_2}{5(1 - e^{-C_2/x})} = 0 \qquad (1)$$

where $x = \lambda_{max} T$. For $\lambda \approx 1\ \mu\text{m}$, $T \approx 10^3$ K, we have

$$\frac{C_2}{\lambda T} \approx 10$$

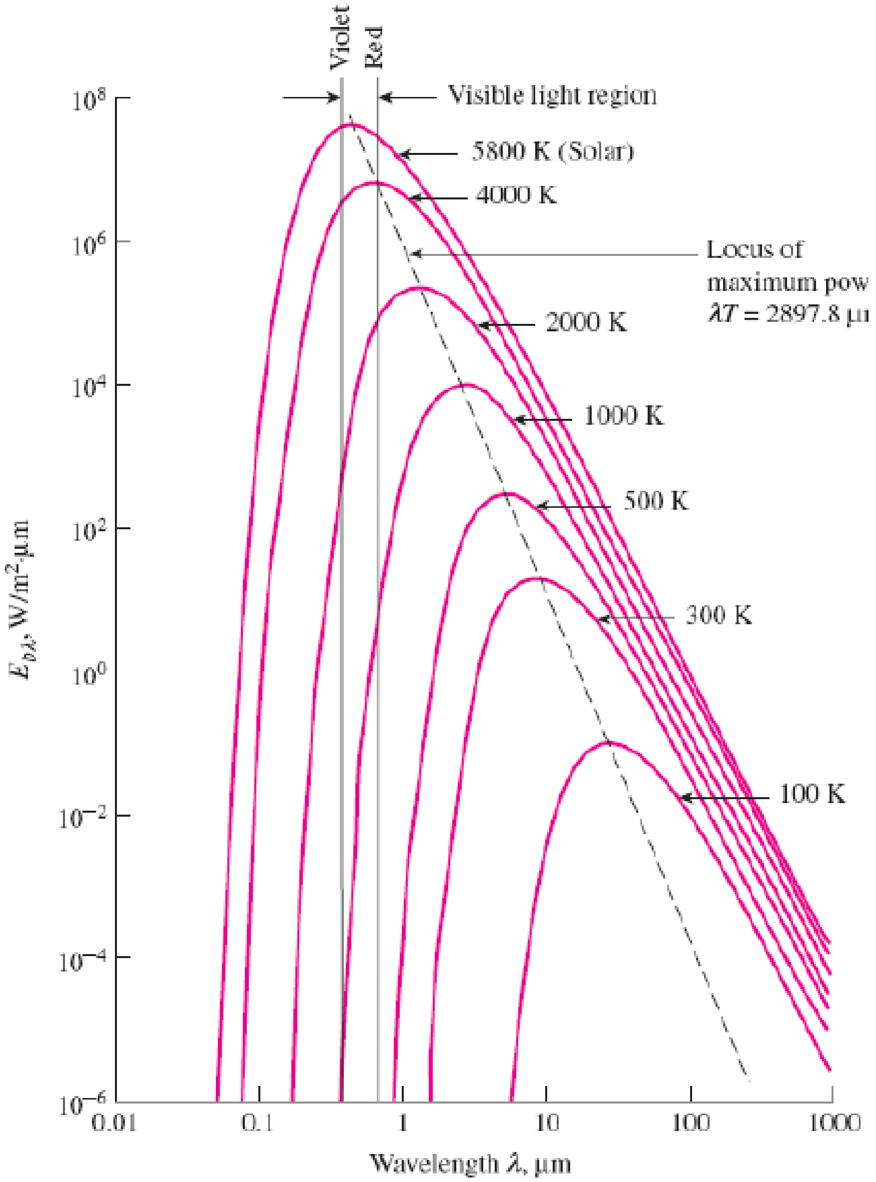
Thus, as a first approximation, we neglect $e^{-C_2/x}$ in comparison to 1, and (1) gives

$$x = \frac{C_2}{5} = 2877\ \mu\text{m}\cdot\text{K}$$

which is quite close. If this value is used as the first approximation, x_0, in *Newton's iterative method*, a single iteration gives

$$x_1 = \frac{1 - 6e^{-5}}{1 - 7e^{-5} + e^{-10}}\left(\frac{C_2}{5}\right) = \frac{0.9595}{0.9528}(2877) = 2897\ \mu\text{m}\cdot\text{K}$$

Salient observation:



- The emitted radiation is a continuous function of *wavelength*. At any specified temperature, it increases with wavelength, reaches a peak, and then decreases with increasing wavelength.
- At any wavelength, the amount of emitted radiation *increases* with increasing temperature.
- As temperature increases, the curves shift to the left to the shorter wavelength region. Consequently, a larger fraction of the radiation is emitted at *shorter wavelengths* at higher temperatures.
- The radiation emitted by the *sun*, which is considered to be a blackbody at 5780 K (or roughly at 5800 K), reaches its peak in the visible region of the spectrum. Therefore, the sun is in tune with our eyes.
- On the other hand, surfaces at $T < 800$ K emit almost entirely in the infrared region and thus are not visible to the eye unless they reflect light coming from other sources.

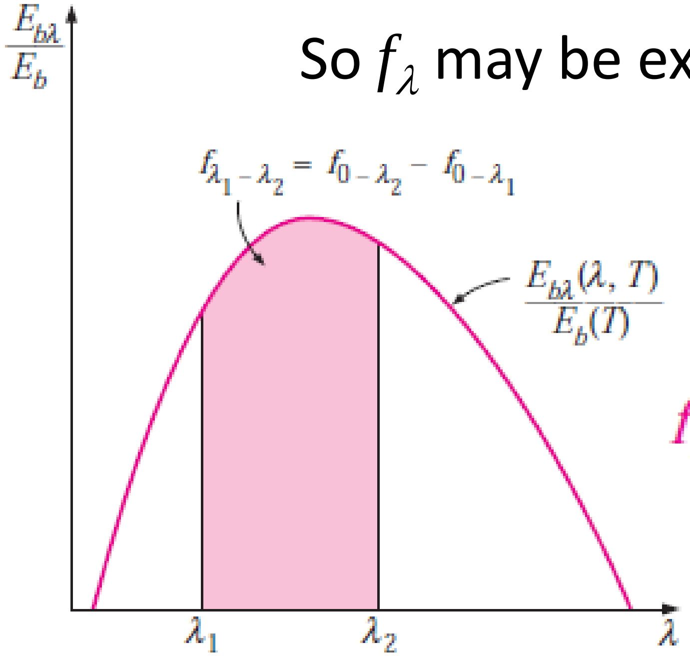
Blackbody radiation function

Fraction of the radiation emitted from a blackbody at temperature T in the wavelength band from λ = 0 to λ

$$f_\lambda(T) = \frac{\int_0^\lambda E_{b\lambda}(\lambda, T)\, d\lambda}{\sigma T^4}$$

$$f_\lambda = \frac{1}{\sigma T^4}\int_0^\lambda E_{b\lambda} d\lambda = \frac{1}{\sigma T^4}\int_0^\lambda \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)}d\lambda = \frac{1}{\sigma}\int_0^{\lambda T}\frac{C_1}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)}d(\lambda T)$$

So f_λ may be expressed as a function of λT



$$f_{\lambda_1-\lambda_2}(T) = f_{\lambda_2}(T) - f_{\lambda_1}(T)$$

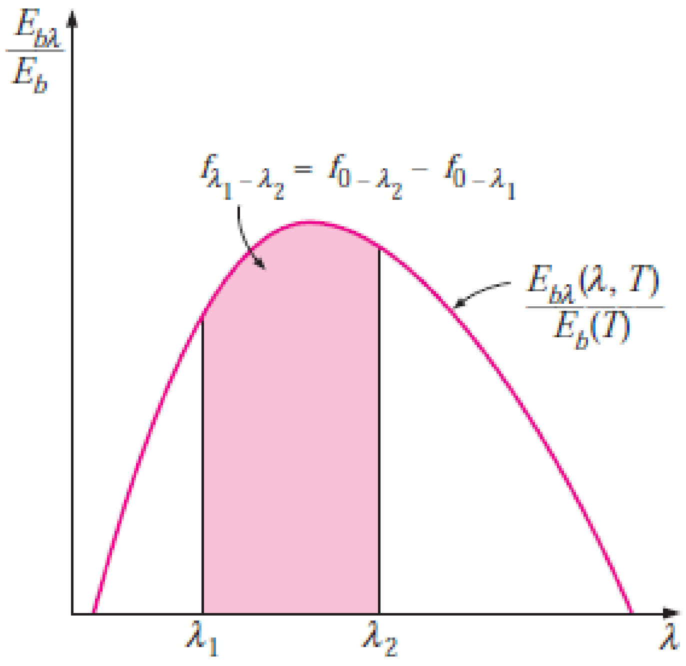
ly radiation functions f_λ

λT, μm · K	f_λ	λT, μm · K	f_λ
200	0.000000	6200	0.754140
400	0.000000	6400	0.769234
600	0.000000	6600	0.783199
800	0.000016	6800	0.796129
1000	0.000321	7000	0.808109
1200	0.002134	7200	0.819217
1400	0.007790	7400	0.829527
1600	0.019718	7600	0.839102
1800	0.039341	7800	0.848005
2000	0.066728	8000	0.856288
2200	0.100888	8500	0.874608
2400	0.140256	9000	0.890029
2600	0.183120	9500	0.903085
2800	0.227897	10,000	0.914199
3000	0.273232	10,500	0.923710
3200	0.318102	11,000	0.931890
3400	0.361735	11,500	0.939959
3600	0.403607	12,000	0.945098
3800	0.443382	13,000	0.955139
4000	0.480877	14,000	0.962898
4200	0.516014	15,000	0.969981
4400	0.548796	16,000	0.973814
4600	0.579280	18,000	0.980860
4800	0.607559	20,000	0.985602
5000	0.633747	25,000	0.992215
5200	0.658970	30,000	0.995340
5400	0.680360	40,000	0.997967
5600	0.701046	50,000	0.998953
5800	0.720158	75,000	0.999713
6000	0.737818	100,000	0.999905

Example 1

The temperature of the filament of an incandescent lightbulb is 2500 K. Assuming the filament to be a blackbody, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also, determine the wavelength at which the emission of radiation from the filament peaks.

Visible range: $\lambda_1 = 0.4\ \mu\text{m to } \lambda_2$



From the Blackbody radiation function table :

$$\lambda_1 T = (0.40\ \mu\text{m})(2500\ \text{K}) = 1000\ \mu\text{m}\cdot\text{K} \longrightarrow f_{\lambda_1} = 0.000321$$
$$\lambda_2 T = (0.76\ \mu\text{m})(2500\ \text{K}) = 1900\ \mu\text{m}\cdot\text{K} \longrightarrow f_{\lambda_2} = 0.053035$$

Therefore, the fraction of radiation in [0.4<λ<0.76]:

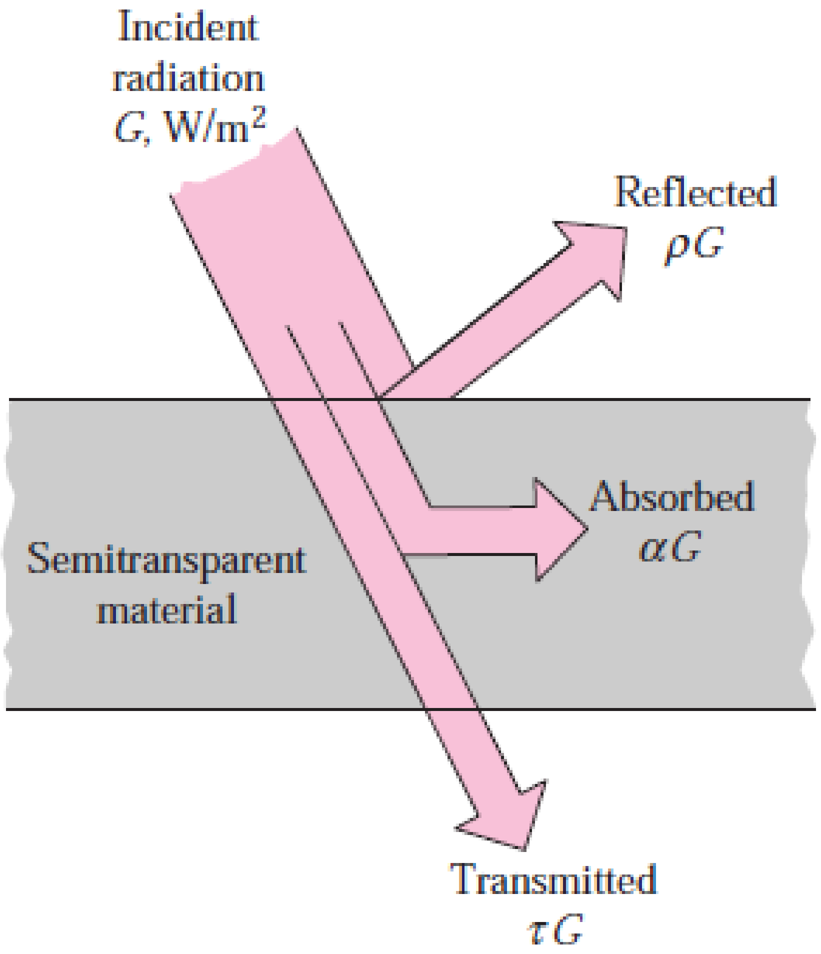
$$f_{\lambda_1-\lambda_2} = f_{\lambda_2} - f_{\lambda_1} = 0.053035 - 0.000321 = 0.0527135$$

For $\lambda_{\text{max power}}$:

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K}$$

$$\rightarrow \quad \lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{2500\ \text{K}} = 1.16\ \mu\text{m}$$

Radiation properties: Absorptivity, reflectivity and transmissivity



Absorptivity: $\alpha = \dfrac{\text{Absorbed radiation}}{\text{Incident radiation}} = \dfrac{G_{abs}}{G}, \qquad 0 \le \alpha \le 1$

Reflectivity: $\rho = \dfrac{\text{Reflected radiation}}{\text{Incident radiation}} = \dfrac{G_{ref}}{G}, \qquad 0 \le \rho \le 1$

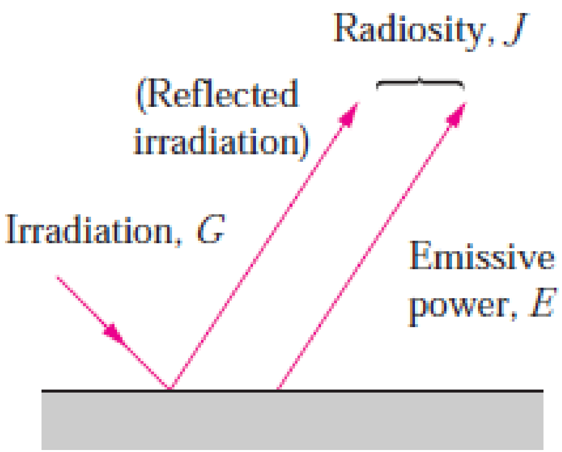
Transmissivity: $\tau = \dfrac{\text{Transmitted radiation}}{\text{Incident radiation}} = \dfrac{G_{tr}}{G}, \qquad 0 \le \tau \le 1$

$$G_{abs} + G_{ref} + G_{tr} = G$$

$$\alpha + \rho + \tau = 1$$

For opaque bodies $\alpha + \rho = 1$

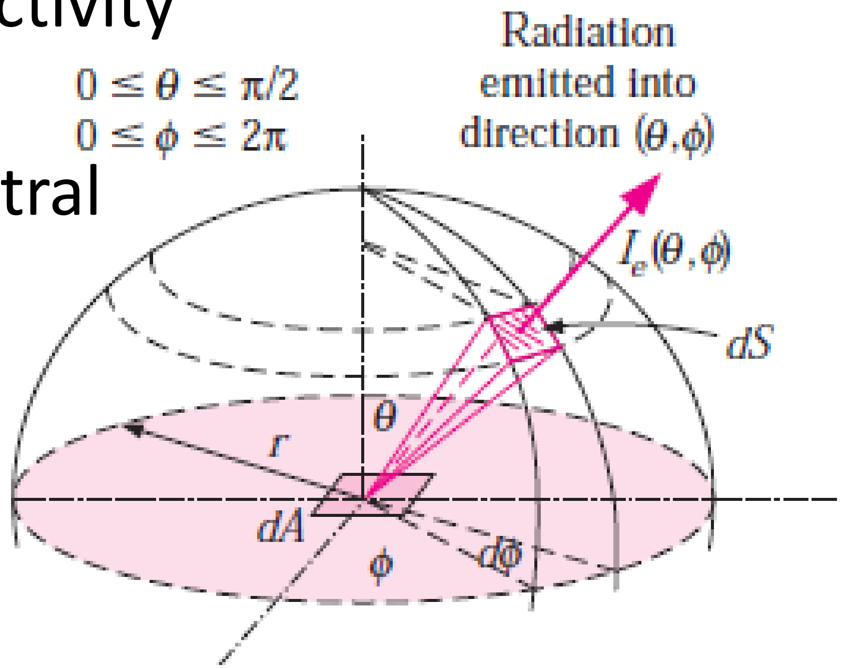
Radiosity (J)



- Rate at which radiation energy leaves a unit area of a surface in all direction

$$J = \rho G + \varepsilon E_b$$

- For diffuse emitter and diffuse reflectors, the reflectivity and emissivity do not depend on the orientation
- But, actual surfaces may have directional and spectral properties also



Radiation properties: Emissivity

Spectral Directional Emissivity: $\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T)=\dfrac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{b\lambda}(\lambda,T)}$ [Emissivity in a particular (θ,ϕ) at a given λ]

Total Directional Emissivity: $\varepsilon_{\theta}(\theta,\phi,T)=\dfrac{I_{e}(\theta,\phi,T)}{I_{b}(T)}$ [Emissivity in a particular (θ,ϕ) integrated over all λ]

Spectral Hemispherical Emissivity: $\varepsilon_{\lambda}(\lambda,T)=\dfrac{E_{\lambda}(\lambda,T)}{E_{b\lambda}(\lambda,T)}$ [Emissivity in a particular λ integrated over all direction]

Total Hemispherical Emissivity:

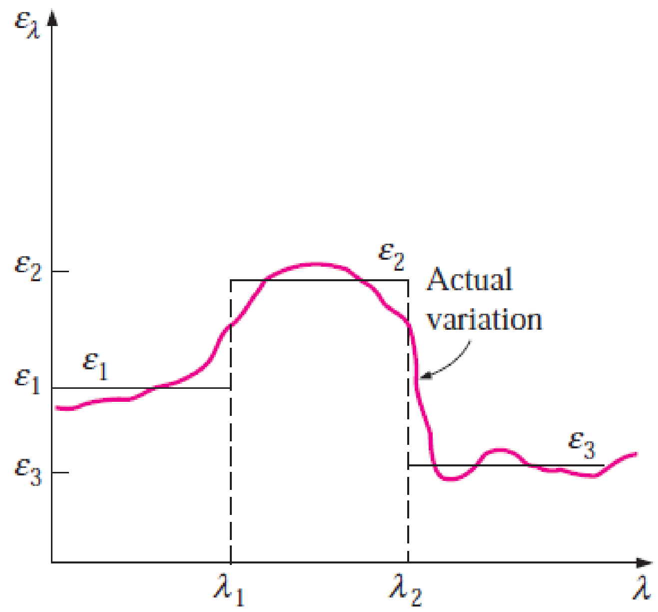
$$\varepsilon(T)=\frac{E(T)}{E_b(T)}=\frac{\int_0^\infty \varepsilon_\lambda(\lambda,T)E_{b\lambda}(\lambda,T)\,d\lambda}{\sigma T^4}$$

The ratio of the total radiation energy emitted by the surface to the radiation emitted by a blackbody of the same surface area at the same temperature

Spectral variation of ε(λ)

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda}\, d\lambda}{E_b} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda}{E_b} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda}\, d\lambda}{E_b}$$

$$= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T)$$

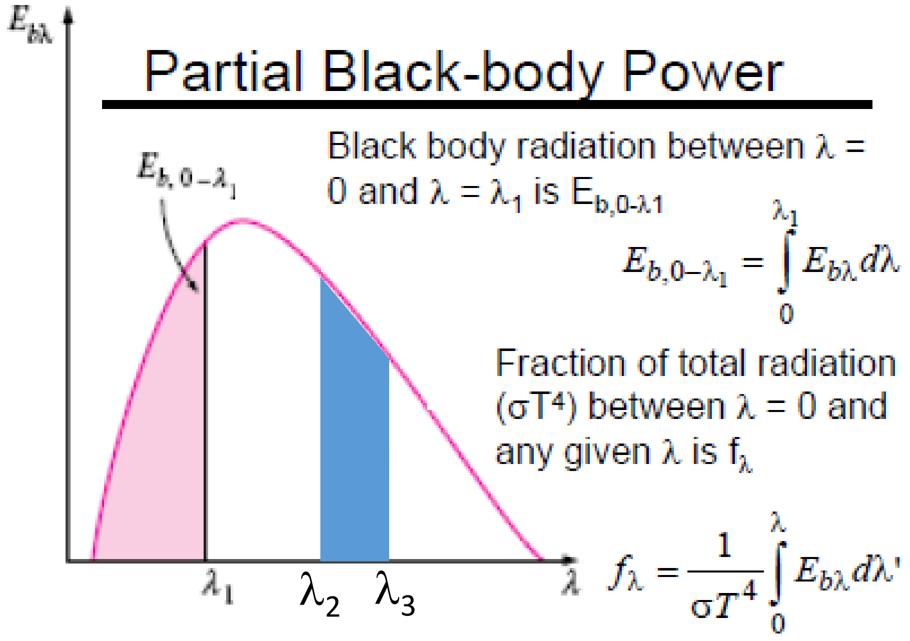


Blackbody Radiation Function:

$$f_\lambda(T) = \frac{\int_0^{\lambda} E_{b\lambda}(\lambda, T)\, d\lambda}{\sigma T^4}$$

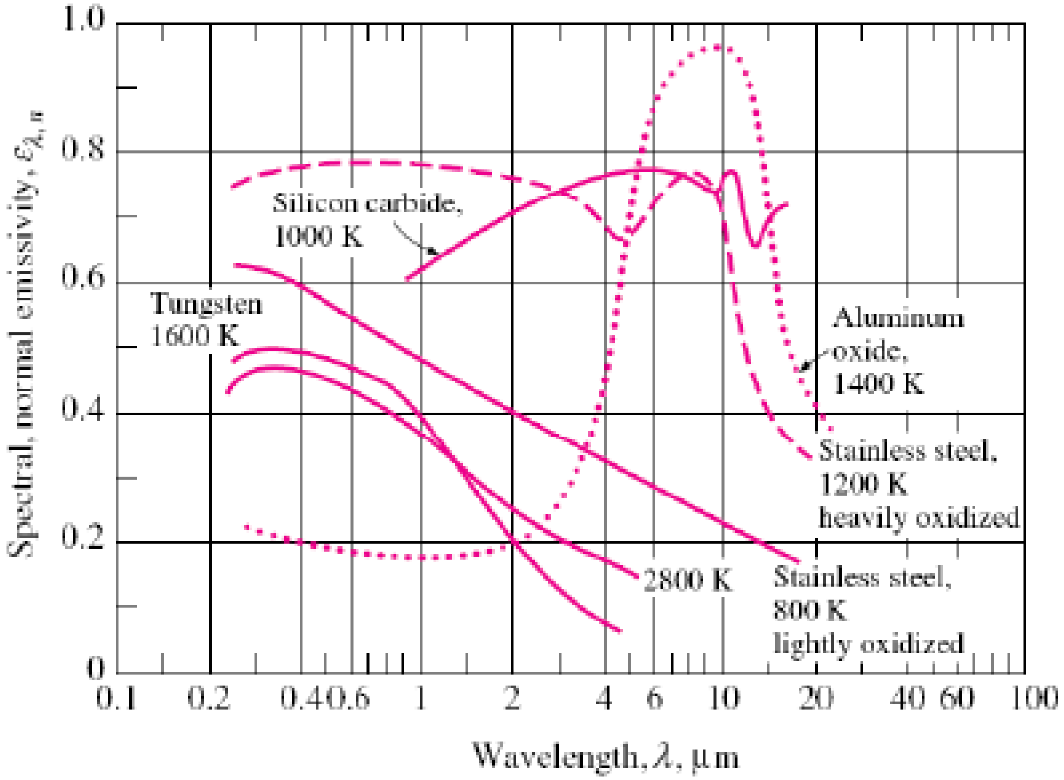
Blackbody Radiation fraction between two wavelengths:

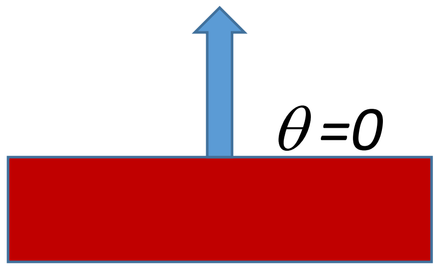
$$f_{\lambda_2-\lambda_3}(T) = f_{\lambda_3}(T) - f_{\lambda_2}(T)$$



Spectral and Temperature variation of emissivity

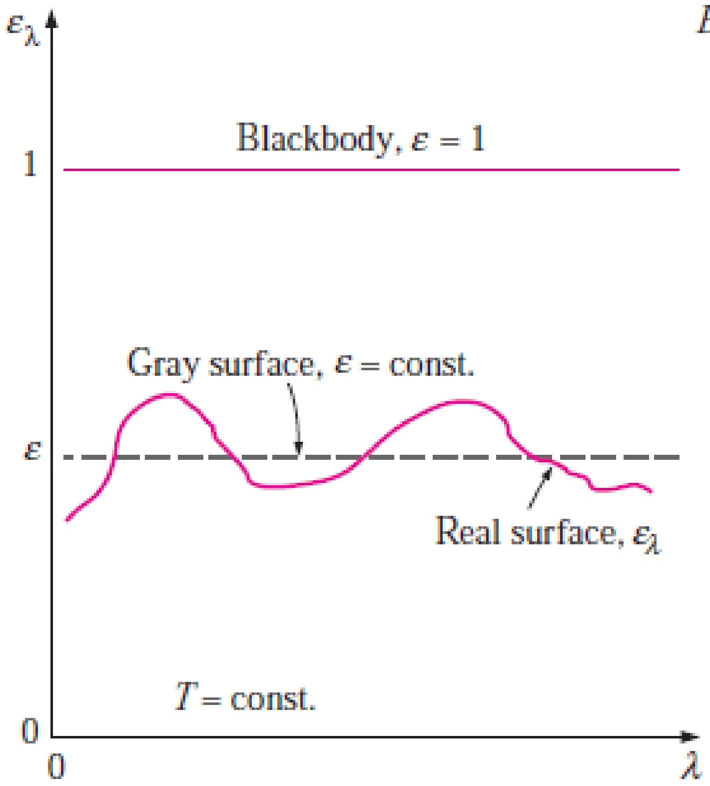
$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T)=\frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{b\lambda}(\lambda,T)}$$

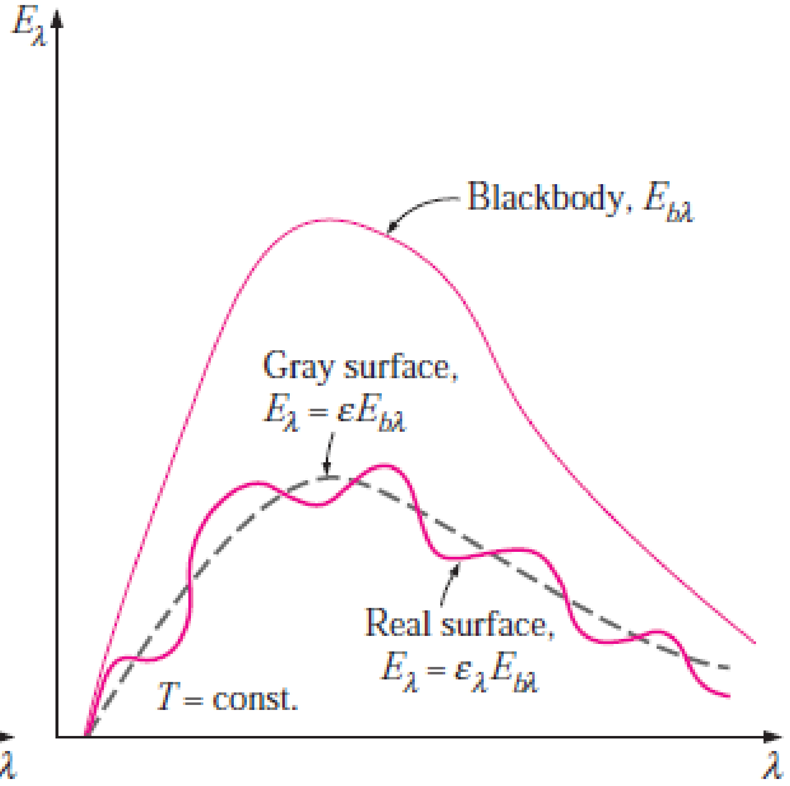




Spectral normal emissivity $\varepsilon_{\lambda,\theta}(\lambda,\theta=0,T)$

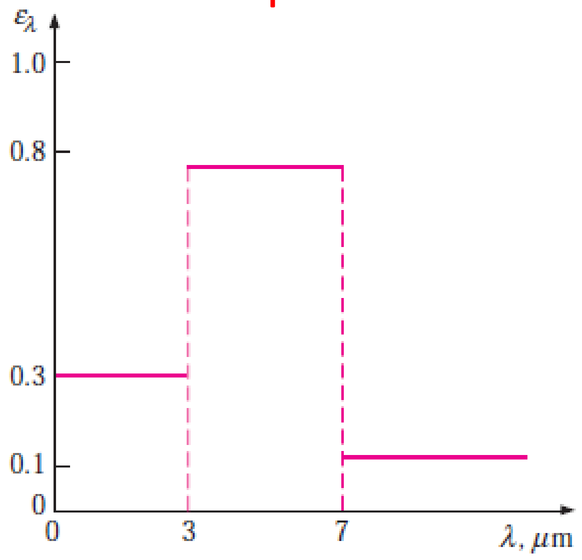
Blackbody vs gray surface





A surface is said to be ***diffuse*** if its properties are *independent of direction*, and ***gray*** if its properties are *independent of wavelength*.
The *gray* and *diffuse* approximations are often utilized in radiation calculations.

Example 3



The spectral emissivity function of an opaque surface at 800 K is approximated as in the Figure

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.3, & 0 \le \lambda < 3\ \mu\text{m} \\ \varepsilon_2 = 0.8, & 3\ \mu\text{m} \le \lambda < 7\ \mu\text{m} \\ \varepsilon_3 = 0.1, & 7\ \mu\text{m} \le \lambda < \infty \end{cases}$$

Determine the average emissivity of the surface and its emissive power.

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda}\, d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda}\, d\lambda}{\sigma T^4}$$
$$= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T)$$
$$= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$

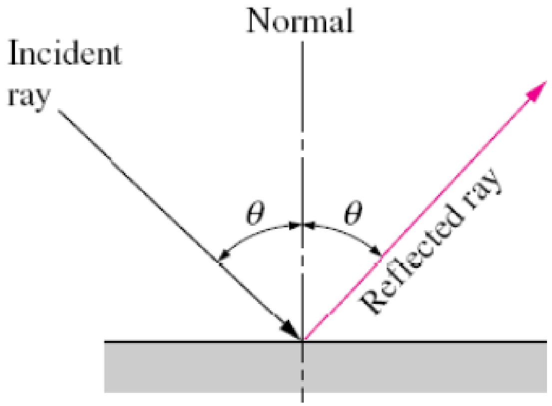
$$\lambda_1 T = (3\ \mu\text{m})(800\ \text{K}) = 2400\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_1} = 0.140256$$
$$\lambda_2 T = (7\ \mu\text{m})(800\ \text{K}) = 5600\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_2} = 0.701046$$

$$\varepsilon = 0.3 \times 0.140256 + 0.8(0.701046 - 0.140256) + 0.1(1 - 0.701046)$$
$$= 0.521$$

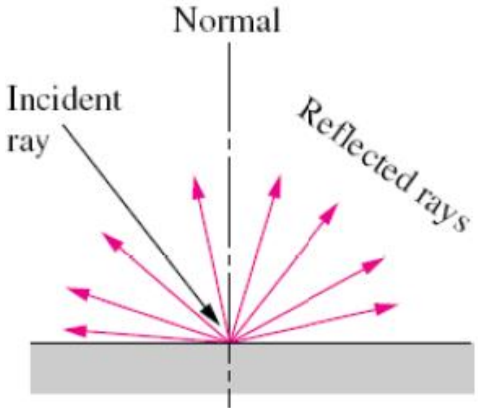
$$E = \varepsilon \sigma T^4 = 0.521(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(800\ \text{K})^4 = 12{,}100\ \text{W/m}^2$$

Directional and Spectral dependence of reflectivity

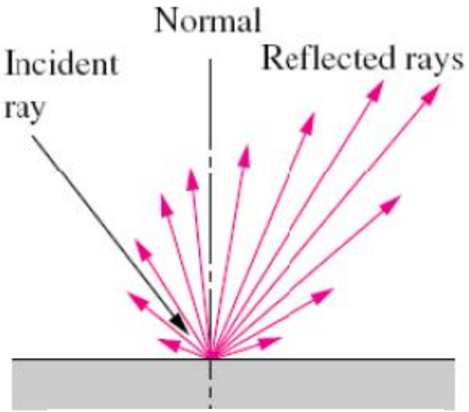
- Directional reflectivity



Specular (mirror)

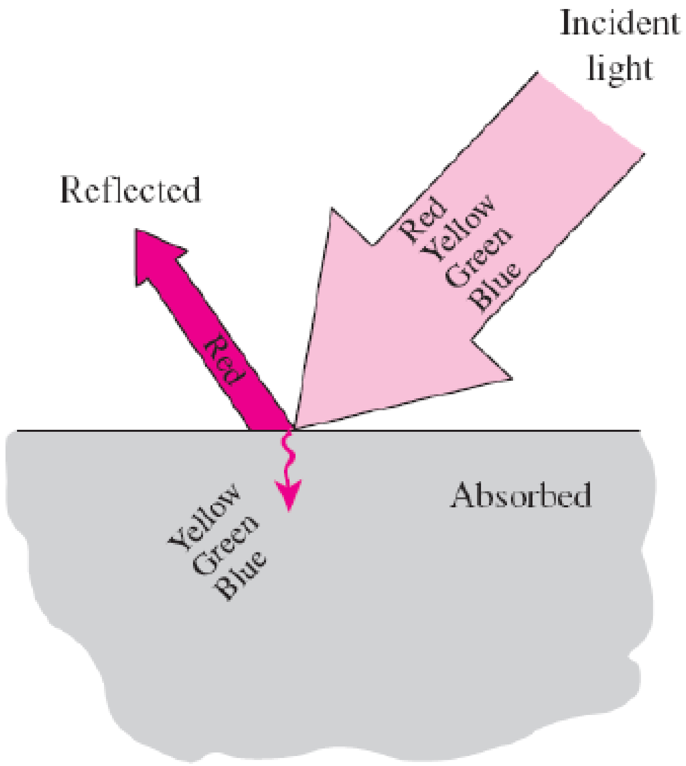


Diffuse

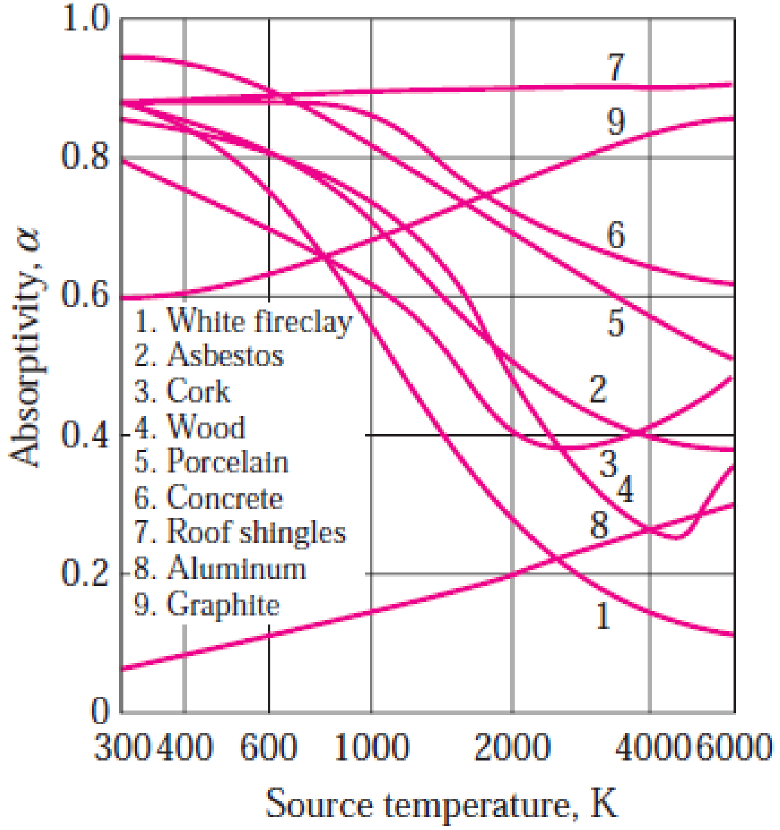


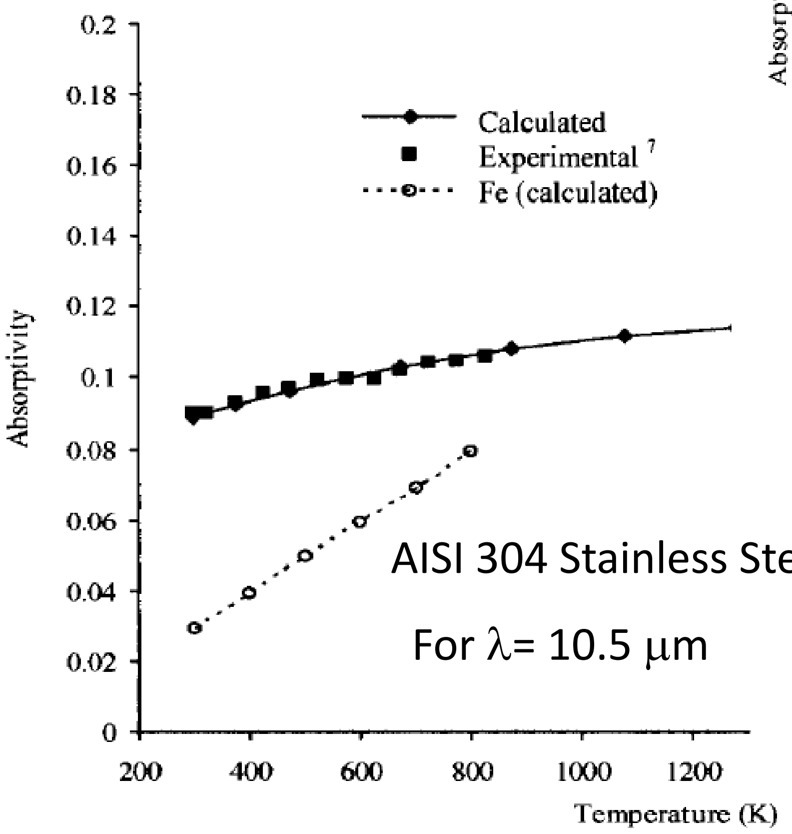
Actual or irregular

- Spectral reflectivity



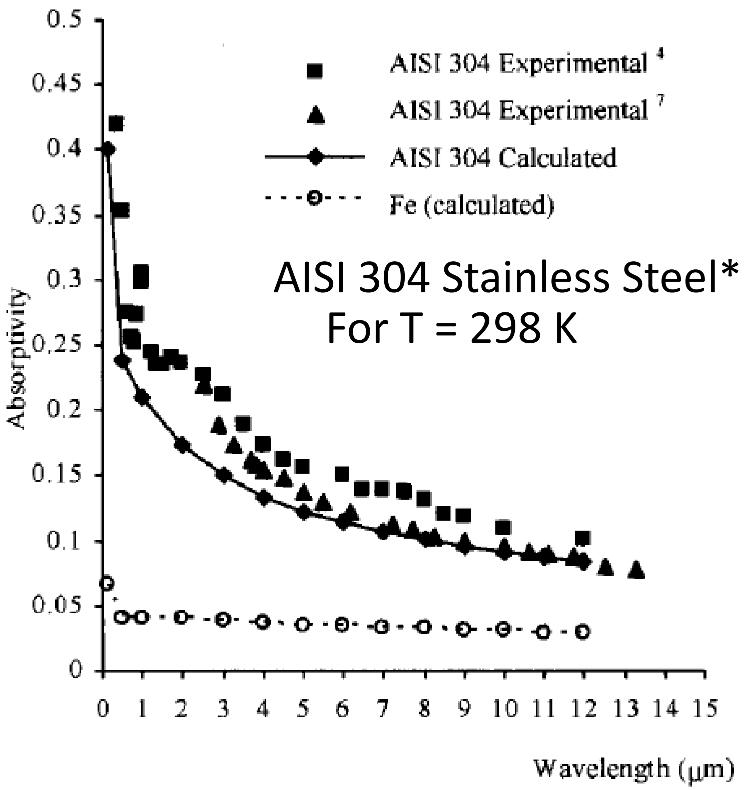
Spectral and temperature dependence of Absorptivity





AISI 304 Stainless Steel*

For λ= 10.5 μm

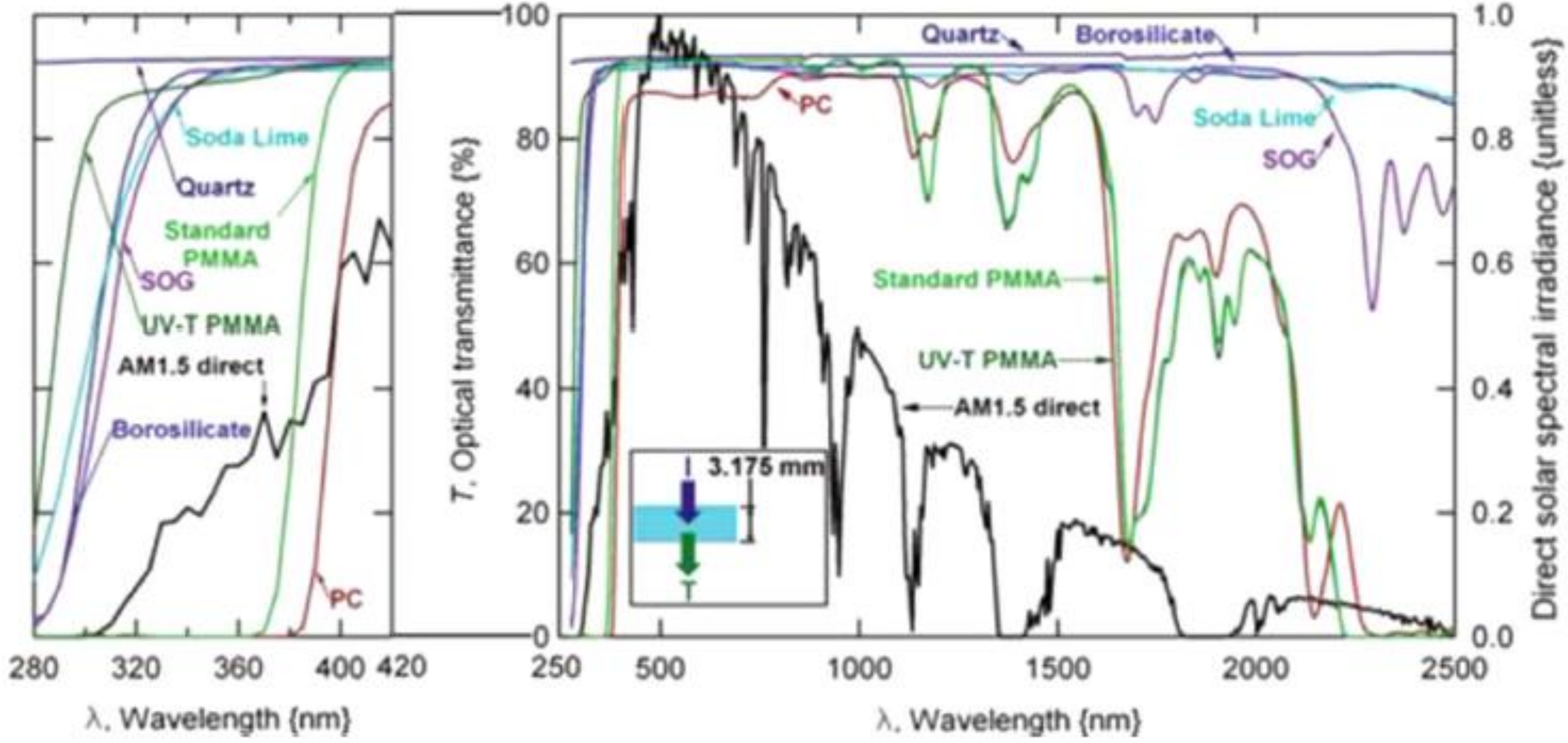


AISI 304 Stainless Steel*
For T = 298 K

*Source: Boyden and Zhang, J. Thermophysics and Heat Transfer, 20 (2006) 1—15.

Transmissivity

Spectral transmissivity of different glasses for SPV Cells



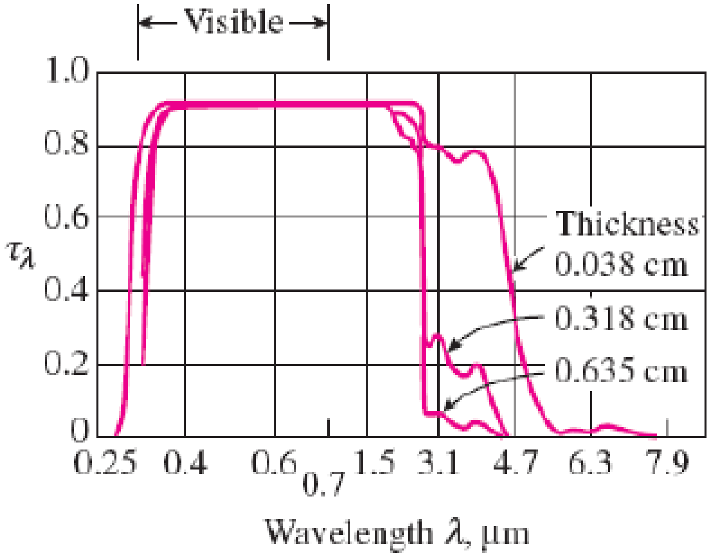
Source: Miller et al., [Optical Engineering, 50(1)](), 013003 (2011). doi:10.1117/1.3530092

The Greenhouse Effect

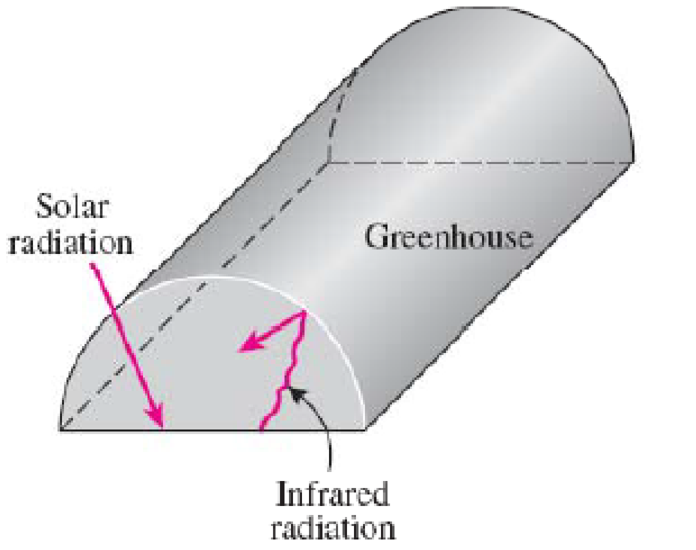
Glass has a transparent window in the wavelength range 0.3 μm < λ < 3 μm in which over 90% of solar radiation is emitted. The entire radiation emitted by surfaces at room temperature falls in the infrared region (λ > 3 μm).

Glass allows the solar radiation to enter but does not allow the infrared radiation from the interior surfaces to escape. This causes a rise in the interior temperature as a result of the energy buildup in the car.

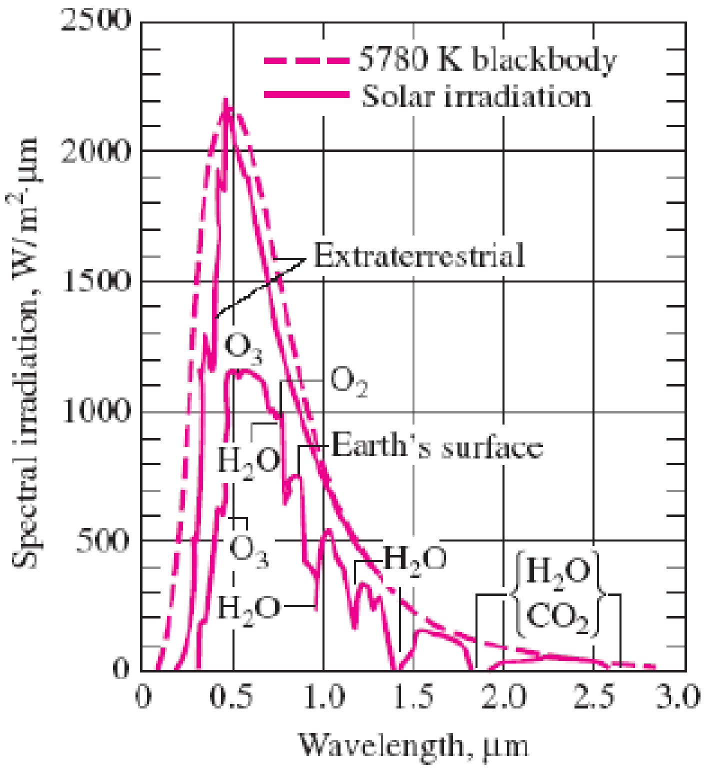
This heating effect, which is due to the nongray characteristic of glass (or clear plastics), is known as the **greenhouse effect**.



The spectral transmissivity of low-iron glass at room temperature for different thicknesses.

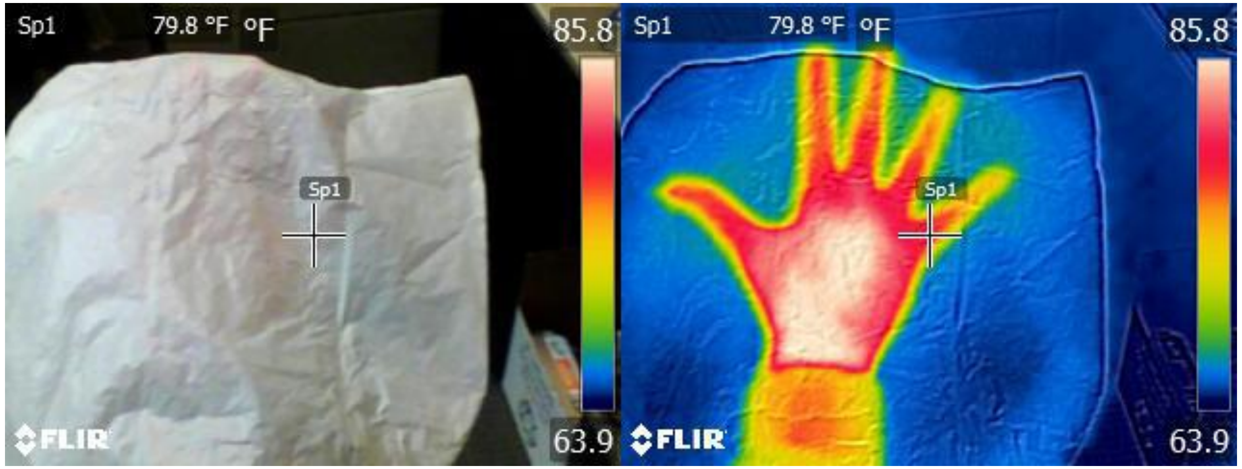


A greenhouse traps energy by allowing the solar radiation to come in but not allowing the infrared radiation to go out.

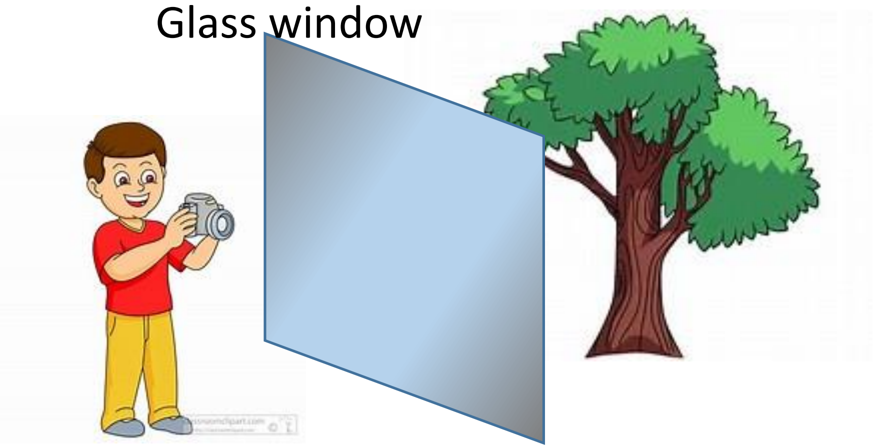


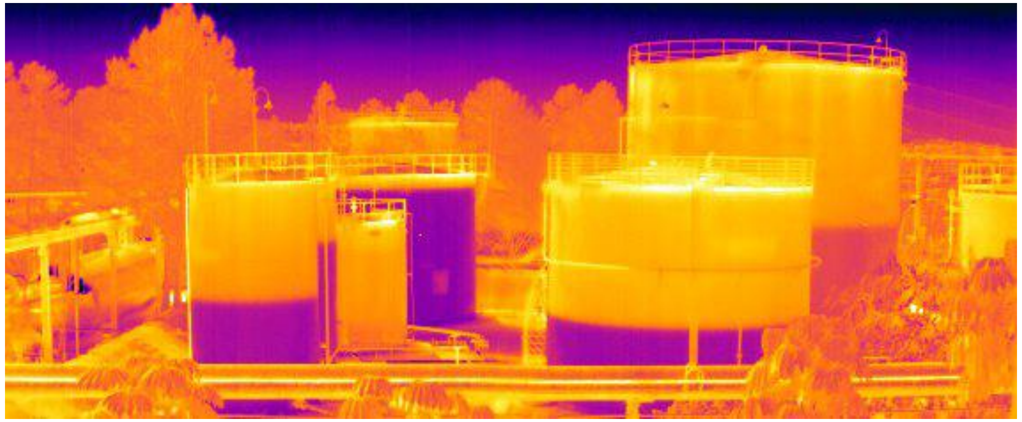
Spectral distribution of solar radiation just outside the atmosphere, at the surface of the earth on a typical day, and comparison with blackbody radiation at 5780 K.

Optically opaque does not mean thermally opaque

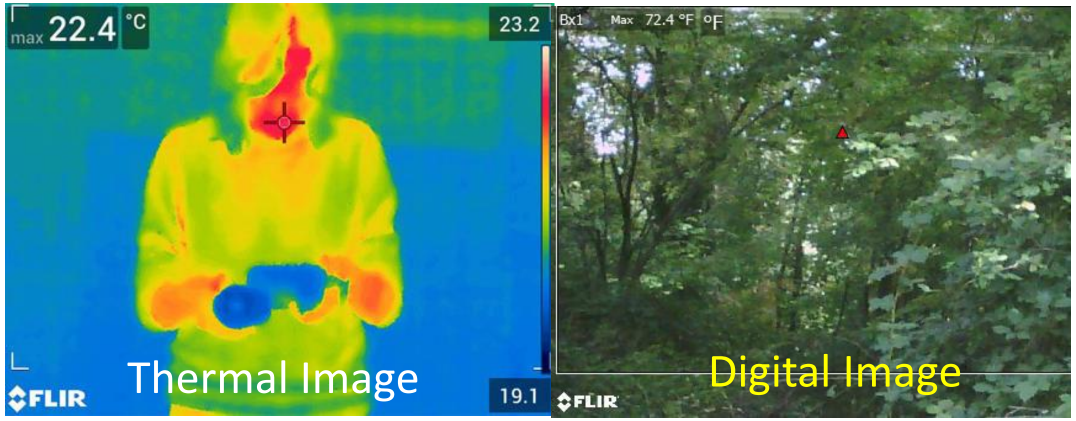


The hand inside the nearly opaque (to visible wavelength) plastic bag is clearly visible in an IR Camera





Liquid level in oil tanks can be measured from outside using IR imaging (metal temperature in contact with the oil being different)

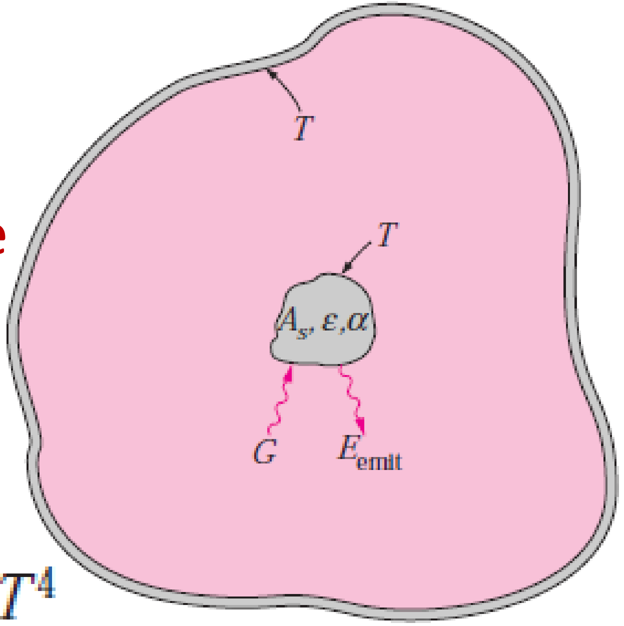


The digital camera sees through the glass the trees outside, while th
thermal camera sees the reflected heat of the photographer

*Source: Flir website (https://www.flir.in/discover)

Kirchoff's Law of Radiation

- A small real object (having ε and α) of area A_s in a large enclosure
- Both the object and enclosure are at the same temperature T



  - Radiation incident on the small body (from the blackbody enclosure): $G = E_b(T) = \sigma T^4$
  - Radiation absorbed by the small body: $G_{abs} = \alpha G = \alpha\sigma T^4$
  - Radiation emitted by the small body: $E_{emit} = \varepsilon\sigma T^4$
  - Thermal equilibrium of the small body: $A_s \varepsilon\sigma T^4 = A_s \alpha\sigma T^4$

$$\varepsilon(T) = \alpha(T)$$

Total hemispherical emissivity = Total hemispherical absorptivity

The same also applies for a specific wavelength: $\varepsilon_\lambda(T) = \alpha_\lambda(T)$

A few practical examples of Kirchoff's Law

- Good absorbers are good emitters also, and vice versa.
  - Black object with rough surface absorbs more, they will emit also more
  - Highly absorbing surfaces get heated more in day-time and at night they radiate more at night
- When a shiny metal ball having some black spots on its surface is heated to a high temperature, and viewed in dark, these spots glow brighter (emitting more). The absorptivity of the black spots were more, and so is their emissivity.
- When a green glass is heated to a very high temperature, it emits light that is richer in red.