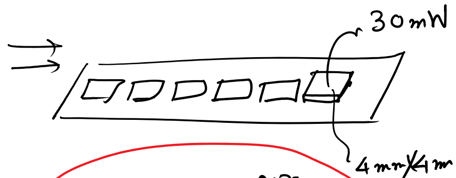
# Forced Convection

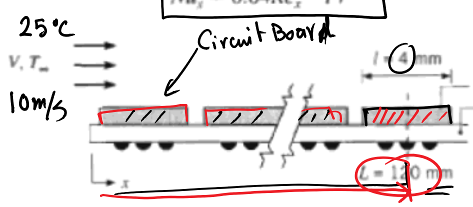
Monday, March 29, 2021 9:58 AM

25°C




**6.26** Forced air at $T_\infty$ = 25°C and $V$ = 10 m/s is used to cool electronic elements on a circuit board. One such element is a chip, 4 mm by 4 mm, located 120 mm from the leading edge of the board. Experiments have revealed that flow over the board is disturbed by the elements and that convection heat transfer is correlated by an expression of the form

$$\boxed{Nu_x = 0.04 Re_x^{0.85} Pr^{1/3}}$$




Estimate the surface temperature of the chip if it is dissipating 30 mW.

$$Nu_x = 0.04 Re_x^{0.85} Pr^{1/3}$$

$\boxdot\ T_\infty$


$$q'' = h(T_s - T_\infty)$$

$$\frac{30 \times 10^{-3}\ W}{4 \times 4 \times 10^{-6}\ m^2} = h(T_s - T_\infty)$$

or $$\boxed{\frac{30}{16} \times 10^3 = h(T_s - 25)}$$

? ?

How to find h ?

Let's find $Nu_x$ first.

$$Nu_x = 0.04 Re_x^{0.85} Pr^{1/3}$$

$Re_x$ = ? $\quad$ $Pr$ = ? $\quad$ $\rho_{air}$, $\mu_{air}$, $k_{air}$, $C_p$

$$Re_x = \frac{\rho U x}{\mu}$$

$$Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

$$\alpha = \frac{k}{\rho C_p}$$

$$T_f = \frac{1}{2}\cancel{(T_s + T_\infty)} \Rightarrow \text{Let } \boxed{T_f = T_s}$$

?

Let us assume

$\rho$ = 1.2 kg/m³

$\nu$ = 10⁻⁵ m²/s

$C_p$ = 1.004 kJ/kgK = 1004 J/kgK

$k$ = 0.03 W/mK

$$Re = \frac{Ux}{\nu} = \frac{\rho U x}{\mu} \quad x = 120\ mm$$

$$Re_x = \frac{10 \times 0.12}{10^{-5}} = 1.2 \times 10^5$$

$$Pr = \frac{1.2 \times 10^{-5} \times \cancel{1004}\ 1000}{0.03} = 0.4$$

$$Nu = 0.04 \times (1.2 \times 10^5)^{0.85} \times (0.4)^{1/3}$$

$$= 611.95$$

$$h = \frac{k\,Nu}{x} = \frac{0.03 \times 611.95}{0.12} = \underline{\underline{153}}\ W/m^2$$

$$q'' = h(T_s - T_\infty)$$

$$\Rightarrow T_s = \frac{q''}{h} + T_\infty \Rightarrow \frac{30}{16} \times 10^3 / 153 + 25$$

$$T_s = 12.25 + 25\ °C$$

$$= 37.25°C$$

Steps

1) Identify the film temp & the fluid properties or use values from tables $\boxed{\nu, C_p, k, \rho}$ $\boxed{P_r, \nu, k}$

2) Calculate $Re$, $Pr$

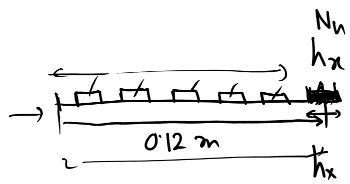
3) Identify the appropriated $Nu_x$ correlation

4) Find out $h_x = \dfrac{k\,Nu_x}{x}$

(Local Nu) and HTC

To find average HTC

$$\boxed{Nu_x = 0.04\, Re_x^{0.85}\, P_r^{1/3}}$$




4m

$$h_x = \frac{k\,Nu_x}{x}$$

$$= \frac{0.04}{k} \times \frac{U^{0.85}}{\nu^{0.85}} \frac{x^{0.85}}{x} P_r^{1/3}$$

$$= \left(\frac{0.04 \times 10^{0.85} \times (0.4)^{1/3}}{0.03 \times (10^{-5})^{0.85}}\right) \frac{x^{0.85}}{x}$$

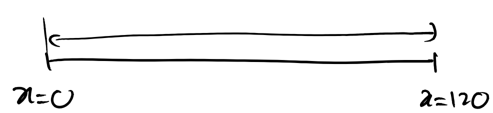
$$= (A)\, x^{-0.15}$$

$$\boxed{h_x = A x^{-0.15}} \quad \bar{h}_x = \frac{1}{x}\int_0^x h_x\, dx$$

$$h_x = A x^{-n}$$

$$\bar{h}_x = \left(\frac{h_x}{1-n}\right)$$

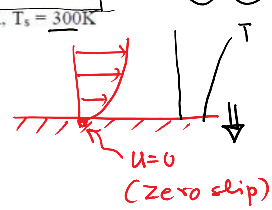
$$\boxed{\bar{h}_x = \frac{A x^{-0.15}}{1-n}} \quad \boxed{x = 0.12}$$




7. In a particular application involving air flow over a heated surface, the boundary layer temperature distribution may be approximated as $\boxed{\dfrac{(T - T_s)}{(T_\infty - T_s)} = 1 - \exp\left(-Pr \dfrac{U_\infty y}{\nu}\right)}$ ← T- profile

where y is the distance normal to the surface and Pr=0.7. If $T_\infty$=400K, $T_s$ = 300K and $U_\infty/\nu$ = 5000 $m^{-1}$, what is the surface heat flux? Find Nu

$\frac{m}{s} \cdot \frac{s}{m^2} = \frac{1}{m}$




$$q'' = -k \left.\frac{dT}{dy}\right|_{y=0}$$

$$= -k\, Pr \left(\frac{U_\infty}{\nu}\right)(T_\infty - T_s)$$

$$= \underbrace{}_{0.03} \quad \underbrace{}_{0.7} \quad 5000 \quad K$$

$$T = T_s + (T_\infty - T_s)\left[1 - \exp\left(-Pr\frac{U_\infty}{\nu} y\right)\right]$$

$$\frac{dT}{dy} = (T_\infty - T_s)[\ldots$$

$= 0.03\ \frac{W}{mK}$ $\quad 0.7 \quad 5000\ \frac{1}{m} \quad K$

$= -\ W/m^2$

$T = T_s + (T_\infty - T_s)\left[1 - \exp\left(-Pr\frac{U_\infty}{\nu}y\right)\right]$

$$\left.\frac{dT}{dy}\right|_{y=0} = (T_\infty - T_s)\left[+\frac{Pr\,U_\infty}{\nu}\exp\left(-\frac{Pr\,U_\infty}{\nu}y\right)\right]_{y=0} = Pr\frac{U_\infty}{\nu}(T_\infty - T_s)$$

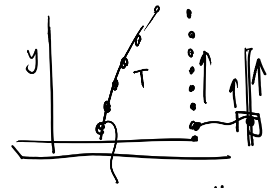
$$h = \frac{q''}{(T_s - T_\infty)} = \frac{-k\,Pr\frac{U_\infty}{\nu}\cancel{(T_\infty - T_s)}}{\cancel{(T_s - T_\infty)}} = k\,Pr\frac{U_\infty}{\nu}$$

$$Nu = \frac{hL}{k} = \cancel{k}\,Pr\left(\frac{U_\infty}{\nu}\cdot\frac{L}{\cancel{k}}\right) = Pr\cdot Re$$

$$\boxed{Nu = 1.\ Re^1 . Pr^1}$$

For flat plate

$$\boxed{Nu = 0.332\,Re^{0.5}\,Pr^{1/3}}$$




$T(y) \Rightarrow q''$

Nu

h

## Heat Transfer for a const. wall Heat flux Case

$$Nu_x = \frac{hx}{k} = 0.453\,Re_x^{0.5}\,Pr^{1/3} \quad \text{Laminar (isoflux plate)} \quad (1)$$

$$\boxed{Nu_x = \frac{hx}{k} = 0.0308\,Re_x^{0.8}\,Pr^{1/3}} \quad \text{Turbulent (isoflux plate)} \quad (2)$$

**Example** A flat surface of an electronic equipment dissipates $5\ W/m^2$ heat. The air flow over the 10 mm x 10 mm chip is maintained at 1 m/s. Find the chip surface temp.

$T_\infty = 25°C$

(1) Find $T_f$ to evaluate the properties

Assume a $T_s$ first.

Let $T_s = 50°C$

$T_f = 37.5°C$

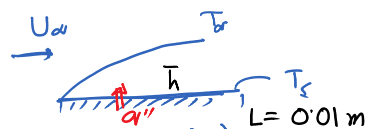
Look up in the property table to find

$\rightarrow \mu, C_p, k, \rho$ (fn. of $T_f$)

Using these properties we find

$Re = \frac{\rho U L}{\mu}$

Laminar or Turbulent. — If Laminar then use eqn (1); If Turbulent




$Nu = f(Re, Pr)$

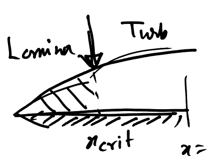
$T_f = \frac{T_s + T_\infty}{2}$ ?

$\mu(T_f)$

$C_p(T_f)$

$k(T_f)$

$\rho(T_f)$




$$\bar{h} = \frac{1}{L}\left[\int_0^{x_{crit}} h_{x,lam}\,dx + \int_{x_{crit}}^{L} h_{x,turb}\,dx\right]$$

$$\dot{q}_{total} = \left.\dot{q}_{tot,turb}\right|_0^L - \left.(\dot{q}_{turb} - \dot{q}_{lam})\right|_0^{x_{crit}}$$

$$\bar{h} = \frac{1}{L}\left[\int^{L} h_{x,turb}\,dx - \int^{x_{crit}}(h_{x,} - h_{x,lam})\,dx\right]$$

$$\bar{h} = \frac{1}{L}\left[\int_0^L h_{x_{turb}}\,dx - \int_0^{x_{crit}} (h_{x_{turb}} - h_{x_{lam}})\,dx\right]$$

$$h_{x\,turb} = \frac{k\,Nu_x}{x} \qquad h_{x\,lam} = \frac{k\,Nu_{x\,lam}}{x}$$

$$q'' = \bar{h}\,(T_s - T_\infty)$$

$$\Rightarrow \boxed{T_s = \frac{q''}{\bar{h}} + T_\infty} \text{ — } \boxed{\text{1st iteration}}$$

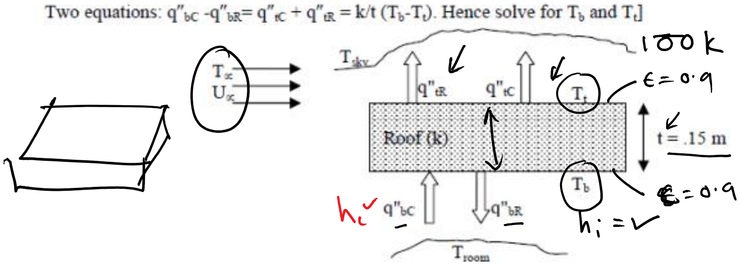
$\boxed{100^\circ C}$

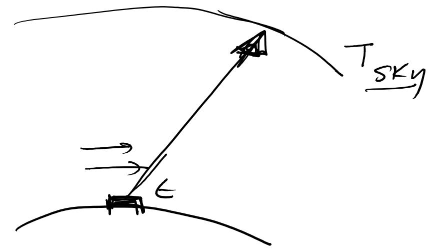
$$T_f = \frac{25+100}{2} \Rightarrow$$

The roof of a house consists of a 15 m × 20 m and 15 cm thick flat concrete slab (k= 2 W/mK). The convection heat transfer on the inner surface of the room is 5 W/m². On a clear winter night, the ambient air is reported to be at 10°C, while the night sky temperature is at 100K. Interior of the house and the internal surfaces of the walls are maintained at a constant temperature of 20°C. The emissivity of both surfaces of the concrete roof is 0.9. Considering both radiation and convection heat transfer, determine the rate of heat transfer through the roof, when the ambient air is blowing at 60 km/h blowing over the roof. along the 15m side [28 kW]

[Hint: $q''_{bR} = \varepsilon(T_{room}^4 - T_b^4)$, $q''_{bC} = 5$, $q''_{tR} = \varepsilon(T_t^4 - T_{sky}^4)$, $q''_{tC} = \bar{h}\,(T_t - T_\infty)$: involves unknown $T_b$ and $T_t$

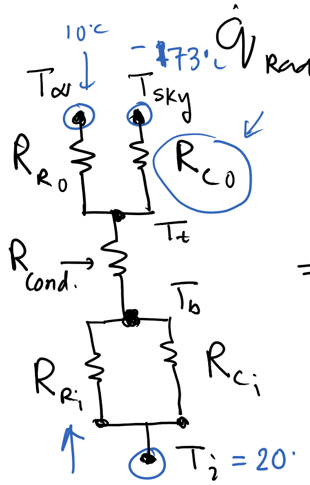
Two equations: $q''_{bC} - q''_{bR} = q''_{tC} + q''_{tR} = k/t\,(T_b - T_t)$. Hence solve for $T_b$ and $T_t$]




$$R_{c,t} = \frac{1}{h_{ct} A} \quad ?$$

$$R_{cond} = \frac{L}{kA} = \frac{0.15}{2\times(15\times 20)} \;\frac{K}{W}$$




$$\dot{q}_{Rad} = A\sigma\epsilon\,(T_s^4 - T_{sky}^4)$$

$$= A\sigma\epsilon\,(T_s^2 + T_{sk}^2)(T_s + T_{sky}) \times (T_s - T_{sky})$$

$h_R \cdot A$

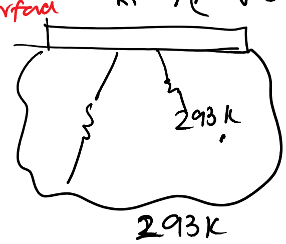
$T_t$

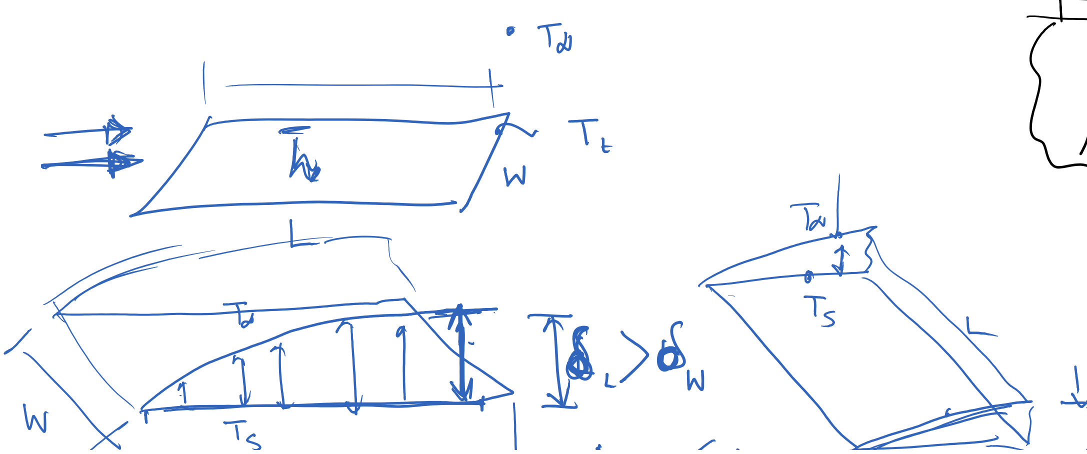
$$\Rightarrow R_{R_0} = \frac{1}{h_{R_0} A}$$

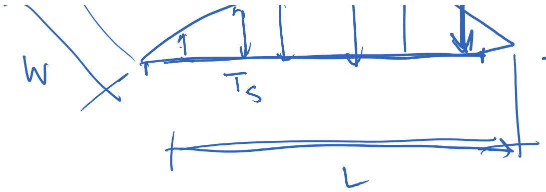
$$R_{R_i} = \frac{1}{h_{R_i} A}$$

$T_b$

lower surface $R_{R_i} = 1/\left(A\sigma\epsilon\,(T_s^4 + T_i^4)(T_s + T_i)\right)$






$$Nu_L < Nu_W$$